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Investigation of numerical and analytical solutions of 1D steady and transient flow in unsaturated layered soils

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ABSTRACT

This study presents a comprehensive numerical simulation and analytical modeling of steady and transient flow in heterogeneously saturated soils. Solving the Richards equation involves challenges due to the nonlinearity between the pressure and the water content. Two constitutive models are used to describe the complex relationships between moisture content, hydraulic head, and hydraulic conductivity, namely the Gardner model and the Van Genuchten-Mualem model. Through detailed simulations, four configurations are investigated, including the capillary barrier effect, bi-layered soil dynamics in paddy fields, and infiltration in multi-layered soil in both saturated and unsaturated steady-state regimes. The comparison between numerical results and the established analytical model shows a good agreement. It underscores the usefulness of the model to reproduce saturated and unsaturated seepage flow in bi-layered and multi-layered soils. Moreover, it captures the dynamic interactions between infiltration and evaporation during the transient phase. These findings offer insights for water resource management, land subsidence mitigation, and environmental sustainability.

KEYWORDS **Numerical. Analytical. Steady. Transient. Unsaturated. Layered soil.**

INTRODUCTION

Infiltration processes within variably saturated soils represent an underexplored field of research with far-reaching implications across diverse scientific disciplines. The behavior of water as it circulates through soils, whether in single-layered or multi-layered configurations, underlies a spectrum of environmental and geotechnical phenomena (Baver *et al.*, 1972; Liakopoulos, 1965). Mastering the complexities of water infiltration in soils is not just a matter of theoretical and academic interest but also holds substantial practical significance across several fields such as groundwater management, agriculture, environmental science, and civil engineering.

Background and problem statement

A pivotal challenge in comprehending variably saturated soils is decoding the nonlinear relationships governing moisture content, hydraulic properties, and hydraulic head. In the literature, recent research endeavors have made noteworthy strides in unraveling these intricate connections (Bastian and Helmig, 1999; Rybak et al., 2015; Srivastava and Yeh, 1991), shedding light on how water circulates through porous media under variable conditions.

One particularly captivating aspect of this research domain revolves around the exploration of capillary barrier phenomena within bi-layered soils. These phenomena arise due to abrupt shifts in soil properties at the interface between two layers. Investigating the dynamics of water flow and pressure profiles within such systems offers valuable insights into soil behavior, with direct applications in agriculture and geotechnics.

Significance and objectives

The advent of analytical and numerical models has revolutionized the capacity to simulate both transient and steady infiltration in single and multi-layered soil systems. These models consider an array of factors, encompassing soil heterogeneity, hydraulic conductivity variations, and types of boundary conditions (Haverkamp *et al.*, 1977). They provide invaluable perspectives on water transport under diverse scenarios.

Moreover, recent studies (Baca *et al.*, 1997; Beliaev and Schotting, 2001; Helmig and Huber, 1998; Khire *et al.*, 1997; Lee and Kim, 2022; Rybak *et al.*, 2015; Shah *et al.*, 2022) highlight the pivotal role of advanced numerical techniques, such as finite element methods and multigrid solvers, to simulate variably saturated flow. These computational tools provide precision in capturing the dynamic behavior of water infiltration. Philip (1957, 1969) established a nonlinear quasi-analytical solution for Richards equation, focusing particularly on vertical flow within a semi-infinite homogeneous unsaturated soil. By using a hybrid Lagrangian-Eulerian approach, Lin *et al.* (1997) developed FEMWATER, a 3D finite element computer model to simulate highly nonlinear flow problems such as infiltration under extreme, initially dry soil conditions. Sabetamal *et al.* (2022) used the general-purpose commercial software package, Abaqus, for coupled hydro-mechanical modeling of unsaturated soil problems. Suk and Yeh (2009) developed a multidimensional finite-element particle tracking method to solve unsteady-state flow problems.

Other researchers have delved into the realm of vertical infiltration, both in steady and transient states, within randomly heterogeneous unsaturated soils. Their investigations consider parameters which are log-normal distributed and spatially correlated random fields, using numerical method and/or analytical techniques such as 3D random perturbation methods. Noteworthy studies in this field include those conducted by Khaleel *et al.* (2002), Lu and Zhang (2004), Lu *et al.* (2007) and Zhang (2002). Previously, Srivastava and Yeh (1991) developed an analytical solution of 1D vertical infiltration in homogeneous and bi-layered soil under unsteady-state condition, where steady state has been used as initial condition. Jacques *et al.* (2018) provide an example of using the HPx computer software to predict the reaction-transport behavior of the soil organic matter, incorporating spatio-temporal variations in both water content and water fluxes.

It is important to note the existence of various contributions with the aim of optimizing water usage, especially in arid and semi-arid regions. One method deserving special attention is the two-layer paddy cultivation method. This technique is often used to conserve water while maintaining a relatively dry top layer of soil to ensure that the rice roots have access to water from the lower layer (Mallareddy *et al.*, 2023).

Unique contributions and novelty

In this context, this study offers several novel contributions:

i) Detailed case studies: Multiple case studies were carried out, namely the capillary barrier phenomenon, the two-layer rice field problem, transient infiltration in single-layer soil, and stationary infiltration in multi-layer soil. These case studies provide a comprehensive understanding of different infiltration scenarios and their practical implications.

ii) Advanced numerical simulations: Utilizing Comsol Multiphysics software, we conducted extensive numerical simulations of water infiltration within variably saturated soils, spanning a wide array of conditions, from steady-state to unsteady flow regimes. This approach highlights the conceptual and numerical challenges inherent in modeling stationary and transient flows within homogeneous and heterogeneous media with variable saturation.

iii) Validation with analytical solutions: The proposed numerical model is validated using existing analytical solutions (Lu and Zhang, 2004; Ross, 1990), which explore the phenomenon of capillary barrier diversion along a horizontal interface. This validation ensures the robustness and accuracy of our model.

The results of this study not only demonstrate the model's capability to simulate and analyze the coupling of flow processes in transient phases but also underline the practical significance of these findings in agriculture, environmental science, and civil engineering. Future research directions may involve extending these simulations to three-dimensional domains, considering more complex soil geometries and heterogeneity, and exploring the coupling of variably saturated flow with other subsurface processes, such as solute transport and heat transfer.

MATHEMATICAL MODELS

Variably saturated flow is governed by the Richards equation. Its nonlinearity arises from the intricate interdependence of hydraulic permeability with water content and, consequently, capillary pressure. Richards equation combines the Buckingham equation, designed for saturated porous media with variable hydraulic permeability, with the continuity equation. It can be expressed in three forms using either pressure head, h(L), or moisture content, θ (Bergamaschi, 1999). For a numerical purpose the formulation based on fluid pressure is employed, such us:

$$C(h).\frac{\partial h}{\partial t} = \nabla\left(k(h).\nabla\left(h - \frac{\vec{g}}{g}\vec{z}\right)\right) \quad (1)$$

where $C(h) = \frac{\partial \theta}{\partial h}$ is the moisture capacity (L^{-1}), $D(\theta) = \frac{k(h)}{C(h)}$ is the unsaturated diffusivity (L^2/T), $k(h)$ is the unsaturated hydraulic permeability (L/T), h is the capillary pressure, and z is the vertical coordinate, T(s) is the time.

Solving the Richards equation proves challenging due to the strong nonlinearity between pressure head, h, and moisture content, θ. To overcome this challenge, various models have been proposed in the literature to describe the water retention curve, θ (h), and the evolution of unsaturated hydraulic conductivity, $k = k(h)$, namely:

Exponential model of Gardner (1958):

$$\begin{cases} S_e(h) = e^{\alpha(h-h_b)} \\ k(h) = k_s.e^{\alpha(h-h_b)} \end{cases} \quad (2)$$

where α is a shape parameter (pore size distribution of the medium) and h_b is the bubbling pressure.

The Modified Mualem–Van Genuchten (MMVG) model (1980):

$$\begin{cases} \theta(h) = S_e(h)(\theta_s - \theta_r) + \theta_r \\ k(h) = k_s.S_e(h)^{1/2}[1 - (1 - S_e(h)^{1/2})^m]^2 \end{cases} \quad (3)$$

where k_s is the saturated hydraulic conductivity (m/s), θ_s and θ_r are the saturation and the residual moisture content (%), respectively, and S_e (%) is the saturation rate expressed as:

$$\begin{cases} S_e(h) = \left[\frac{1}{1 + (\alpha_{VGM}.h)^n}\right]^m \\ m = 1 - \frac{1}{n} \end{cases} \quad (4)$$

where α_{VGM} is the inverse capillary length parameter related to the moisture content, θ(h), and n is a parameter related to the distribution of the pore size in the porous medium.

MATERIAL AND NUMERICAL APPROACH

The Gardner model is used in numerical and analytical studies. In the numerical modeling, the Van Genuchten model and Brooks-Corey model are used depending on the case studied. Each model is suitable for specific types of soil and/or flow conditions. Table 1 summarizes the cases studied under different flow conditions and the various infiltration flow problems. In this study, the finite element analysis software COMSOL Multiphysics was utilized to model and simulate intricate interactions within the investigated system. This involved studying phenomena like water infiltration and evaporation in variably saturated soils, encompassing a diverse range of conditions, from steady-state to unsteady flow regimes. Figure 1 displays the adopted process to solve the Richards equation.

COMSOL Multiphysics leverages the Finite Element Method (FEM) as its foundational numerical technique. This method discretizes the computational domain into finite elements, allowing for the approximation of field variables over each element using basic functions. By formulating variational principles from the weak forms of the Partial Differential Equations (PDEs), FEM facilitates an accurate simulation of physical phenomena, including fluid flow in porous media like that described by the Richards equation.

Capillary barrier phenomenon

The numerical study of the capillary barrier phenomena in bi-layered unsaturated soil is carried out to illustrate the effect of the soil discontinuity at the interface. This effect occurs when there is a sudden transition in the soil properties at the interface of two layers. One must first examine the transient phase numerically to determine the corresponding reached steady state. Subsequently, this state can be further analyzed both numerically and analytically.

This domain sketch involves a vertical column with a height of 6m (Fig. 2), consisting of two different layers. The lower layer consists of coarse soil (sand), and the upper layer consists of fine soil (silt), separated by a horizontal interface at the midpoint of the domain. The upper layer is subjected to an infiltration flow rate, q. In order to balance computational efficiency with calculation precision, a mesh size of 37.5cm was selected and the domain was discretized into 17 nodes. The initial time step is 0.0275h, and each successive time step size is doubled, without exceeding 2.4h.

Transient phase

The numerical solution for 1D unsteady-state infiltration in bi-layered soil using Gardner's exponential function is

TABLE 1. Summary of the various infiltration flow problems and cases studied under different flow conditions

N°	Study case	Layer type	Saturation type	Flow nature	Initial condition	Boundary conditions: Top	Boundary conditions: Bottom
01	Capillary barrier phenomenon	Bi-layer	Variably Saturated	Transient	Hydrostatic pressure	Infiltration	Free surface of the aquifer
				Steady	X		
02	Bi-layered paddy field problem	Bi-layer	Saturated	Steady	X	Fixed water sheet height	
			Variably Saturated				
03	1D Transient Infiltration	Single-layer	Variably Saturated	Transient	Hydrostatic pressure	Infiltration then evaporation	
04	1D Steady State Infiltration	Multi-layer	Variably Saturated	Steady	X	Infiltration	

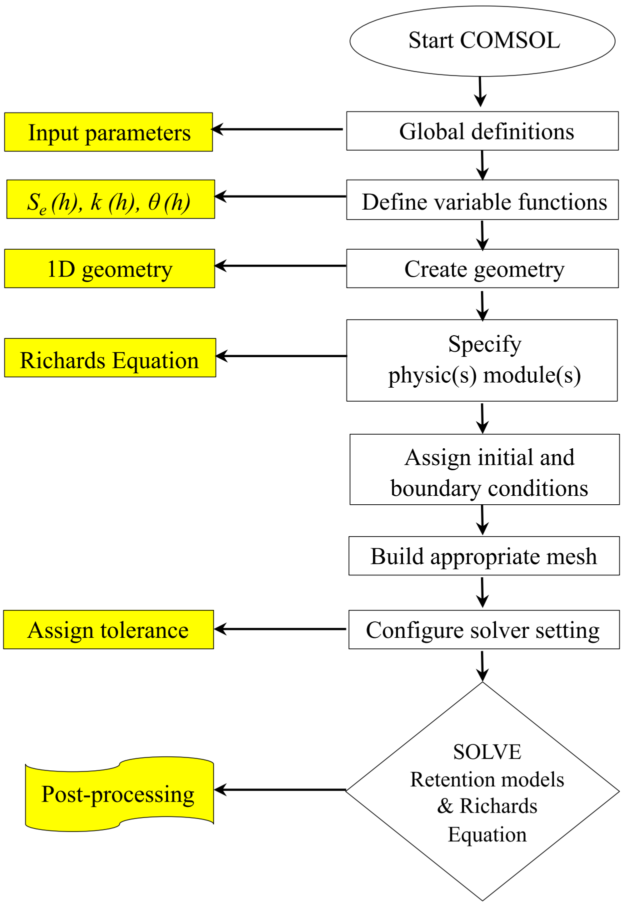

FIGURE 1. Flowchart of the numerical solution steps for the Richards equation.

examined. The transient phase begins with a hydrostatic state as the initial flow condition. During the transient phase, an infiltration flow of -7x10^{-7}m/s (negative sign indicating infiltration) is applied to the upper boundary, while zero pressure is applied to the lower boundary of the column as:

$$h(0,z) = -z \quad (5)$$

$$q(t,L) = -7x10^{-7} \quad (6)$$

$$h(t,0) = 0 \quad (7)$$

Steady-state phase

Using Gardner's exponential law and relying on the principle of pressure continuity at the interface between the two soils (*z*= 3m), the water pressure head in coarse-grained soil, h^c, is equal to the water pressure head in the fine-grained soil, h^f. This relationship is expressed as:

$$K_r^c(z=0) = K_r^{f\,\frac{\alpha_c}{\alpha_f}}(z=0) \quad (8)$$

where (K_r^c, α_c) and (K_r^f, α_f) are the relative hydraulic conductivity and the shape parameter of the coarse and fine soils, respectively. Table 2 summarizes the hydro-mechanical parameters of the bi-layered unsaturated soil.

Bi-layered paddy field problem

In this steady-state case, a bi-layered soil is subjected to infiltration flow under a fixed water sheet height, h_0. This bi-layered soil comprises a vertical column with a height of 0.6m, consisting of two distinct layers (Fig. 3). The lower layer is composed of coarse soil, while the upper layer consists of fine soil. These two layers are separated by a horizontal interface located at the midpoint of the column. Furthermore, the bi-layered soil is in contact with a water table at the lower boundary of the coarse soil layer.

Two cases can occur: saturated flow and variably saturated flow. In the former case, the study focuses on 1D bilayer infiltration in steady state under fixed surface water layer, h_0, at the top of soil and in the presence of a water table at the bottom of the soil. The surface water layer h_0 represents a flood or submersion condition (*e.g.* bed irrigation). Here, it is assumed that pressure, h(z), remains everywhere greater than the air inlet pressure, h_b, and therefore the hydraulic conductivity, K(h), is equal to the hydraulic conductivity at saturation for the two media. An analytical solution is then easily obtained by applying Darcy's law to both media, with a continuity condition at the interface. This results in the following pressure expressions, $h^c(z)$ and h(z), for each medium as well as the infiltration rate, q (m/h):

Coarse grained soil: $0 \leq z \leq I$

$$h^c(z) = -\left(\frac{q}{K_s^c}+1\right)z \tag{9}$$

Fine grained soil: $I \leq z \leq L$

$$h^f(z) = -\left(\frac{q}{K_s^f}+1\right)(z-I)+h_0 \tag{10}$$

where L is the total length of the bi-layered soil, and I is the elevation of the interface. By applying the pressure continuity condition at the interface (z= I):

$$h^f(z=I) = h^c(z=I) \tag{11}$$

the following expression is derived:

$$q = \frac{-(L+h_0)K_s^c.K_s^f}{I.K_s^f+(L-I)K_s^c} \tag{12}$$

yielding q= $-8.51\text{x}10^{-04}$m/h. This value was confirmed through numerical analysis employing the Brooks and Corey model (1964). Table 3 summarizes the hydro-mechanical parameters of the bi-layered paddy field under saturated flow.

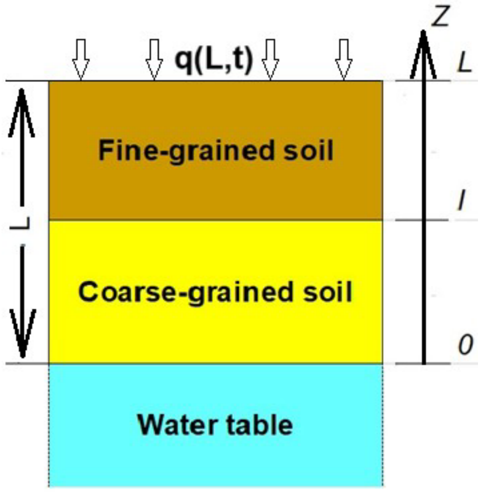

FIGURE 2. Schematic representation of the capillary barrier problem, (q(L, t)= 7.10^{-7}m/s, L= 6m, l= 3m).

In the case of variably saturated flow, it is assumed that the fine medium is fully saturated while the coarse medium remains unsaturated ($h_c < h_b^c$). Therefore, the analytical solution is different for both media. Darcy's law is applied for the fine medium, whereas the Kirchhoff transform is employed to linearize the Gardner's function for the coarse medium. Thus, the pressure expressions for each medium are as follows:

Fine grained soil: $I \leq z \leq L$

$$h^f(z) = -\left(\frac{q}{K_s^f}+1\right)(z-I)+h_0 \tag{13}$$

Coarse grained soil: $I \leq z \leq L$

$$h^c(z) = \frac{1}{\alpha_c} ln\left[\left(\frac{q}{K_s^c}+e^{-\alpha_c.h_b^c}\right)e^{-\alpha_c.z}-\frac{q}{K_s^c}\right]+h_b^c \tag{14}$$

In the case of a variably saturated soil, $q_0 = 2.5\text{x}10^{-3}$m/h, which value was chosen based on values observed in field studies (Giudici, 2023; Green and Ampt, 1911; Vogel *et al.*, 2001). Note that this rate was likely selected to represent a moderate to high infiltration scenario, which is relevant for studying the dynamics of flow in unsaturated soils. Table 4 summarizes the hydro-mechanical parameters of the bi-layered paddy field under saturated flow.

TABLE 2. Hydro-mechanical parameters of the bi-layered unsaturated soil

Soil and flow parameters		Fine layer	Coarse layer
flux infiltration at soil top	(m/s)	7 x 10^{-7}	7 x 10^{-7}
saturated hydraulic conductivity	K_s (m/h)	3.6 x 10^{-6}	1.5 x 10^{-4}
exponential model parameter	α	0.45	4.5
bubbling pressure	h_b	0	0

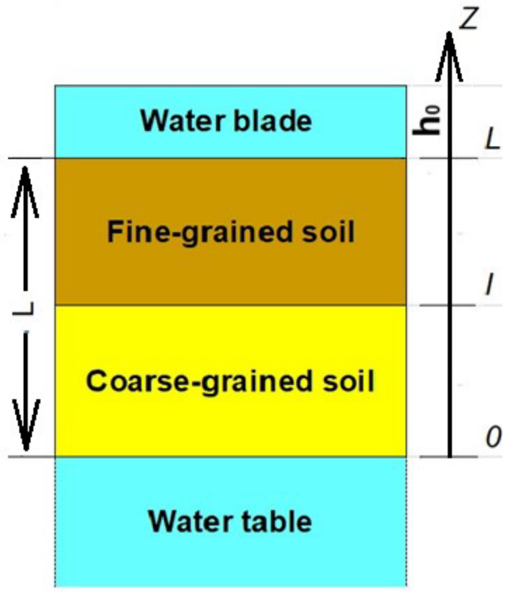

FIGURE 3. Schematic representation of the bi-layer paddy field problem (L= 0.6m, l= 0.3m, h_0= 0.2m).

1D transient infiltration followed by evaporation in single-layered soil column

In this case, 1D unsteady-state infiltration in single-layered soil is numerically treated. A column of 3m is considered (Fig. 4), containing soil with a saturated hydraulic permeability of 10^{-6}m/s, a saturation water content of 0.55, and a residual water content of 0.1. The soil is subject to the infiltration flow of 0.06m/day from its upper boundary during the first week, followed by an evaporation process of 0.005m/day (climate ranging from humid to semiarid) in the subsequent week. The soil is limited below by a water table. The transient phase begins with a hydrostatic state where the initial and boundary flow conditions are:

$$h(t,0) = 0 \tag{15}$$

$$h(0,z) = -z \tag{16}$$

$$q(t,L) = \begin{cases} q_f if 0 < t \leq 7days \\ q_e if 7 < t \leq 14days \end{cases} \tag{17}$$

Using the Van Genuchten model, the study domain is discretized into 100 elements. The initial time step is set at 0.0037day and increases until it reaches 1d (maximum value). Table 5 summarizes the hydro-dynamical parameters of infiltration-evaporation process in single-layered soil column.

1D steady-state infiltration in multi-layered soil column

In this case, 1D steady-state infiltration in a multi-layered soil column was examined using the Van Genuchten (1980) model. The columns comprise three layers of 2m each (Fig. 5). Both top and bottom layers consist of loamy fine sand, while the middle layer contains of clay loam. Table 6 summarizes the hydrodynamic properties of the soil.

In the presence of a water table at the bottom of the soil, this last is subject to the infiltration flow from its upper boundary as:

$$h(z=0) = 0m \tag{18}$$

$$q(z=L) = 1.610^{-6} m/s \tag{19}$$

The analytical study presented in this case is based on the linearization of Gardner's function using the Kirchhoff transformation as described by Lu and Zhang (2004). Depending on the layer contents, the expressions for water pressure are given as follows:

Bottom coarse-grained soil layer:

$$h^c(z) = \frac{1}{\alpha_c} ln\left[\left(\frac{q}{K_s^c} + e^{-\alpha_c . h_b^c}\right) e^{-\alpha_c . z} - \frac{q}{K_s^c}\right] + h_b^c \tag{20}$$

Fine-grained soil layer:

$$h^f(z) = \frac{1}{\alpha_f} ln\left[\left(\frac{q}{K_s^f} + e^{-\alpha_f . h_b^f}\right) e^{-\alpha_f (z-z_1)} - \frac{q}{K_s^c}\right] + h_b^f \tag{21}$$

Top coarse-grained soil layer:

$$h^c(z) = \frac{1}{\alpha_c} ln\left[\left(\frac{q}{K_s^c} + e^{-\alpha_c . h_b^c}\right) e^{-\alpha_c (z-z_1)} - \frac{q}{K_s^c}\right] + h_b^c \tag{22}$$

where z_1 is the upper level of the lower soil layer (z_1= 2m); z_2 is the upper level of the fine-grained soil layer (z_2= 4m), h_b^f is the bubbling pressure in fine layer (h_b^f=0), h_b^c is the

TABLE 3. Hydro-mechanical parameters of the bi-layered paddy field under saturated flow

Soil and flow parameters		Fine grained	Coarse grained
Thickness of layer	(m)	0.3	0.3
Pressure water at soil top	h_0 (m)	0.2	0.2
Saturated hydraulic conductivity	K_s (m/h)	3.2 x 10^{-4}	1.4 x 10^{-1}
Exponential model parameter	α	0.11	1.1
Bubbling pressure	h_b (m)	-0.64	-0.23

TABLE 4. Hydro-mechanical parameters of the bi-layered paddy field under variably saturated flow

Soil and flow parameters		Fine grained	Coarse grained
Thickness of layer	(m)	0.2	0.8
Pressure water at soil top	h_0 (m)	0.2	0.2
Saturated hydraulic conductivity	K_s (m/h)	3.2 x 10^{-4}	1.40 x 10^{-1}
Exponential model parameter	α	0.14	10.7
Bubbling pressure	h_b (m)	-0.64	-0.33

TABLE 5. Hydro-dynamical parameters of infiltration-evaporation process in single-layered soil column

Thickness of soil	(m)	3
Saturated hydraulic conductivity	K_s (m/s)	10^{-6}
Flux infiltration at the top soil	q_f (m/d)	0.06
Flux evaporation at the top soil	q_e (m/d)	0.005
Van Genuchten air-entry suction parameter	A	0.69
Van Genuchten pore size distribution	N	2.65

TABLE 6. Hydraulic parameters for multi-layered soil submitted to infiltration

	Symbol	Loamy fine sand	Clay loam
Residual liquid volume fraction	θ_r^c	0.0286	0.1060
Porosity	θ_s^c	0.3658	0.4686
Hydraulic conductivity	K_s^c	6.26 x 10^{-5}	1.57 x 10^{-6}
Van Genuchten air-entry suction parameter	α_c	2.12	1.04
Van Genuchten pore size distribution	n_c	2.4	1.3

bubbling pressure in fine layer (h_b^c=0), α_c is the exponential model parameter for the coarse-grained soil (α_c= 2.12) and α_f is the exponential model parameter for the coarse-grained soil (α_f= 1.04).

RESULTS AND DISCUSSION

Capillary barrier phenomenon

The results of analytical and numerical simulations are presented and discussed to investigate various aspects of steady and unsteady flow in unsaturated soils. The evolution of the water pressure is calculated at 0day, 2days, 6days, 11days and 12days. Using Gardner's model, the numerical results reveal that the steady state is reached after 11days (Fig. 6A).

The analytical resolution differs for both media as:

Coarse soil: By linearizing Gardner's function using the Kirchhoff transformation (Lu and Zhang, 2004), the following expression is obtained for $h(z)$:

$$h(z) = \frac{1}{\alpha_c} ln\left[\frac{-q}{K_s^c} + \left(\frac{q}{K_s^c} + e^{-\alpha_c.h_b^c}\right) e^{-\alpha_c.z}\right] + h_b^c \quad (23)$$

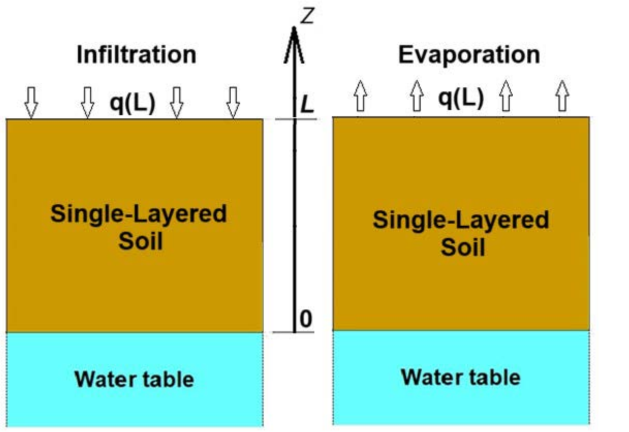

FIGURE 4. Schematic representation of single-layered soil subjected to infiltration followed by evaporation, (q(*L*,t)= 0.06m/day, *L*= 3m).

where h_b^c is the bubbling pressure in the coarse-grained layer. In our case, $h_b^c = 0$.

Fine soil: Based on the analytical solution given by Ross (1990):

$$h(z) = \frac{1}{\alpha_f}\ln\left[\left(\left(\frac{q}{K_s^c}\right)^{\frac{\alpha_f}{\alpha_c}} - \frac{q}{K_s^f}\right)e^{\alpha_f.(z_1-z)} + \frac{q}{K_s^f}\right] + h_b^f \quad (24)$$

where z_1 denotes the interface coordinate and h_b^f is the bubbling pressure in the fine layer. In our case, $h_b^f = 0$.

Figure 6B shows the comparison between the numerical and the analytical model of pressure head, which is a combination of a pressure term in coarse soil, in which the pressure is established by linearization of Gardner's function, and a pressure term in fine soil, in which the pressure term is inferred from the analytical solution of Ross (1990). In addition, Figure 6B shows that the bi-layered soil is fully unsaturated ($h<h_b= 0$) and that the superposition of the analytical and numerical curves indicates that a capillary barrier phenomenon is perfectly reproduced. The analysis shows the formation of a capillary barrier at the interface, significantly affecting water infiltration into the deeper layer. This effect is crucial in understanding water distribution and management in layered soils.

Figure 7 illustrates the impact of the capillary barrier during the infiltration process. Capillary forces exert a significant influence on this phenomenon. The fine upper layer tends to amass water, thereby hindering its infiltration into the coarse lower layer. In this dynamic process, water ingress into the coarse soil layer transpires when the matric pressure at the base of the fine soil layer increases to a specific point, P_f, which is reached during the transient phase after 11 days and corresponds to the moisture content, Θ_f, near the saturation level of the fine soil layer. This point is associated with a pressure of -1.2m (as depicted in Fig. 7B).

Given the principle of continuous water pressure within porous materials, equilibrium necessitates that the water pressure at the top of the coarse soil layer also attains -1.2m (which occurs after 11days; Fig. 7A) before any water can infiltrate the coarse soil. The equivalent point for the coarse soil layer is denoted as P_g, with an associated water content of Θ_g, approximating the residual moisture content of the coarse soil layer. It is worth noting that even once P_g is achieved, the ingress of water into the coarse soil layer transpires at a notably sluggish rate. This reduced rate is attributed to the relatively lower hydraulic conductivity of the fine soil layer in comparison to the coarse soil layer, which diminishes the velocity of water penetration.

Bi-layered paddy field problem

Figure 8A shows the comparison between the numerical and the analytical model of the pressure profile, h(z). The excellent correlation between both curves proves that the problem is perfectly simulated. The pressure value at the interface h= -0.3m confirms that the two-layer system is totally saturated because this pressure is greater than the air inlet pressures in both layers. Figure 8B shows that the pressure profiles obtained numerically and analytically

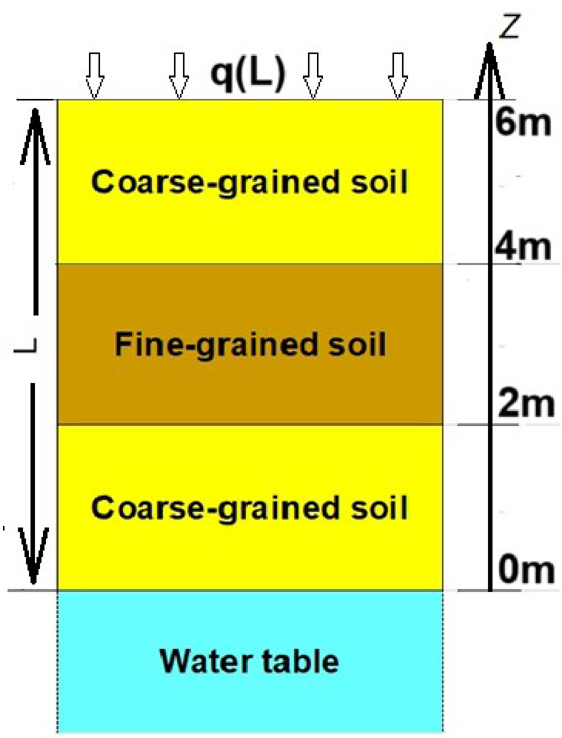

FIGURE 5. Schematic representation of multi-layered soil column subjected to infiltration, (q(z= *L*)= 1.6 10^{-6}m/s).

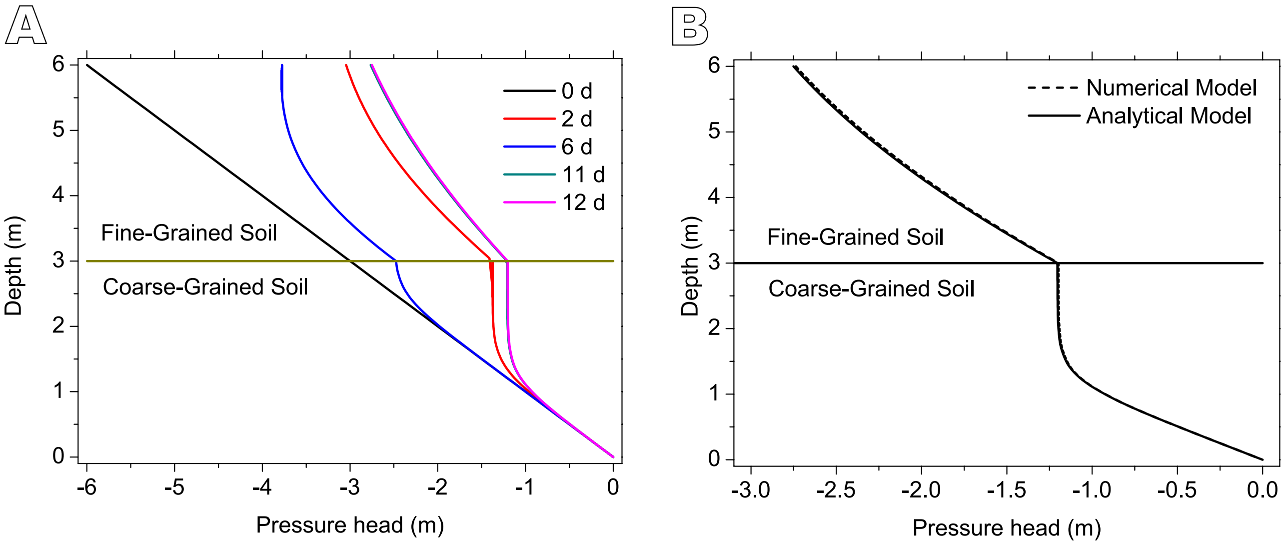

FIGURE 6. A) Temporal evolution of pressure for the capillary barrier phenomenon in 1D column and B) comparison with the steady analytical solution at *t*= 12days.

are superposed. It is found that coarse soil is desaturated at 0.3m < z < 0.8m, because h(z) < h_b^c and fine soil is fully saturated because h(z) > h_b^f. This confirms that the desaturation assumption for the coarse medium and the saturation of the fine medium are accurately represented.

1D Transient infiltration followed by evaporation in single-layered soil column

The simulation of this case includes output times for infiltration rate of 0.06m/day at 0days, 0.25days, 1day, 3days and 7days. Subsequently, the simulation provides output times for evaporation rate of 0.03m/day at 9days, 11days, 13days and 14days. Figure 9 shows the succession of water pressure profiles, starting from an initial hydrostatic state between the water table and the soil surface, across the succession of infiltration (under a zero-water layer) and the evaporation. After a rainfall week, the evaporation phase occurs over the next week. The water pressure at the bottom is maintained at zero (pressure of the free surface of the aquifer). On the other hand, the water pressure at the top of the soil increases during the first week due to the infiltration. At the onset of the second week, the evaporation phase commences, leading to a gradual decrease in water pressure at the soil's uppermost layer.

1D Steady-state infiltration in multi-layered soil column

Figure 10 illustrates the accumulation of seepage water at the interface between the fine soil layer and the bottom soil layer, attributed to the lower hydraulic permeability of the fine soil layer when compared to that of the coarse soil layer. In general, there is a reasonably good correlation between the numerical model and the analytical model in this case. The slight discrepancy between the curves resulting from the application of the numerical Van Genuchten model and the analytical Gardner model can be attributed to several factors, with the nature of the soil playing a crucial role. One potential factor contributing to the difference between the curves generated by the Van Genuchten and Gardner models lies in the inherent realism of these two approaches. The Van Genuchten model is more realistic and complex compared to the Gardner model. It incorporates more detailed parameters to describe water retention and hydraulic conductivity, allowing for a better representation of soil reality. The Van Genuchten model is based on parameters such as matric potential and saturation,

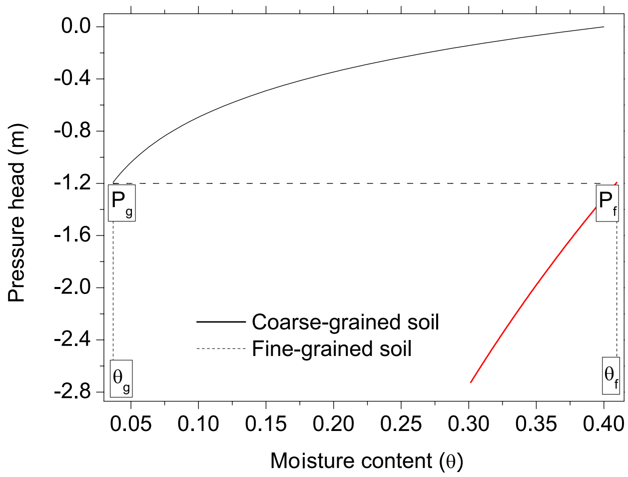

FIGURE 7. Water retention curve at *t*= 12days.

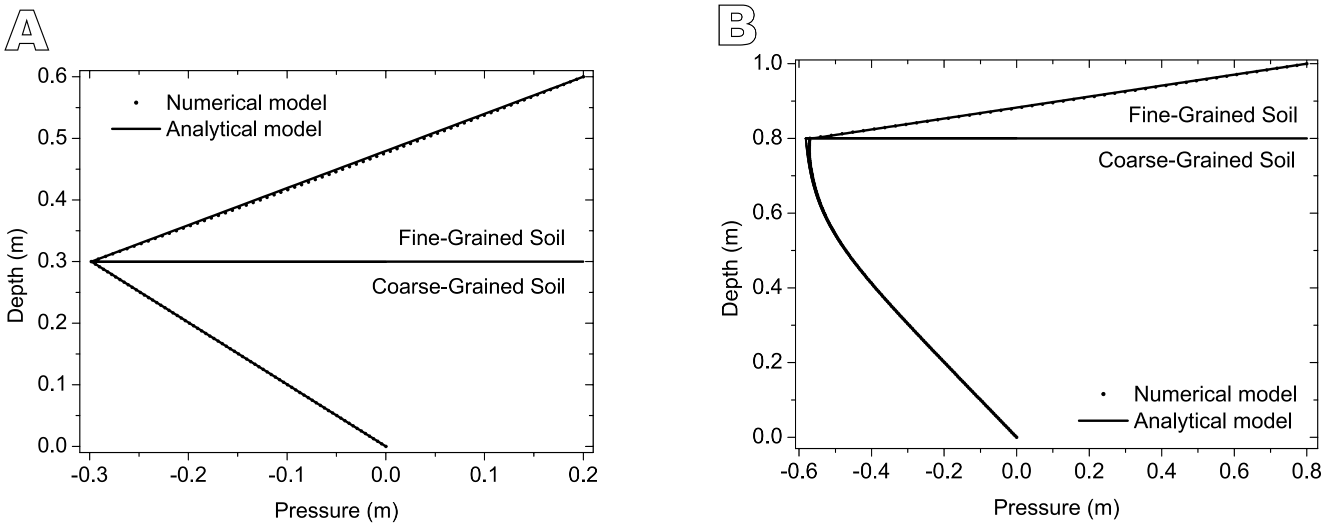

FIGURE 8. Pressure profile in bi-layered paddy A) saturated, B) variably saturated.

which are relatively easy to measure and, as such, provides a more realistic reflection of soil properties, particularly in varying saturation conditions. In contrast, the Gardner model, being simpler and based on an exponential equation that relies on the permeability relative parameter, which can be challenging to measure accurately, especially in variably saturated environments. This limitation can lead to inaccuracies in representing the nuances of soil behavior.

The nature of the soil is also a key factor. Each model was developed taking into account specific soil types. The Van Genuchten model is often used for more complex soils, such as multi-layered soils, where its complexity may better correspond to reality. In contrast, the Gardner model may be less accurate for these types of soils.

CONCLUSIONS

The numerical model presented in this study is adequate to model seepage flow through heterogeneously saturated soils, with a particular emphasis on the transient infiltration. By leveraging the Richards equation and advanced numerical techniques, one can investigate phenomena such as capillary barrier, paddy field problem in bi-layered soils and infiltration in multi-layered soils and the impact of various factors on infiltration dynamics. To assess the accuracy and validity of our model, numerical results in both saturated and unsaturated steady-state regimes were compared to established analytical solutions based on the models by Lu and Zhang (2004) and Ross (1990). The comparison shows a good agreement, and the key findings from this study are summarized as follows:

i) Capillary barrier phenomenon: The model successfully reproduced the capillary barrier effect at the interface between fine and coarse soil layers. This effect is critical in understanding water distribution in layered soils, which has significant implications for designing efficient irrigation systems and preventing waterlogging in agricultural fields.

ii) Bi-layered paddy field dynamics: The numerical simulations accurately captured the pressure profiles and saturation states in bi-layered paddy fields. This provides valuable insights for optimizing water management practices in rice cultivation, ensuring that soil moisture

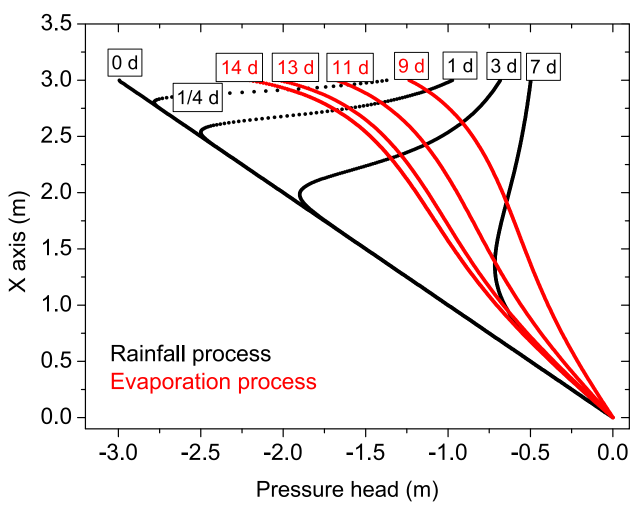

FIGURE 9. Pressure profiles during rainfall and evaporation processes in single-layered soil.

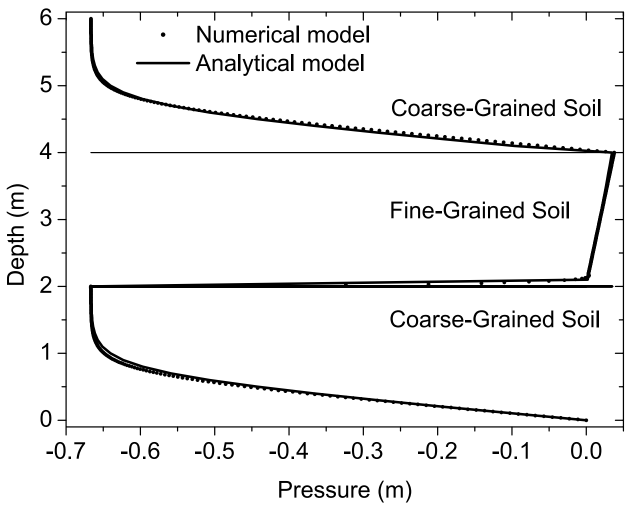

FIGURE 10. Pressure profiles in multi-layered soil.

levels are maintained to support crop growth while minimizing water usage.

iii) Transient infiltration and evaporation: The dynamic interactions between infiltration and evaporation were well-represented in single-layered soil columns. This understanding is vital for predicting soil moisture fluctuations and designing sustainable irrigation schedules that consider water input and water loss due to evaporation.

iv) Steady-state infiltration in multi-layered soils: The comparison between numerical results using the Van Genuchten model and analytical solutions based on the Gardner model demonstrated good agreement. This highlights the robustness of our model in simulating complex soil-water interactions in multi-layered systems, which is essential for effective drainage design in civil engineering projects.

These results also highlight the capability of the model to simulate the coupling of flow processes in transient phase, particularly the dynamic interplay between infiltration and evaporation. It is also important to note the effect of air inlet pressure on flow behavior as it can have significant implications for different flow processes.

The hydrology field of unsaturated soil is of paramount importance in diverse applications, ranging from agriculture and environmental science to civil engineering and geotechnics. Accurate numerical simulations play a crucial role in enhancing our understanding of these complex processes and can inform decision-making in practical scenarios.

Future research should involve extending these simulations to three-dimensional domains, considering more complex soil geometries and heterogeneity. It should also explore the coupling of variably saturated flow with other subsurface processes (*e.g.* solute transport and heat transfer). Advancing our knowledge of time-dependent flow in unsaturated soils can result in improved solutions for addressing real-world challenges related to water resource management, land subsidence, and environmental sustainability.

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