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USING THE RESCALED RANGE ANALYSIS FOR THE STUDY OF HYDROLOGICAL RECORDS: THE RIVER TER AS AN EXAMPLE

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SUMMARY

Water discharges into the Ter river have a strong influence on the water chemistry and on the dynamics of benthic communities. For this reason, the temporal patterns followed by monthly runoff from 1954 to 1988 have been studied. From a statistical point of view, discharges show a very regular seasonal cycle, but in practice, climatic events introduce a source of variability on runoffs. To determine whether the series of 420 monthly discharges fluctuates with some regularity or at random, a rescaled range analysis was made. The persistence (H) of the series of discharges was measured by the equation $R / S = (T / 2)^H$. In fact, H is a measure of the existence of clear trends or periodicities in the records of persistent stocastic processes. An additional measure of the persistence was obtained by means of the relationship D = 2 - H, where D is the local fractal dimension. For comparative reasons, a random series of the same length, mean and standard deviation was generated and analysed in the same way as the Ter discharges were. The results were H = 0.68 and D = 1.38 and H = 0.44 and D = 1.52, for the Ter data and for the random series respectively. These results show that the simulated series is a case of ordinary Brownian motion while the Ter discharges series has an intermediate value of persistence. Because of the value of H in the Ter series, there are periods whose values were estimated by the autocorrelation function and the periodogram of the series. The results show the existence of short fluctuations of 3, 6 and 12 months determining the seasonality of the annual cycle, and large fluctuations with periods of 5.5, 8.6, 10.1 and 11.7 months which can be considered an expression of cycles of circa 7 and 11 years.

KEY WORDS: River discharge, fractal dimension, Hurst exponent, fractional Brownian motion, persistence.

. . . In the case of the landscape, rain falling at random and moving from each point of the surface along the steepest profile, through erosion and convergence of runoffs, generates a secondary "hydrologic" relief. Will the fractal quality be preserved ? (MARGALEF, 1988)

INTRODUCTION

In Mediterranen ecosystems, annual cycles are always described in terms of regular fluctuations associated with seasonal climatic changes. Nevertheless, this pattern is usually modified by irregular climatic fluctuations that cause sudden changes in their dynamics. Temporal ponds and rivers are among the most affected ecosystems because they are mainly controlled by the periodicity and intensity of the rains. The turnover time of the water acts as a key factor in most of the processes taking place (SCHINDLER, 1988; ARMENGOL, 1988).

In rivers, strong rains increase the intensity of the advective transport of solid

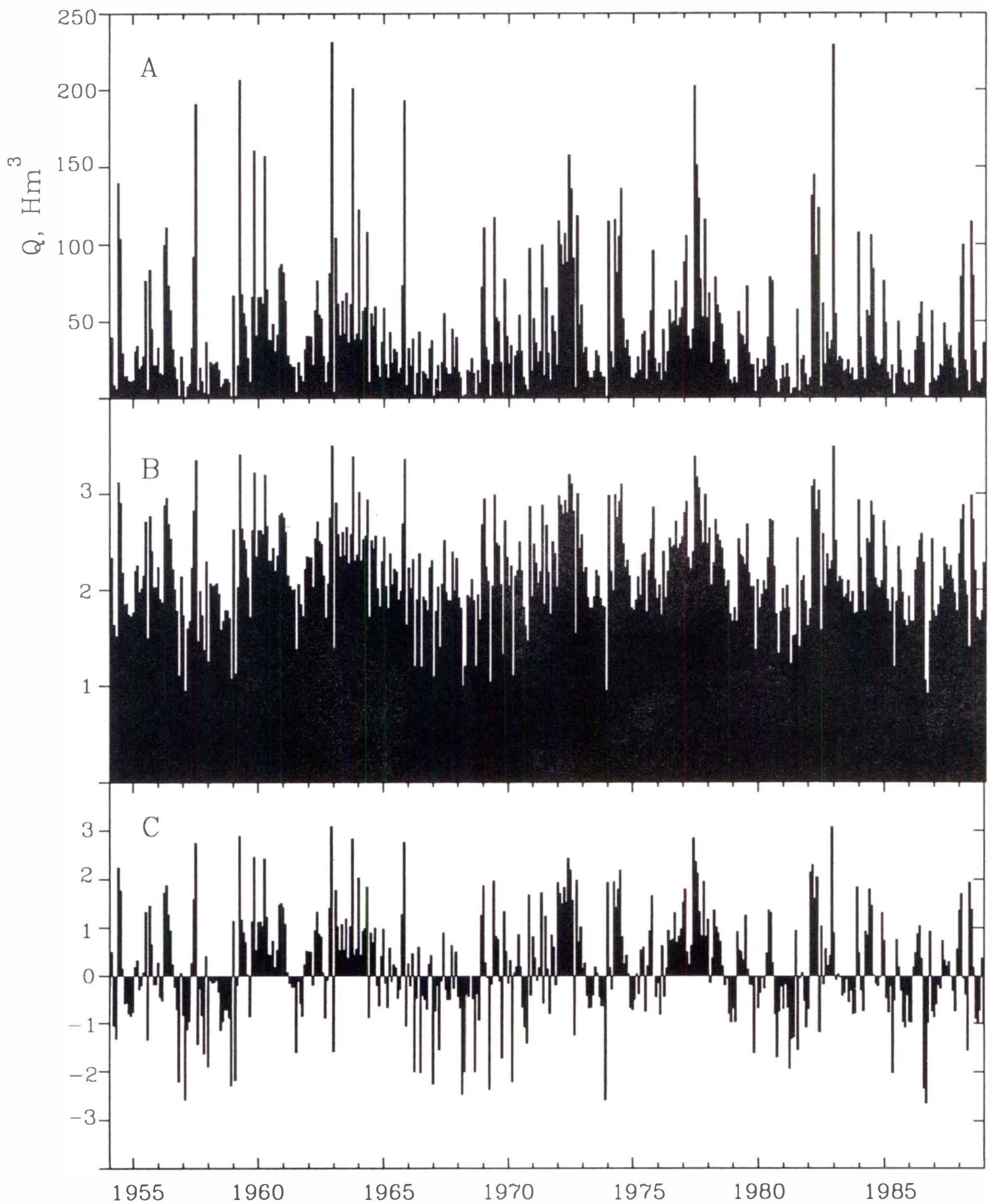

FIGURE 1. River Ter monthly discharges for the 1954 - 1988 period. A) original series of data in Hm^3 $month^{-1}$. B) normalized series by $Y = X^{0.22}$. C) the standardized sequence.

and dissolved substances. Because of this, the spates act as huge inputs of external energy that simplify the structure of the river. Most of the organisms are carried down and the chemical composition of water becomes more similar along the river. The annual cycle of the communities can be seen as a sequence of populations with short life span, well adapted to seasonal changes but able to reconstruct the community after each event (MARGALEF, 1980). If longer periods of time are considered, changes in the communities can be seen as a sequence of periods of organisation (secondary successions) followed by quick disruptive episodes (backward steps) (MARGALEF, 1977).

In the Ter river, water flow is the most important factor determining the annual variability in the environmental conditions (SABATER & ARMENGOL, 1986) and in the dynamics of phytobenthic (SABATER *et al.*, 1988) and macroinvertebrate (PUIG *et al.*, 1987) communities. These relationships are well established for short periods of time, such as the annual cycle, but for longer periods it is assumed that climatic fluctuations have a similar effect. If we accept these relationships for the Ter river, the long-term study of a record of hydrological events as large as possible can provide us with some insight into how it

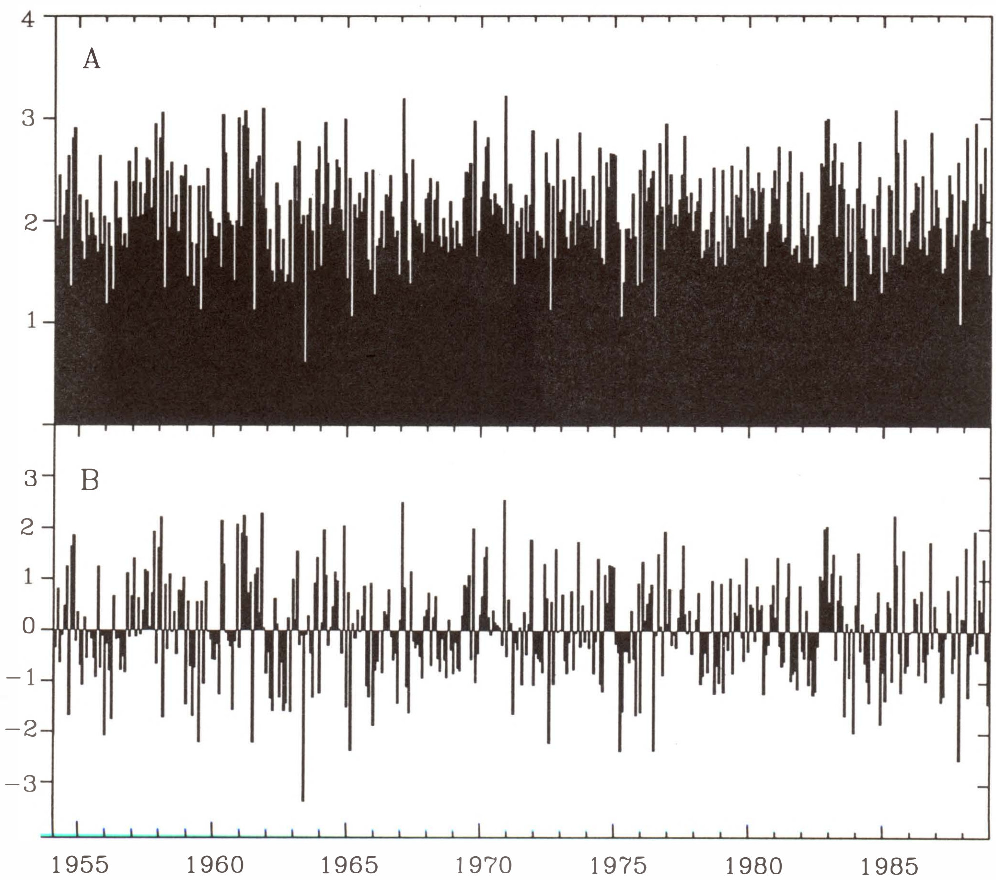

FIGURE 2. Simulated sequence of 420 values (35 years x 12 months) normally distributed with the same mean and standard deviation as the data of figure 1B . B) the standardized series.

functions from an ecological point of view. For longer periods it is well established that the water discharges affect the chemical pollution and the eutrophication levels in the reservoirs situated in the middle stretch of the river (VIDAL, 1977, 1987, 1989; ARMENGOL *et al*., 1986).

Long-term records of hydrological events show erratic behaviour at different time scales (FEDER, 1988). These records can be studied by means of the rescaled range analysis (HURST *et al*., 1965) and characterized by an exponent H, called the Hurst exponent, which is a measure of the persistence or "memory" of one series (MANDELBROT & VAN NESS, 1968). MANDELBROT (1985) has shown that this record is a self-affine fractal curve with a dimension D = 2 - H. For a practical perspective, the H exponent measures how far a temporal series of data departs from a random series.

We have used a record of 35 years of monthly average discharges to analyse the persistence in the data sequence. The aim of this study is to determine the temporal trend followed by the series and how this key factor carried weight on the river considered as an ecosystem. It is well known that wet and dry years appear in clusters, the "Joseph effect" (MANDELBROT & WALLIS, 1969); for this reason the existence of temporal "cycles" has been studied here by means of autocorrelation and spectral analysis.

MATERIAL AND METHODS

The record of hydrological data used in this study is composed by the monthly discharges at Roda de Ter station from 1954 to 1988 (Fig.1). The data were obtained from the Dirección General de Obras Hidráulicas gauging yearbooks (D.G.O.H., 1972, 1973, 1974 and 1975; D.G.O.H., 1976) for the period 1965-1977, and were provided by the Servei Hidrològic de la Generalitat de Catalunya for the periods 1954-1965 and 1978-1988.

Roda de Ter is situated in the middle stretch of the Ter river, 88.6 Km from the source (river length 200,3 Km), where it has a drainage area of 1,476.8 Km^2 (48.11 % of the total). At this point the Ter is a fourth order river. This station was selected because it is situated above the reservoirs, thus excluding their regulatory effect, and at the same time the drainage area is large enough to avoid the effect of sudden discharge due to a local rainfall.

The hydrological characteristics of the

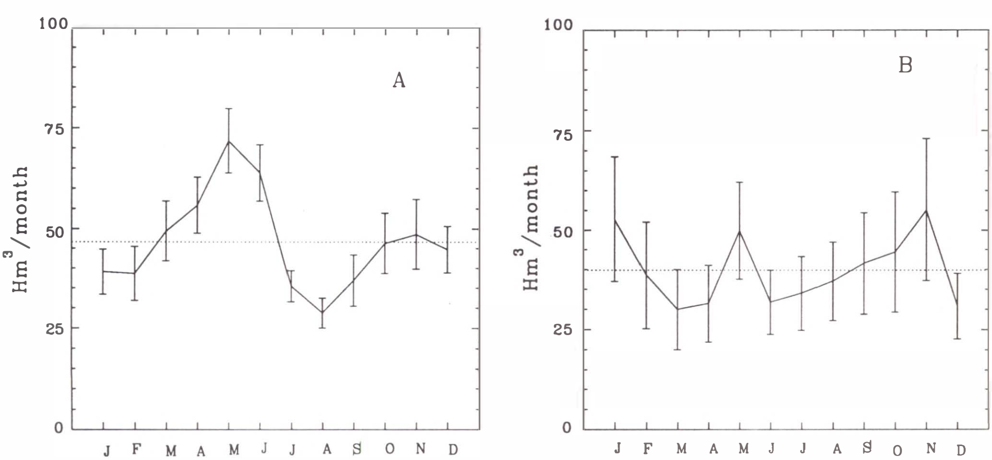

FIGURE 3. Average annual cycle showing the mean monthly discharges with 95 % confidence interval: A) for Ter discharges (Fig. 1B) and B) for random series (Fig. 2 A).

Ter are typical of the Mediterranean region, but the proximity of the Pyrenees also has an influence. The annual cycle is characterized by two periods of maximum water flow; the first, in May (28.6 ± 6.0 Hm^3 $month^{-1}$, $p < 0.05$ for all the data of this paper), is the highest due to the effect of the spring rains plus the thaw of the snow accumulated in the Pyrenees, while the second, in November, is less important (17.6 ± 5.7 Hm^3 $month^{-1}$) and exclusively due to the autumn rains.

Two minima are situated in between, the lower in August (11.4 ± 2.6 Hm^3 $month^{-1}$) and the other in February (14.2 ± 3.9 Hm^3 $month^{-1}$). The average monthly water flow for the period studied was 41.3 ± 3.74 Hm^3 with ranges between 231.4 Hm^3 and 0.70 Hm^3, while, if the data are grouped by years, a mean of 554.6 ± 35.7 Hm^3 is obtained with a maximum of 1180 Hm^3 in 1972 and a minimum of 258 Hm^3 in the year 1957.

The Ter hydrological record does not follow a Gaussian distribution (the kurtosis is 4.912 and the skewness is 2.001). For this reason, it is not possible to use many of the parametrical statistical methods. In order to transform the original data into a new series with a near normality distribution, a power transformation has been sought by means of the Box & Cox

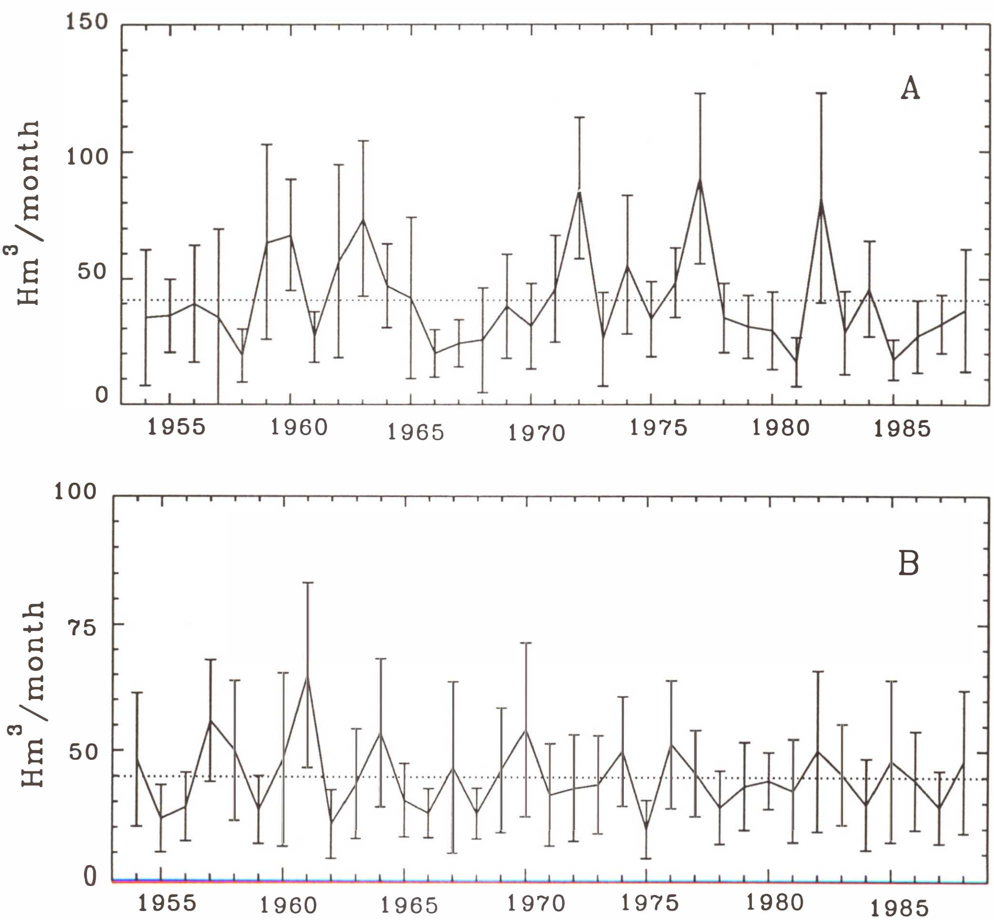

FIGURE 4. Mean annual water flow with 95 % confidence interval: A) for Ter series (Fig. 1B) and B) simulated sequence (Fig. 2A).

method (BOX & COX, 1964). The closest transformation to near normality of the data is $y = x^{0.22}$. The new series (Fig. 1) has a mean of 2.112 and an standard deviation of 0.449, while the kurtosis and the skewness measure 0.058 and 0.028, respectively. Finally, the series used in the analysis has been standardized (Fig. 1).

To illustrate how far the hydrological series departs from a random record, a new Gaussian series of the same length, mean and standard deviation has been constructed, but following a random walk process (Fig. 2).

For comparison of the two series the seasonal and annual distribution of the data are shown in figures 3 and 4.

RESULTS

Not all the rainfall runs immediately into the rivers; a certain amount of water remains stored in the watershed and contributes to the discharge in the driest months. In the same way, during dry periods, the waterflow is higher than expected due to rainfall. As a first approach, there is a buffer effect on discharge due to the storage capacity of the drainage basin. The rescaled range analysis measures the importance of this effect by means of the Hurst exponent.

If $x_1, x_2, \ldots x_T$ is a series of monthly discharges for a period of time T, it may be characterized by the mean ($\bar{x}$), the standard

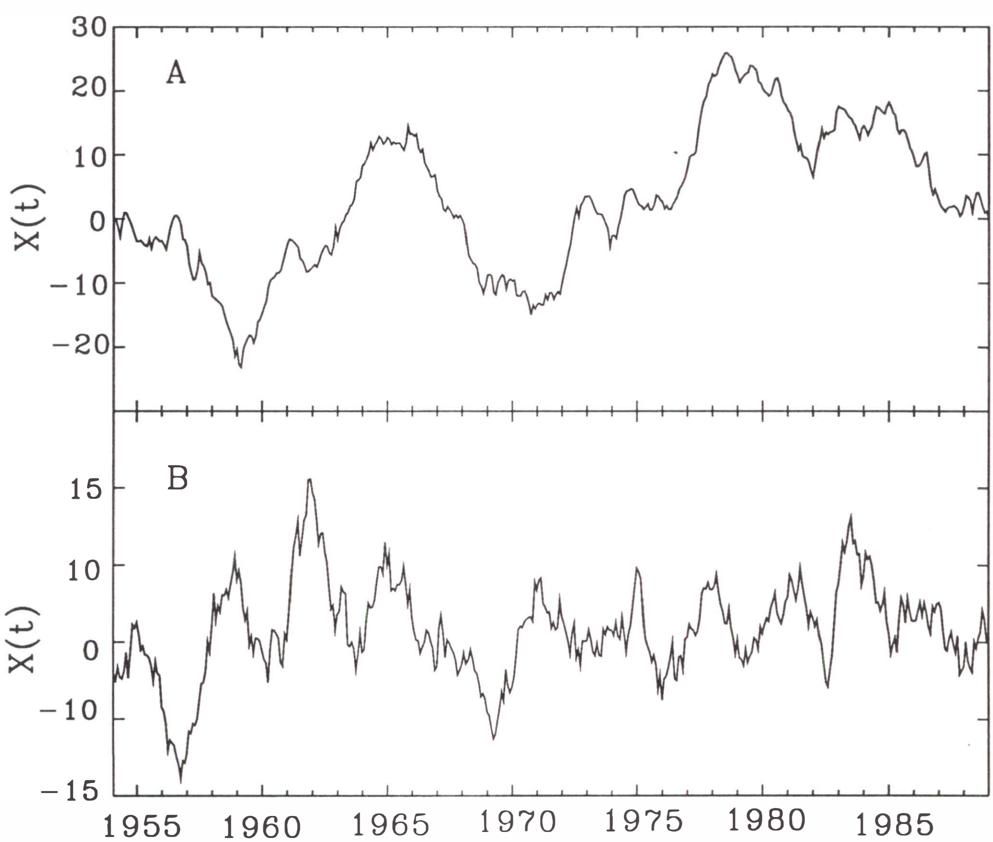

FIGURE 5. Accumulated departures from (zero) mean for: A) the Ter hydrological data, and B) the random series.

deviation (S) and the range (R), which is the difference between the maximum and the minimum departures of the water flux (x_i) from the mean ($\bar{x}$). Formally,

$$\bar{x} = \sum_{i=1}^{T} (x_i) / T \quad (1)$$

$$S = [1/T (\sum_{i=1}^{T} (x_i-\bar{x})^2)]^{1/2} \quad (2)$$

$$x(t) = \sum_{i=1}^{T} (x_i-\bar{x}) \quad (3)$$

$$R(T) = \operatorname{Max}_{i=1}^{T} x(i) - \operatorname{Min}_{i=1}^{T} x(i) \quad (4)$$

The rescaled range, R/S, is well described by the empirical relation (HURST *et al.*, 1965):

$$R / S = (a\ T)^H \quad (5)$$

where H is the Hurst exponent.

In figure 5 the accumulated departures from the mean, x (t), are plotted against time for the standardized discharges of the Ter river (Fig. 5a) and for the random series (Fig. 5 b). The figures obtained are two cases of *fractional Brownian motion*, and are generalized random functions, x (t), each one characterized by the exponent H, which ranges between 0 and 1 (MANDELBROT & VAN NESS, 1968; MANDELBROT, 1987). Fractional Brownian motions are very useful to describe time series followed by various natural phenomena such as river discharge, rainfall, tree rings, distribution of varves in lake sediments or the frequency of sun spots (FEDER, 1988). This kind of motion can also be used to generate new surfaces like watersheds or continents, mountain profiles, the shape of lakes or the form, number and distribution of islands in archipelagos (MANDELBROT, 1975). VOSS (1985) has introduced an algorithm called *successive random addition* for generating this kind of curve for a given value of H, although there are many other ways of doing so (e.g. KIRKBY, 1987). From the practical point of view, the case H = 0.5 is ordinary Brownian motion, and when $0.5 < H \leq 1$ the curves show a *persistence* as high as the value of H. Finally, it has been demonstrated (MANDELBROT, 1985) that these curves are statistically self-affine scaled sets and their fractal dimensions are not uniquely defined. In other words, there is no global fractal dimension, but there are local ones and this is because the fractional Brownian motions are scaled in two variables; in this case river discharge and time. For each case the fractal dimension is:

$$D = 2 - H \quad (6)$$

At this point it is interesting to describe how figures 5a and 5b were obtained. We began with two series of data and after their transformation into fractional Brownian motions they had the appearance of profiles or chains of mountains. In fact, these are generated irrespective of whether the data series are real or simulated. Can we consider that these profiles reflect something intrinsic or characteristic of the basin?

Obviously they cannot be considered as an idealized reconstruction because we have time series and the basin has a spatial plan. Nevertheless, our tentative answer is affirmative. Can the path of the runoff to the rivers be imagined from figures 5a and 5b? The answer is yes, we only have to imagine how the water runs through the artificial landscapes of figure 5. The random series (Fig. 5 b) shows a more abrupt relief, high peaks and narrow valleys, suggesting a quick response to rainfalls and low storage capacity. In other words, there is no persistence or memory. On the other hand, the real series (Fig. 5 a) has a more even profile, with large plateaus and wide valleys, where water can be retained or stored for more time, meaning persistence or memory.

In practice the Hurst exponent is calculated fitting the power equation $R/S=(aT)^H$ to a set of values of R/S and T obtained from different time lags. In figure 6 the results for the Ter hydrological data, H = 0.68, and random series, H = 0.44, are shown. As was expected, the random series has a value of H near 0.5 which corresponds to ordinary random motion. On the other hand, the Ter data show an intermediate value of H which is very similar to the value found by HURST (1965), H = 0.72 using 94 series of discharges ranging between 10 and 100 years of data. For comparison, the Loire, using the mean monthly flow between 1863 and 1966, has H = 0.69, the Rhine at Basle for the period 1808-1966 has H = 0.5, while for the Nile the value of 0.9 is the highest measured to date (MANDELBROT & WALLIS, 1969).

It is possible to achieve a complementary measure of the Hurst exponents for the hydrological and random series by means of the relationship D = 2 - H, in which D is the local fractal dimension of each record

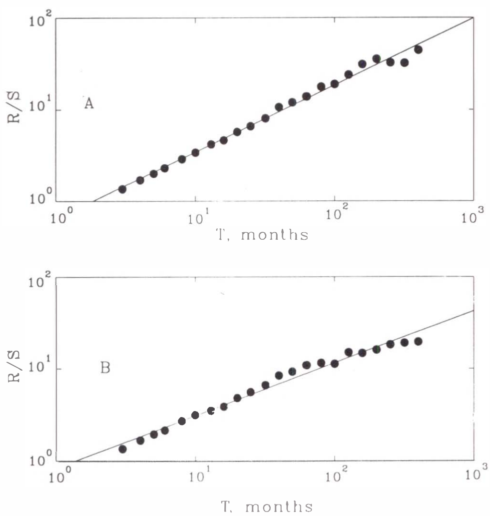

FIGURE 6. R / S as a function of lag T (filled circles) for the series of figure 1 B (A) and 2 A (B). The lines represent fits of the Hurst law $R / S = (aT)^H$ to the observed R / S. For the Ter series (A), H is 0.68, with a correlation coefficient of 0.955, while for the random series H is 0.44, with a value of r = 0.915.

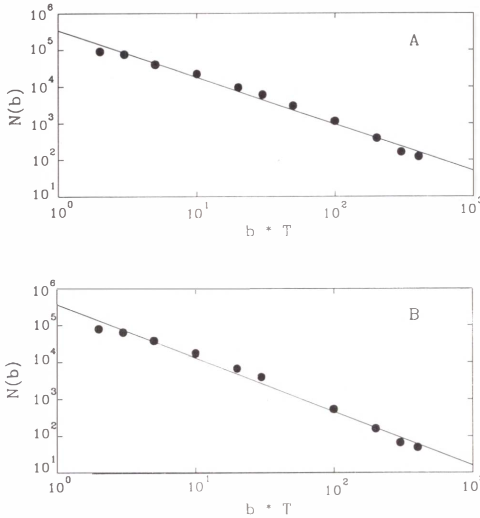

FIGURE 7. Number of "boxes" N (b) as a function of the time scale obtained for the accumulative normalized departures of the Ter series (Fig. 5A), and of the simulated sequence (Fig. 5B). In both cases a minimum "box" of T = 1 month and a = 0.001 Hm^3 have been used and its size was increased by 13 different multiplicative values of b. The curves are fitted to the equation $N(b) = b^{-D}$. In the Ter series (A) the box dimension is 1.38 with a correlation coefficient of 0.997. In contrast, in the random series (B) these values are 1.52 and 0.998 respectively.

of accumulated departures, x(t) (Fig. 5), considered as a Brownian function. As these functions are self-affine records scaled in two variables, x(t) and t, only local fractal dimensions depending on time lags can be calculated. The box counting method to determine the fractal dimension considers that if we cover the x (t) records with a grid of boxes of width bt in time, and of length ba in ordinates, *the box dimension* (D) is defined by the equation,

$$N(b; a', t) = b^{-D} \qquad (7)$$

where N (b; a', t) is the number of boxes we need for covering the record, a and t are the minimum values of the parameters x (t) and t respectively, while b is a multiplicative factor which determines the increasing size of the boxes. In practice, the box dimension (D) is obtained by fitting the equation (7) to a series of values of N(b) and b by linear regression.

Figures 7a and 7b show the results we obtained using the box-counting method from data in figure 5. Because time is an integer, the minimum box width selected was t = 1 month, while a value of 0.001 Hm^3 was chosen for height. The obtained results for these values of a and t are indicated in figures 7 a and 7 b. The fractal dimension for the Ter hydrological record is 1.38 and for the random series 1.52. Both results comply with the expected fractal values according to equation (6) showing slight differences due to the statistical approach.

Since both H and D change when another scale of boxes is taken, the evolution of the box dimension for other sizes of boxes is considered. The value of D remains almost constant when the height of the box is between 0.01 and 0.0001 Hm^3. However, D grows very quickly when the time scale is changed and values higher than two months are considered as the minimum width of the boxes. In figure 8 the values of D are always higher for random series than for the Ter discharge series, but both curves grow asymptotically towards a theoretical maximum of D = 2 (MANDELBROT, 1987). In conclusion, there is a loss of the sensitivity of method when the selected time lag is longer than the minimum time intervals of the data.

High values of persistence can be considered as a measure of the strength of the periodical cycles throughout time - the "Joseph effect"- but they do not give information about the frequency of such cycles. To determine these frequencies, we have calculated the autocorrelation function (ACF) and the periodogram of the original series of hydrological data. The results of the ACF are shown in figure 9. Because of the noise of the series, the ACF (Fig. 9 a) does not show any characteristic period. To avoid this effect, the series was filtered by moving average (MA) process using three different spans: 3 months (MA3), 6 months (MA6) and 1 year (MA12). In each case, a certain amount of information is lost but the most important cycles persist (Fig. 9 b-d). The results show the existence of high correlations for periods of 1, 2 and 3 years, then there is a gap until around 5.5, 8.6, 10.1 and 11,7 years. These longer term fluctuations can be considered as an expression of the cycles of *circa* 7 and 11 years that are characteristic in many climatic events in all the Mediterranean region (MARGALEF, 1977). Because of the high correlations obtained for short time periods, these cycles do not appear in the ACF. For this reason the periodogram and the spectral density of the Ter discharges, logarithmically transformed, have been calculated by Fourier analysis. In the spectral density a Tukey window (DENMAN, 1975) with a span of 12 was used. The results (Fig. 10) show little difference from the values obtained by ACF for long-term cycles, that is 5.8, 8.7 and 11.7 years. Nevertheless, this method gives a more precise measure of the importance of seasonal cycles. The cycles of 3 and 6 months are the most persistent,

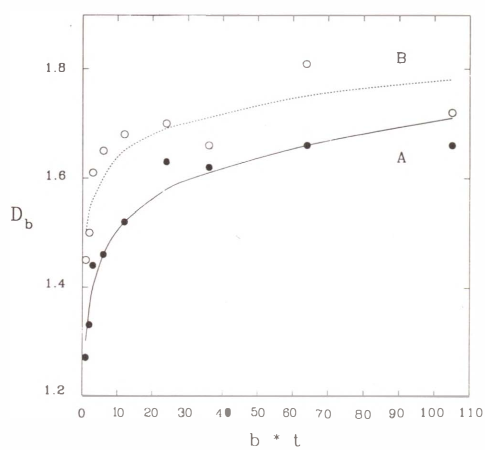

FIGURE 8. Evolution of the box-dimension as a function of the time scale of the "boxes": A) for the Ter data (filled circles) and B) for the random sequence (hollow circles). In both cases a minimum value of 0.001 for x (t) have been taken. The data are fitted to a logarithmic equation to show their trend.

but there are also other short cycles of around these periods which can be produced by small differences in the seasonality of each year.

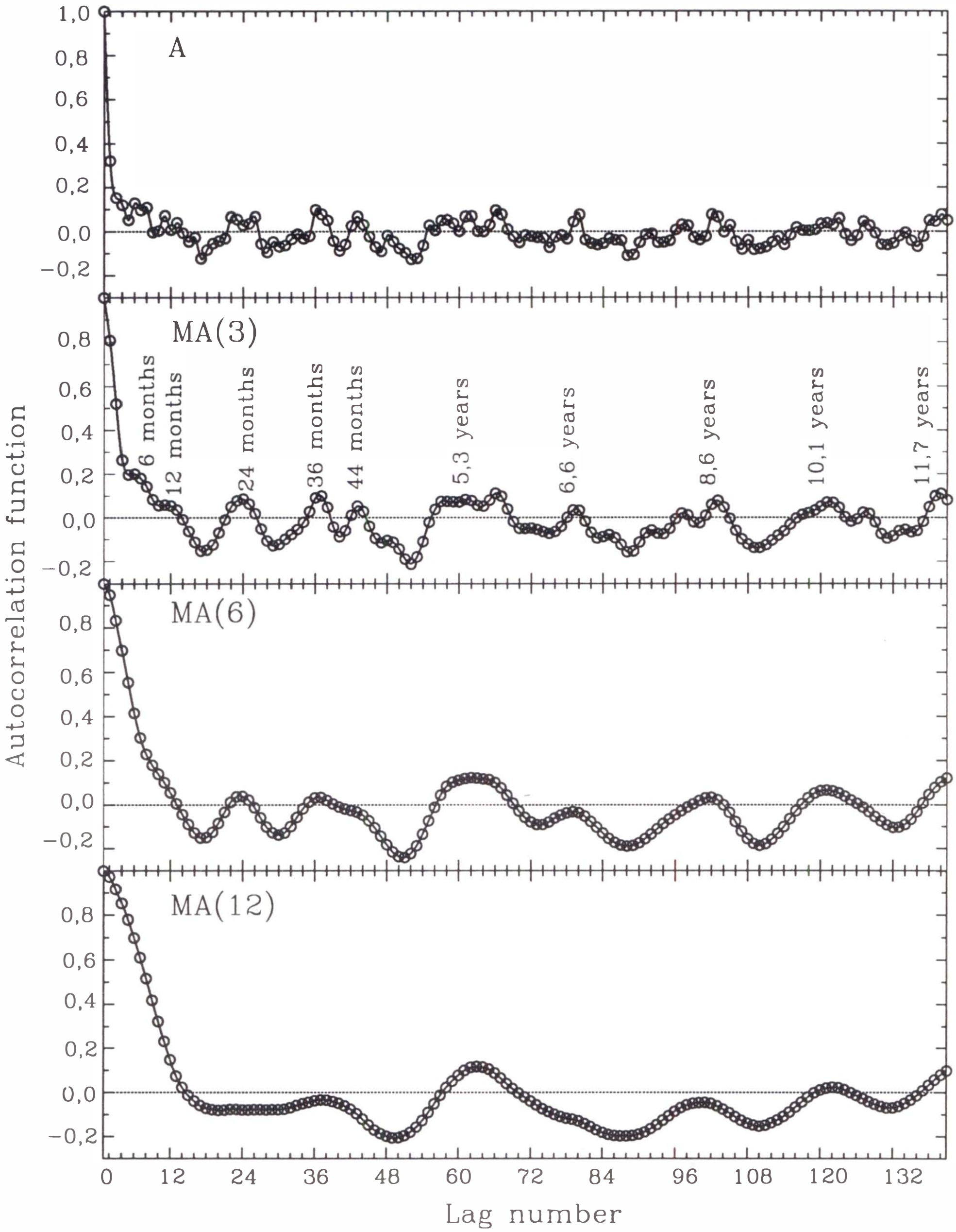

FIGURE 9. Autocorrelation functions of the River Ter monthly water flow (A) and the same series but filtered by a moving average process with a 3 month span MA (3), 6 month span MA (6) and a 1 year span MA (12). The periods which represent the most conspicuous peaks are indicated.

DISCUSSION

To date, the morphology of the catchment has been considered the main cause of the persistence in hydrological fluctuations. This is the simplest version of the storage model: the water runs off more slowly in the floodplains and even reliefs than in mountainous areas with high slopes. In a more realistic model the discharges are the sum of a series of independent processes in decreasing order of importance. For instance, the first factor may be the existence of natural stores of geological origin (karsts); the second, morphology (the slopes); the third, climatic factors (snow accumulation); the fourth, microclimatic factors (soil moisture), and so on, until all possible factors have been taken into account. The fractal nature of the basin may also contribute to the fractal behavior of the river discharge (FEDER, 1988). The result is that the discharge of a river depends not only on recent precipitation but also on earlier rainfalls and, at a theoretical limit, it will be independent of recent rainfall.

In lakes, the intensity of the fluctuations produced by climatic oscillations is regulated by their size (GOLDMAN *et al.*,

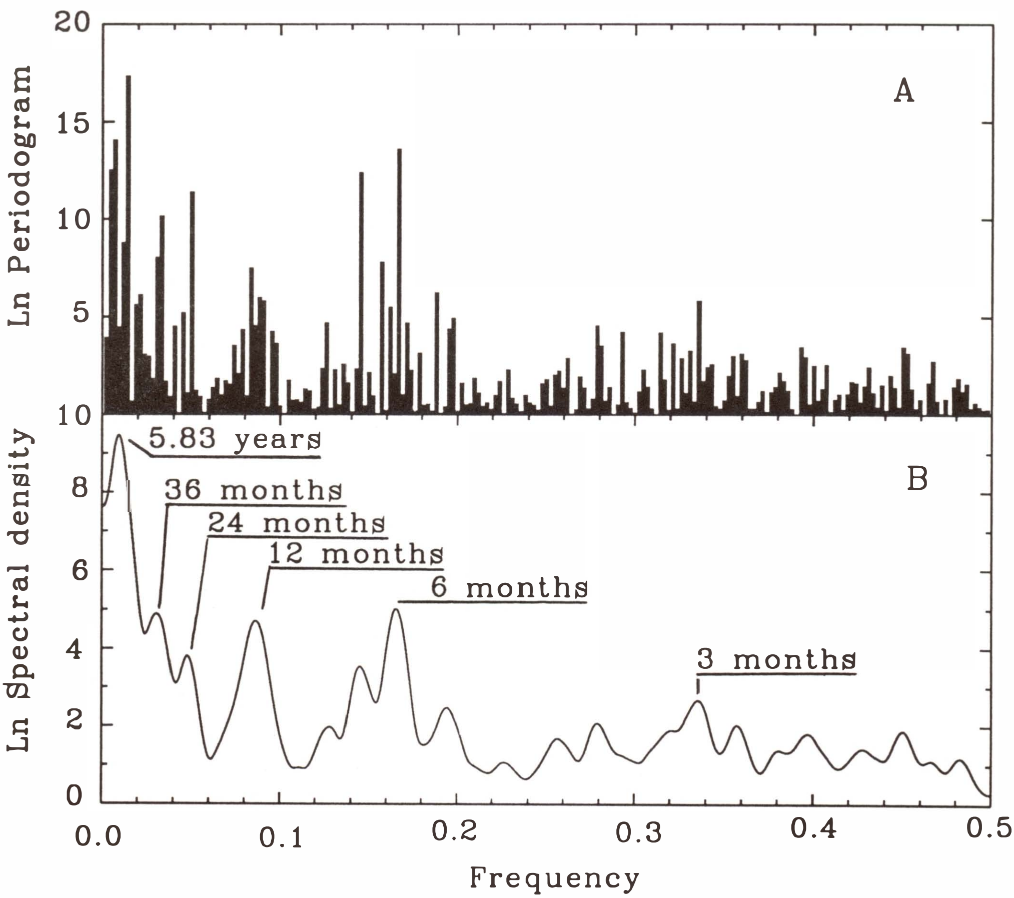

FIGURE 10. Periodogram (A) and spectral density (B) plots of the results obtained by a Fourier analysis from the original Ter data, logarithmically transformed. The periods which correspond to the largest cycles are indicated.

1988; POWELL, 1989). In a similar way, the size of the catchment is an important factor determining the persistence of the hydrological series due to the possibility of integrating events taking place in different subbasins.

It is interesting to consider the effect of various human activities on the basin to determine what effects may be expected on persistence. The channeling of rivers leads to destruction of floodplains; the pumping of groundwater destroys various natural stores, while the construction of reservoirs increase them. In a similar way, the areas without vegetation and with eroded soils do not have water retention capacity and the persistence is close to values of 0.5.

As for the communities, hydrological events are a key factor in determining their persistence in the river. According to MARGALEF (1980), the organisms make their own Fourier analysis to foresee the environmental fluctuations. In fact, first they make a factorial analysis to identify the key factor and the correlated ones. Accordingly, communities in a river are an example of physical - biological coupling. The species try to couple their biological cycles to temporal scales of variability of physical (= climatic) events. First of all, they need to know if prediction is possible. If it is, that means $0.5 < H < 1$ or $0 < H < 0.5$, and then natural selection can act. This coupling is as strong as the value of H.

In conclusion, it is easy to be a prophet in the Nile with $H = 0.9$, and it may be possible to have some chance in the Ter ($H = 0.73$), but it is better not to bet on rivers with a value of H close to 0.5.

ACKNOWLEDGEMENTS

We express our gratitude to Professor R. Margalef for giving us an idea of the value of the study of environmental fluctuations in understanding the structure and dynamics of communities. His ideas (MARGALEF, 1977, 1980, 1983, 1988) were a stimulus for the study of the Ter river hydrological series. It is a pleasure to dedicate this paper to him.

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