Building a community of practice to promote inquiry about geometry: A study case of pre-service teachers interacting online1

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Summary

This study is part of a longitudinal research project that aims to analyze the implications of online interaction in the training of mathematics teachers. A semantic analysis of written texts produced in a specific environment was performed. It highlights that an interactive dynamic, carried out in the hypertextual journey through the environment spaces and the discussion with the group, elements of community of practice were important to describe a way to understand the learning. Since inquiry is a powerful issue that can grow naturally from online interaction (synchronous or asynchronous) and contribute to developing critical pre-service teachers knowledge, research around learning in this particular setting should pay attention to inquiry.

Keywords

Community of Learning, Pre-service teachers, Online interaction, Geometry

Introduction

With the development of Information and Communication Technologies (ICT), the interest in online professional training by Internet tools has increased. The massive presence of ICT in our daily and professional lives has contributed in different ways, to the constitution of new paths of interaction and learning. Some studies indicate that widening the presence of real life problems and ICT resources can bring about meaningful changes in cognition. In Brazil, research on the potential of ICT in graduate programs is still scarce. Geometry has hardly been present in grounding curricula, in spite of its importance in student's personal growth, whether in basic or higher education. There is an undeniable need for research analyzing interactive processes in online training in geometry in undergraduate courses.

From different parts of the world, work on communities of learning has awoken an interest in researchers interested in professional development. Some authors have shown that one way of

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professional development takes place through communities of practice (Graven, 2003; Wenger, 1998). Nevertheless, the study of learning in virtual communities as contributing to a social construction of knowledge is still scarce in Brazilian higher education (Borba & Villareal, 2005) and researchers in this field are looking to build theoretical models that can help them understand interaction that takes place in these particular settings. The present study focused on aspects of the learning of pre-service mathematics teachers drawing from their interaction carried out in a virtual environment. Elements of community of practice were identified in online interactions around geometry problems. Some issues concerned about pre-service teachers learning with ICT are also addressed.

Through this study, we are viewing the interactive process of pre-service teachers from the perspective of a community of practice (Wenger, 1998). Our study highlights the need for the analysis of learning in virtual communities of practice, in an Internet mediated communicative process, to be grounded on the analysis of the various interaction processes carried out through training activities. Besides, it provides examples of elements in the learning of geometry, which were developed in the community of pre-service teachers (Bairral & Zanette, 2005).

Our analytical process confirmed that elements of community of practice did help us carry out the first stage of a descriptive analysis (Bairral, 2005b). Nevertheless, these elements of community practice are not enough to deeply analyze the influence of interaction on the pre-service teachers knowledge. We think that looking for different influences in the interaction (for instance, the typology of tasks, the time of reflection, the role of trainer, attitudes and intentionality of the participants, the discursive aspects of the different communicative acts) that take place within an inquiry environment is more challenging and powerful. Since we have seen that in online interaction professional content knowledge is hypertextual and collaboratively constructed, we cannot analyze those elements separately.

Making communities for online mathematics learning

We have adopted a sociocultural perspective, which considers learning as a situated activity born out of the meanings constructed through participation in specific communities of learning. In such scenarios, apprentices build knowledge in a wide net of meaning, which emerges from personal and contextual interaction (Cole, 1996; Wenger, 1998). Wenger provides a theory of learning in which the primary unit of analysis is neither the individual nor social institutions but communities of practice. The account systematically explores the intersection of four learning components (meaning, community, identity and practice), which provide a conceptual framework for analyzing learning as social participation. According to Wenger, meaning is a way of talking about our ability to experience the world as meaningful. Community is a way of talking about the social configurations in which our enterprise is defined and our participation is recognizable as competence. Identity is a way of talking about how learning changes who we are. Practice is a way of talking about shared historical and social resources, frameworks and perspectives that sustain mutual engagement in action. In a given practice,

as the participant wishes to belong, Wenger considers identity a key element. These four components are deeply interconnected and mutually defining. We can feature a given community of practice if we set forth the relationship of the participants with one another, as well as among them, their activity and the world they inhabit (Lave & Wenger, 1991). The concept of community of practice does not imply the physical presence of the participants, nor does it require a precise definition of social borders (p. 98). Wenger also points up that participation in a community of practice implies a voluntary attitude. In mathematics education, some studies have developed an analysis of social practice in the perspective of communities of practice. Matos (1999) understands that the analysis of mathematics learning at school as a situated phenomenon requires adopting the perspective that learning and knowing are part of social practice. It is on this basis that he discusses concepts related to the notion of community of practice, using them to analyze the social practice of students in a mathematics class. Matos identifies aspects related to the objectives, to the shared meanings in interaction and to the established practice. He stresses that research in mathematics education should also face its practice in the context of its social and political responsibility. Graven (2003) argues that, in the initial pre-service training, Wenger's framework of learning does not deal comprehensively with all primary aspects of learning (in all contexts). Graven considers confidence as an overarching fifth component requiring discussion and analysis in its own right. According to Graven, confidence, like meaning, practice, identity and community, is closely intertwined with all other learning components. Graven adds"[confidence] has its specific features that could not be subsumed within the other components" (p. 36). Thus, confidence in relation to learning as mastery involves the insight to know when you do not know, the confidence to admit to this and to look for ability and support from the community. Jaworski (2004), considering the power of inquiry in the process of learning and the importance of dialogue in coming to know, believes that a shift from community of practice to community of inquiry provides a perspective in which a reflective development of teaching by individual teachers results in a developing community (p. 25). According to her, inquiry, in a community of inquiry is more than the practice of a community of practice. Teachers develop inquiry approaches to their practice and together use inquiry approaches to develop their practice. For Jaworski, there are three mutually embedded forms of inquiry regarding mathematics: inquiry in mathematics (pupils in schools learning mathematics through exploration in tasks and problems in classrooms), inquiry in teaching mathematics (teachers using inquiry to explore the design and implementation of tasks, problems and activity in classrooms), and inquiry in research which results in developing the teaching of mathematics (teachers and didacticians researching the processes of using inquiry in mathematics and the teaching of mathematics). In our environment and virtual community we use the second form, that is, inquiry in teaching mathematics. In developing our formative activity, we are constantly challenging the capacities of the future educator to reflect geometrically (explore, classify, construct, represent, prove, model) on his or her own knowledge and the implications of the virtual community over this constructive process. This formative activity has been seen as a process of progressive inquiry (Hakkarainen, 2003; Muukkonen, Lakkala, & Hakkarainen, 2001). Its attempts to engage the pre-service teachers in a self-directed inquiry process by asking the virtual community to 1) state their

initial and often fuzzy questions, 2) produce their own working theories, 3) collaboratively evaluate and redirect their inquiry, 4) search for deepening knowledge, 5) generate subordinate research questions, and to 6) produce in-depth explanations for the whole learning community.

We believe that critical thinking (Bairral & Giménez, 2003) and the increasing use of argumentation through online interactions to develop metacognitive ability (Bairral, 2005c) are closely aligned with the aims of progressive inquiry. Those concepts encourage pre-service mathematics teachers to approach a problem strategically and to actively seek out sources of knowledge, discover biases, sift through increasingly large quantities of relevant or non-relevant information, and formulate and defend their own intellectual positions (Garrison, Anderson, & Archer, 2000). The authors emphasize that it is essential that this process occur in an interactive and social environment (p. 96). Different professional actions are in continuous and critical movement (Bairral, 2003). In the case of pre-service teachers, we have: Access to the environment, carrying out tasks, initial positioning with some type of justification, constructive exchange and integration of perspectives. In the same way, in the case of the researcher (or trainer) there are different actions involved in the process: Facilitation, commitment and integration in the process, differential personal attention, socialization and attention to a collaboration committed with the community.

Research design

Our research takes place at the Group2 of ICT Studies and Research in Mathematics Education (GEPETICEM). When it first came into being, in 2003, the environment (www.gepeticem.ufrrj.br) was proposed as a 22-hour university extension course. In 2004-2005, the setting was reconsidered and included among the activities of the 60-hour course subject "Topics in Spatial Geometry". As an extension course (pilot study) we had 5 students, and when we included it as a subject course in the curriculum, 71 students enrolled.

Our distance education programs take place in a specific virtual learning environment. This setting is being conceptualized as a complex interactive system that, through experience from professional practices promote interaction and knowledge construction (Bairral, 2005a). In summary, the environment gathers:

- The community and its intentions.

- The different tasks that individuals want to solve.

- The discursive aspects developed in each communicative space (e-mail, online forum, chat).

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[2] Research group registered at CNPq (National Counsel of Technological and Scientific Development), Brazil.

- The norms of participations and collaboration established.

- The meaningful (physical, symbolic, computational, cognitive) artifacts.

In structuring the environment, strategic elements were considered (Bairral & Giménez, 2004): affective-attitudinal (photographs, work contract, self-assessment, course plan), communicative (e-mails, discussion forum, chat), motivational (Applets, animation, questionnaire, hipertextual activities, photographs and personal data of colleagues) and informative (links of interest to web pages of varied interests, official documents). Besides off-line encounters, both a theoretical discussion and practical deepening of tasks were also carried about by virtual e-mail interaction, discussion list and final chat. See below for the main page of the scenario and examples of the some interrelated strategic elements.

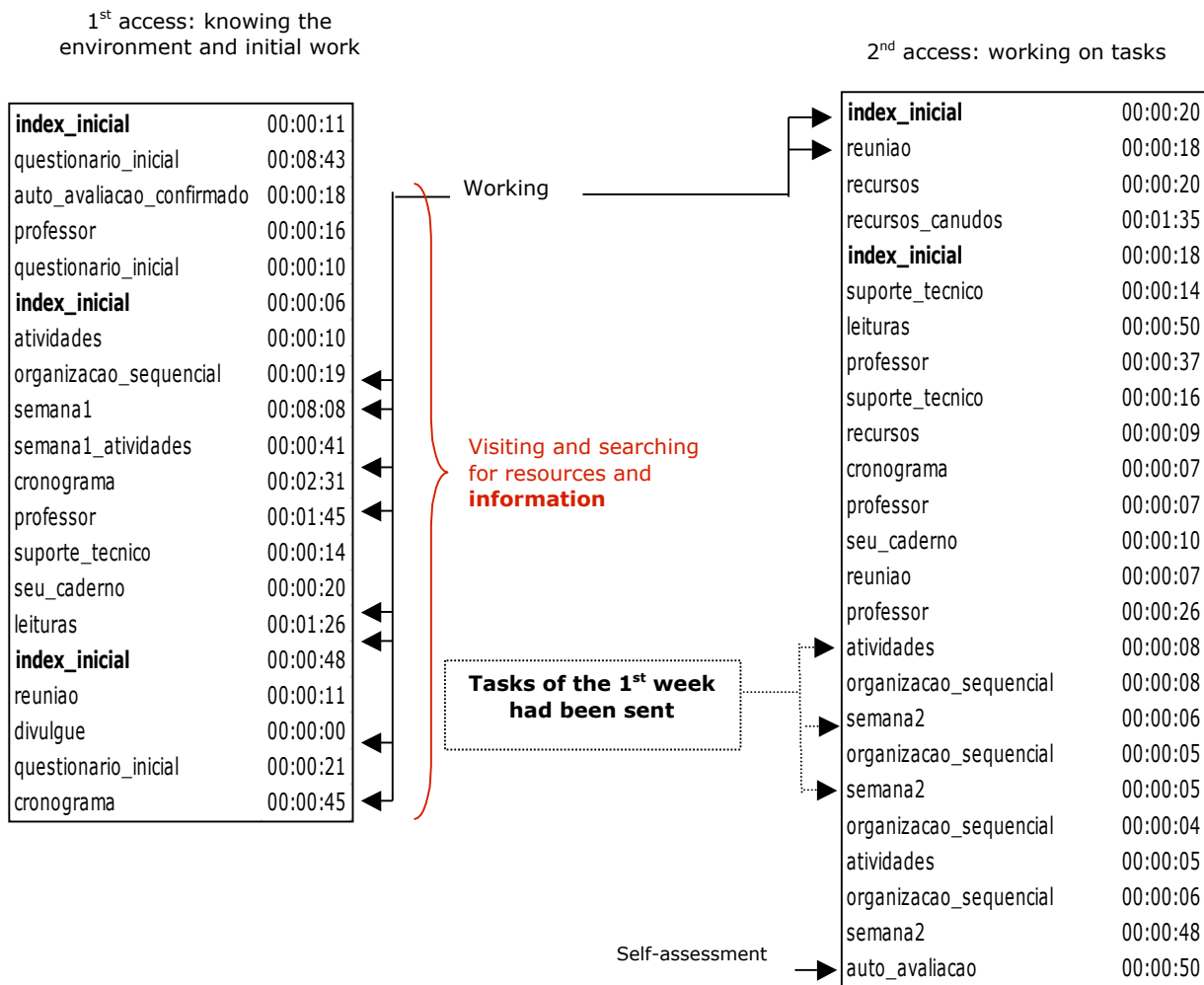Example of strategic elements *attitudinal* (self-assessment, previous questionnaire and work contract, personal note book), *communicative* (e-mails, chat, discussion group), *motivational* (animation and hipertextual activities) and *informative* (links of interest to web pages of varied interests, Brazilians official documents).

Figure 1: The main page of the Virtual Environment (Topics in Spatial Geometry)

We use a variety of tasks. In close connection to those tasks we establish categories prior to experience, which are later to be considered in the analysis. Besides off-line encounters, both a theoretical discussion and practical deepening of tasks were also carried about by virtual e-mail interaction, discussion list and chats.

Data collection and analytical process

One enters the environment by typing in a username and password. Each access is logged in a log-file, where the accessed links are identified, along with the time, frequency and different access steps.

1st access: knowing the environment and initial work

2nd access: working on tasks

index_inicial	00:00:11
questionario_inicial	00:08:43
auto_avaliacao_confirmado	00:00:18
professor	00:00:16
questionario_inicial	00:00:10
index_inicial	00:00:06
atividades	00:00:10
organizacao_sequencial	00:00:19
semana1	00:08:08
semana1_atividades	00:00:41
cronograma	00:02:31
professor	00:01:45
suporte_tecnico	00:00:14
seu_caderno	00:00:20
leituras	00:01:26
index_inicial	00:00:48
reuniao	00:00:11
divulgue	00:00:00
questionario_inicial	00:00:21
cronograma	00:00:45

Working

Visiting and searching for resources and **information**

Tasks of the 1st week had been sent

index_inicial	00:00:20
reuniao	00:00:18
recursos	00:00:20
recursos_canudos	00:01:35
index_inicial	00:00:18
suporte_tecnico	00:00:14
leituras	00:00:50
professor	00:00:37
suporte_tecnico	00:00:16
recursos	00:00:09
cronograma	00:00:07
professor	00:00:07
seu_caderno	00:00:10
reuniao	00:00:07
professor	00:00:26
atividades	00:00:08
organizacao_sequencial	00:00:08
semana2	00:00:06
organizacao_sequencial	00:00:05
semana2	00:00:05
organizacao_sequencial	00:00:04
atividades	00:00:05
organizacao_sequencial	00:00:06
semana2	00:00:48
auto_avaliacao	00:00:50

Self-assessment

Figure 2: Log-file example which one personal access is salved on database

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Information from the log-file has been important to organize and allows an assessment of professional development and provides means to improve the ICT system. We carried out an interpretative case of study. We proposed different activities during the course: carrying out, individually and collectively, tasks, exams, seminars and self-evaluation assignments. Triangulation was done exclusively through spontaneously shared interaction in the Internet communicative spaces: chat, e-mail, and discussion forum. We use the following procedures in data reduction: (1) selection of various contributions throughout the course; (2) identifying implicit elements of community of practice in them; and (3) contrasted them with other discourses of the pre-service teacher along the interactive process.

It is worth remembering that we adopted some principles: *(i)* professional knowledge has to be seen as a construction that cannot be separated from contexts and activities in which they are developed; *(ii)* socialized ideas in virtual environment express an intentionality and personal need of debate, *(iii)* since learning occurs during discursive online interactions, it also involves the reconstruction of identities, *(iv)* the implications of the virtual environment for the professional content knowledge depends on the discourse established in each communicative space, and *(v)* interaction, learning and evaluation are mutually related elements, both in and with the community.

What follows is an example of how we carried out the analysis from the different interaction instances. The texts illustrated here refer to the first unit, the aim of which was to develop geometrical thinking through exploratory activities of spatial figures. Texts are presented just as they have been written, and bold type and underlined parts exemplify aspects considered by Wenger (1998) for the study of learning as social practice. Let us have a look at the analysis on graduate student *sigt*[3].

Pre-service teachers interacting in virtual communities and learning

In the course of the initial questionnaire, when *sigt* tells about her experience with geometry as a student, she comments:

> ¨ (. .) **I have** studied all the geometry that can be given at school, without leaving out <u>any part</u>. I like the part that doesn't have to be <u>memorized,</u> like formulas. **I never** took a specific course in geometry. I have <u>difficulties in</u> drawing spatial figures, and, when it is not a good drawing, I find it <u>hard</u> to identify the parts in the drawing¨.

As *sigt*'s text shows, the preferences are very clearly towards situations that involve more reasoning than formula application, and the acknowledgement of her own difficulties. When this student performed a set of tasks that aimed at analyzing sections and describing flat shapes (see the example below), and after interacting with a study mate, she developed an interesting geometric reflection.

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[3] Nickname

Task: *When we perform a flat cut in a solid, the figure that appears in the cut area is called the cut section. In the cube below, to continue with flat sections, is it possible to obtain a square? Substantiate your answer. In what position should a plane be placed so that the section obtained is shaped as a rectangle? Which is the maximum perimeter of the rectangular plane? Justify your answers.*
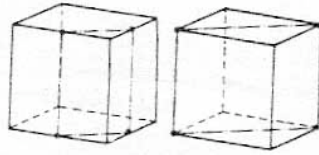

Fig. 1   *Fig 2.*

*(Sigt, by e-mail) ¨It is <u>not possible</u> to obtain a square, because we would have to calculate two opposed diagonals in the cube, then **we would** realize that the values of the diagonals (that would function as edges) would not be the same as the values of the edges that would come with them, forming the square. The plane must be placed in any part of the cube, as long as the side of the rectangle starts from two points in adjacent edges and these points serve as vertices of the rectangle; from there, two edges come out from each vertex, one horizontal and one vertical, connecting with the opposed point to form a rectangle. Maximum perimeter: $2a\sqrt{2} + 2a = 2a(\sqrt{2}+1)$¨.*

In the text above, we can see how *sigt* increases her communication and constitutes her own geometric meanings. Even though her answer for the task contained some "mistakes", because she belonged to the community, she felt confident (Graven, 2003) to socialize and exchange them. The text below, from a conversation in a chat, shows us the concern of the pre-service teachers with their own learning (see *nathalia 09/03/2004 - 18:34:25*), their learning community (*siduarte 09/03/2004 - 18:37:33*) and their continuous need for discussion and deepening (*sigt, 09/03/2004 - 18:38:20*) at performing other tasks (Muukkonen et al., 2001).

(. .)

nathalia (09/03/2004 - 18:34:25) : Did <u>all of you</u> find that the answer to question 3 last week was a pyramid?

siduarte (09/03/2004 - 18:34:48) : I also answered that it was a pyramid.

sigt (<u>09/03/2004 - 18:35:22</u>) : I don' t remember, but I know that one of the answers was a pyramid

nathalia (09/03/2004 - 18:35:32) : I agree with you, sigt! It's **very hard to visualize without** actually holding the solid!

sigt (09/03/2004 - 18:37:27) : one question in the third week speaks about taking the solid and manipulate it, I thought that **was complicated**

siduarte (09/03/2004 - 18:37:33) : we can exchange ideas. . as soon as I find the time to read the questions, I'll put my ideas at the forum… Please, try to <u>help me</u>.

sigt (<u>09/03/2004 - 18:38:20</u>) : I'm trying, but <u>I need to discuss</u> about them, because I keep changing the way I see them. (. .)

While in off line work (conventional classroom classes) there is a concern about the performance of tasks at specific moments, in the online dynamic there is the possibility of doing and reflect which is more time-flexible. In this process, both the teacher and the students become partners and start questioning, for example, the analysis of answers to a task in terms of simply right or wrong. At the moment we can see that inquiry here is an issue that appears naturally. Not only was the trainer (researcher in our case) able to promote inquiry, but also as Jaworski (2004) pointed out, inquiry is both a tool and a way of being and the communities support and grow through the inquiry. In this type of dynamic, the trainer takes on an important role: to encourage and to work on behalf of the students' interests.

There are a variety of conceptual aspects in relation to cognition and learning for each student. When socialized, these concepts favor a collective reconstruction of meaning and learning within the community. In the texts, we can identify different aspects regarding the learning of geometry in the established community: the importance of visualization, motivation to learn and to build didactic material which was heretofore unknown, as well as a careful reporting of difficulties inherent to the development of an activity.

At the moment we presented some examples focused on the questionnaire and task (both of them from synchronous interaction) and chat (asynchronous interaction). Despite the fact that we are aware that there are different intentions in the interactive process, we tried to show that inquiry can grow through this open dynamic, from different perspectives and times of reflection. For that, complicity assumes an important role and trainer should pay attention to this particular issue. Looking at the interaction below, from the discussion forum, we will show that, even in online interaction, elements of community of practice have close interrelation.

Sent Sun, Feb 13, 2005 11:38 am
Subject: Class on Feb 02, 05
I thought it was real nice to have **a practical class** (making material). Visualising **helped me understand** the figures better (the ones presented in the notes), and having something like an art class (children's stuff) was challenging, it was difficult to have that <u>cube get to stand</u>. It was fun, too. I loved having a class like that, after a **calculus class**, it was perfect <u>not having to calculate anything</u> to close the day and start celebrating carnival. Do enjoy carnival!!! Talk to you later. Bye!!!

Sat Feb 26, 2005 3:16 am
Subject: Class with rods
The **class with rods** was very interesting, because **from there** we could realize how important <u>rigidity</u> is for a solid. When we built <u>a cube</u>, we confirmed that its diagonals are very important so it can get its rigidity. The concept that drove home also is that a solid gets more rigid since **we use** triangles in making it.

Sent Sat Mar 05, 2005 10:20 pm
Subject: Sheet with complementary exercises
The sheet helped a lot with the exercises <u>but</u> **I was left with a doubt lurking about** the task, which asked to find the <u>angle</u> formed by two <u>diagonals</u> on different planes. Of course if I draw a third diagonal I'll get an equilateral triangle and the angle will be 60°. **That doesn't quite get into my mind**, I went on making the mistake and I wrote 90° on the test. **Don't you guys make** the same mistake I made. The angle will only be 90° if the <u>diagonals</u> are on the same plane!!! Bye. Love, Su

Sat Feb 26, 2005 3:16 am
Subject: Class with rods

The **class with rods** was very interesting, because **from there** we could realize how important <u>rigidity</u> is for a solid. When we built <u>a cube</u>, we confirmed that its diagonals are very important so it can get its rigidity. The concept that drove home also is that a solid gets more rigid since **we use triangles** in making it.

Sent Tue Mar 15, 2005, 2005 6:44 pm
Subject: Which diagonal?
Hi, Wellington, I'm glad the list helped **you**. Suellem, <u>we need</u> **to pay attention to which diagonal we are referring to**: the cubes? a face's? <u>We know that</u> the diagonals of the face of a cube are perpendicular. So, the angle between them is 90°. Now, Suellem, **I didn't understand,** when you referred to the same plane? Was it that the angle in the exercise had to be 60°, **that you didn't understand**? **Who can help us with this interesting exchange?**

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With the coming to belong to the community (*don't make the same mistake…we need to pay attention…*) the meaning becomes a way to talk about our doubts, questionings and skills we have, or are on our way to developing (*a class… not having to calculate…*). As identity involves a peculiar way in which each individual developed (*…it helped me understand…*), through his or her interaction in the community, a reflection on his/ her own learning process (*that doesn't quite fit into my mind…*), we can identify different perspectives which can support a mutual involvement in the search (*… from there…*) of rebuilding the meaning of the learning situation under analysis (*… the sheet…the calculus class…class with rods…about the diagonal which we referred to…*). Socialized ideas at the forum express an intentionality and personal need for debating in and with the community. Although these interactions take place in non real time, they can be discussed in real time by the subjects whether within or out of the class environment. Nevertheless, given the flexibility, which is peculiar to online work, the students are not constrained by boundaries of physical space or time and they go on exchanging their ideas.

Through the whole the interaction shown above, some asynchronous and other synchronous, we can see that pre-service teachers, the majority of whom do not have previous experience in geometry, were able to explore and deepen their understanding of different conceptual aspects of spatial geometry. They had an opportunity to know and use a variety of technologies and, in this way, understand how these media resources can contribute in different ways to the learning of mathematics. With this purpose, ICT come to contribute to the constitution of one more setting, without substituting off-line programs. The socialization of their ideas made the future teachers feel assured that they belonged in a community where experiences are exchanged.

With the processes of online training mediated through Internet, besides the construction and study of exemplifying models of spatial geometric models, handling and visualizing favored by available ICT resources (software, APPLETS, animation available at Internet, etc.) bring different implications for cognition. For instance, when developing some of their tasks, some pre-graduate students preferred writing and drawing their answers in Word, while others used other drawing programs, and some favored using pencil and paper. Introducing ICT resources as mediating artifacts for learning encourages pre-graduates to reflect on different pedagogical actions, which can be implemented in mathematics lessons. In this process, as Blanton (1998) highlighted, telecommunication will take on an important role: that of becoming a link between theory and practice. We believe that with this type of work we are not reducing geometric learning to the analysis of individual mental capacities or mechanical procedures, which would only tend to increase the marginality and marginalization process of people who are already conceptually marginalized (Matos, 1999). The policies of training teachers online must take on their own role as social and political practice.

Conclusion

Through this research we have studied aspects of the learning of pre-service teachers, drawing from their interaction as it was carried out in a virtual environment. We have analyzed spontaneous interaction in environments where participation was optional, such as chats, e-mail exchange, and participation at the discussion forum. There was an explicit intentionality from the participants to socialize their ideas and problems with the community. On the whole, this exchange took place at physical places and times, which were not coincidental among the participants. The pre-service teachers were able to explore and deepen different conceptual aspects of spatial geometry. They had an opportunity to get to know and use a variety of technologies and, through this, understand how these media resources can contribute in different ways to the learning of mathematics.

As Blanton (1998) pointed up, studies in online education have to specify elements of learning, which bring implications in the nature and quality of participation within given communities. In this perspective, this research confirms aspects of learning identified in online interaction among pre-service teachers. Moreover, although the idea of community implies participating in a system of activity over which participants share understandings about what they do and about what it means for their lives and communities, we understand that this social practice goes beyond this, as it should, to be a transforming one (Matos, 1999). Our studies have pointed out that inquiry in a community of inquiry is actually more than practice in a community of practice (Jaworski, 2004). Through online interaction (synchronous and asynchronous), apprentices (teachers, researchers and trainers) can build knowledge from a wide range of meaning that emerges from personal and contextual interaction, progressive inquiry, with different aims and discursive aspects, and ultimately makes practice more flexible and changeable. So, we are aware that, besides identifying the elements involved in learning in communities of practice, it is crucial to study the challenging and inquiring interactive dynamic which takes place on the hipertextual path through the various spaces of the training environment and in the continuous discussion among the community.

Studying learning in communities of practice, through online interaction, means looking widely at professional activity (Bairral, 2005b). Proposed tasks, motivation and each interlocutor's interest, his or her functions, as well as the multi-discursive components inherent to each contribution, all bear an influence in this dynamic. Therefore, the analysis of learning has to be oriented by the integrated identification of meanings reconstructed from different elements of practice, from the commitment with the community and from the collaboration that establishes itself in the community of belonging. If we take into account the great development that ICT have had over the last few years, this research, besides proposing a virtual environment structure for the training of teachers in our university, has shown our students ways of interacting which differ from the ones they were used to in the conventional classroom. We believe that this introduction, along with the possibility to view it critically,

will bring new perspectives into the use of technological means and their social presence (Garrison et al., 2000).

Since we believe that the online interaction within a community both supports and grows through an inquiry (Jaworski, 2004), we shall close with the excitement of anticipation, as the making and proposing of mathematical tasks and the analysis of the development of metacognitive thought in virtual communities of inquiry with pre-service teachers currently challenges and drives our interest. The role of the educators (teacher, researchers and trainer) in these particular setting is one of these particular issues.

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