Sources of the Planetary Theories in Fahhād’s ʿAlāʾī zīj*: Solving a Medieval Case of Intellectual Fraud*

S. Mohammad Mozaffari
Department of History of Science and Scientific Archaeology.
University of Science and Technology of China (USTC)
s.m.muzaffari@hotmail.com
ORCID: 0000-0001-8677-8718

Abstract: According to the anonymous 13th-century *Shāmil zīj*, Abu ʾl-wafāʾ al-Būzjānī’s (10 June 940–after 25 May 997) planetary theories were adopted in Ibn al-Fahhād’s 12th-century *ʿAlāʾī zīj* without acknowledgement. This puzzling case of plagiarism, noted by the late Prof. E. S. Kennedy in 1956, is solved by means of a graphical approach, together with some astronomical analysis, in order to (A) visualize the perplexing network of the development of planetary theories in medieval Islam and (B) distinguish the types of relations existing between them. The paper describes the new strategies invented in the Islamic period to modify the available planetary theories in order to reconcile them with the observations, called *istidrāk* and *iʿtibār*, which were different from the standard methods set forth in the *Almagest*. The accusations of fraud are shown to be unfounded. It is shown that Ibn al-Fahhād’s solar and lunar theories and his precessional rate of 1°/66^{y} are taken from the Mumtaḥan tradition; his theories of Jupiter and Saturn were the results of his modifications of ʿAbd al-Raḥmān al-Khāzinī’s corresponding theories in the *Muʿtabar zīj* (1121 CE) on the basis of the results he achieved from his observations; his theory of Mars was constructed on the basis of his observations, independent of any other available theory; and his theory of Venus is taken directly and faithfully from Khāzinī.

Key words: Islamic astronomy, observational astronomy, Abu ʾl-wafāʾ al-Būzjānī, Ibn al-Fahhād.

Resum: Segons l’anònim Zīj Shāmil del segle xiii, les teories planetàries d’Abu ʾl-wafāʾ al-Būzjānī (10 de juny de 940–després del 25 de maig de 997) es van adoptar en el segle xii en *ʿAlāʾī zīj* d’Ibn al-Fahhād, del segle xii, sense reconeixement. Aquest cas desconcertant de plagi, assenyalat pel desaparegut Prof. E. S. Kennedy el 1956, es resol mitjançant una aproximació gràfica, juntament amb algunes anàlisis astronòmiques, per tal de

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(A) visualitzar la sorprenent xarxa en el desenvolupament de les teories planetàries a l'Islam medieval i (B) distingir els tipus de relacions existents entre elles. L'article descriu les noves estratègies inventades al període islàmic per modificar les teories planetàries disponibles i conciliar-les amb les observacions, anomenades *istidrāk* i *iʿtibār*, que eren diferents dels mètodes estàndard exposats a l'*Almagest*. Es demostra que les acusacions de frau són infundades. Es demostra que les teories solars i lunars d'Ibn al-Fahhād i la seva taxa de precessió d'1°/66 foren extretes de la tradició *Mumtaḥan*; les seves teories de Júpiter i Saturn van ser el resultat de les seves modificacions de les teories del *zīj Muʿtabar* de ʿAbd al-Raḥmān al-Khāzinī (1121 dc) sobre la base dels resultats que va aconseguir a partir de les seves observacions; la seva teoria de Mart es va construir sobre la base de les seves observacions, independentment de qualsevol altra teoria disponible; i la seva teoria de Venus es pren directament de Khāzinī.

Mots clau: Astronomia islàmica, astronomia observacional, Abu 'l-wafāʾ al-Būzjānī, d'Ibn al-Fahhād.

1. Introduction

In the modern scholarship on the historiography of medieval Islamic astronomy, Abu 'l-Wafāʾ al-Būzjānī (10 June 940–after 25 May 997), a Persian mathematician and astronomer active in Baghdad,[1] is often remembered in relation to the misinterpretation of a passage in his *Majisṭī* VII.2.10[2] by Louis Pierre Eugène Amélie Sédillot (1808–1875, the son of the French orientalist Jean Jacques Emmanuel Sédillot, 1777–1832), which gave birth to the blatantly erroneous notion that Abu 'l-Wafāʾ had discovered lunar *variation*. The controversy over the issue persisted for four decades, before being finally removed by Carra de Vaux (1867–1953).[3]

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[1] His detailed bio-bibliography, with the main focus on his arithmetic and geometry is A.P. Youschkevitch's 1960 entry in *DSB*, Vol. 1, pp. 39–43, which served as a basis for Yvonne Dold-Samplonius's entry in Selin 2016, pp. 14–17, and the entry in *BEA*, pp. 188–189. See also H. Suter's entry in *EI*$_2$, Vol. 1, p. 159. The principal literature on Abu 'l-Wafāʾ's mathematics after 1960 is: Saidan 1974, Kennedy and Mawaldi 1979, Kennedy 1984, Sesiano 1998, Raynaud 2012, and the references mentioned therein.

[2] Abu 'l-Wafāʾ, *Majisṭī*, ff. 99v–100r.

[3] Sédillot 1845–1849, Vol. 1, p. 42ff; Carra de Vaux 1892. A brief discussion of the controversy is given in Dreyer 1906, p. 252–257 and Neugebauer [1957] 1969, pp. 206–207. On Carra de Vaux's life and works, see Inayatullah 1971.

Another contentious matter involving Abu 'l-Wafā' concerns his alleged planetary theories:[4] according to the anonymous *Shāmil zīj*, a 13th-century *zīj* compiled in northwestern Iran (possibly by Athīr al-Dīn al-Mufaḍḍal b. ʿUmar al-Abharī, d. between 660 H/1263 CE and 663/1265), Abu 'l-Wafā''s solar, lunar, and planetary parameter values were deployed in the *ʿAlā'ī zīj* composed in Persian by Farīd al-Dīn Abū al-Ḥasan ʿAlī b. ʿAbd al-Karīm al-Fahhād of Bākū or Shīrwān (*fl. ca.* 1160–1180 CE, preserved in a single, but incomplete, manuscript in the Salar Jung Library in India, no. H17). Al-Fahhād not only did not acknowledge his source, but also claimed to be responsible for the derivation of the fundamental parameters used therein from his own observations:[5]

> This *zīj* has been laid down on the basis of the mean motions corrected by Abu 'l-Wafā' Muḥammad b. Aḥmad al-Būzjānī and his team by their successive observations and examinations released from them after al-Ma'mūn. The author of *al-Zīj al-ʿAlā'ī* unfairly proclaimed that he observed them by the instruments he acquired, while he did not gain any reputation for his observations. I found a table including (*jadwal^{an} mushtamil^{an}*) these mean motions [viz. those used in both *ʿAlā'ī* and *Shāmīl zījes*] in [one of?] al-Būzjānī's works. Then, I adopted them after I found out, by the observations of conjunctions and the methods of experiment (*iʿtibār*), that they are correct.[6]

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[4] Let us clarify some terminology we will use throughout the present paper: by the term «theory» of a planet, we refer to a set of coherent and unprecedented values for the fundamental parameters of a «model» of a planet (including structural parameters or orbital elements: eccentricity, longitude of apogee, and motional parameters: mean motions in longitude and in anomaly and radix mean positions) measured by an astronomer using any imagined method (e.g., derived from observational data or from the process of modification of earlier theories); the underlying planetary «models» in our context are all Ptolemaic. In the *Almagest*, a single term *ὑπόθεσις* («hypothesis», rendered as *aṣl* in Isḥāq-Thābit's Arabic translation) was used for a building block of a planetary model (e.g., eccentric), for a model of a planet's spatial motion, and for a fundamental parameter value (e.g., an eccentricity value).

[5] Anonymous, *Shāmil zīj*, P1: f. 1v, P2: f. 7r, F1: f. 1v, F2: f. 1v, D: f. 1v, Pa1: f. 2v, Pa2: f. 1v, Pa3: f. 7v, C: f. 1r; for a comprehensive review of the manuscripts of this work consulted, see van Dalen 2021b, pp. 517–522. On the reasons behind the hypothesis of Abhārī as the author of the *Shāmil zīj*, see *ibid*, p. 511. It is worth noting, however, that the authenticity of the *ʿAlāʾī zīj* was never challenged in Abharī's *al-Zīj al-mulakhkhaṣ ʿalā al-raṣad al-ʿAlāʾī* (*The abridged zīj based on the ʿAlāʾī observations*), as its title clearly indicates.

[6] «بعد ما رأيتها مصحّحة بمشاهدة القرانات وطرق الاعتبار», as is in MSS P1 (1273/4 CE, earliest), Pa2 (16th ct.), F2 (1347 CE), and D. An alternative reading is found in MSS P2, F1 (14th/15th), Pa1

It is clear that in identifying the source of his serious allegation of plagiarism against Ibn al-Fahhād, the anonymous author of the *Shāmil zīj* only ambiguously refers to «a table including...», a remark that can be found in all the MSS consulted without exception; it is by no means clear how he could assess the accuracy of a set of solar, lunar and planetary theories against observations only on the basis of their underlying mean motions, without having access to the correlated values for the longitudes of the apogees, the radix mean positions in longitude and in anomaly, and the tables of equations (on the key term *iʿtibār*, see below, Sect. 3.1.2).

We are thus confronted with a puzzling case of plagiarism in the medieval intellectual world. To be sure, plagiarism was as prevalent in the Middle Ages as in our time, but had its own peculiarities. Medieval Islamic authors scarcely identified their sources, except when they were subject to criticism of any sort or when the sources used were manuals widely in use or textbooks broadly known and easily accessible to other scholars in the community. The bor- rowings were sometimes made from various sources, summarized in quite different ways, and were subject to later changes to the extent that their exact sources could only be identified by a well-informed scholar. The examples are too numerous to be listed here, and each and every case is too sophisticated to be concisely summarized.[7] In our case, a famous practical astrono- mer has been clearly accused of plagiarism, and the assumed source, Abu ʾl-Wafāʾ, was equally well known. The story of the fraud was noted in the 1950s by the late Prof. E. S. Kennedy,[8] but the problem remains unresolved.[9] This paper aims to explain it.

The bulk of Abu ʾl-Wafāʾʾs astronomy has come down to us, though incompletely, in his work entitled *Majisṭī* (without the definite article «al-»). The work

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(15th/16th ct.), Pa3 (15th ct.), and C (1711/2 CE) as «بعد ما رأيتها مصحّحة بما شهد[ت]ها[ا] القرانات وطرق الاعتبار» = after I found out that the conjunctions and the methods of experiment gave evidence that they are correct.

[7] In fact a scholar might lift an entire work from another one! For instance, Al-Nadīm (ed. Flügel, Vol. 1, p. 275, ed. Tajaddod, p. 334, En. translation, Vol. 2, p. 654) quotes a piece of writing in which it is claimed that Abū Maʿshar plagiarized some astrological treatises actually penned by Sanad b. ʿAlī who had given them to him in person.

[8] See Kennedy 1956, no. 29 on p. 129 and p. 169 for the diagram exhibiting, in a chronological order, the 13th- and 14th-century *zījes* compiled in northwestern Iran and northern Iraq on the basis of the *ʿAlāʾī zīj*. At the time, the only extant partial copy of this work had not yet come to light. It was later found by Sonja Brentjes.

[9] van Dalen 2021b, pp. 512–513.

is extant in a single incomplete manuscript in the Bibliothèque nationale in Paris (Arabe 2494).[10] The *Majisṭī* is divided into seven «books» (*maqāla*), each further divided into sections called «types/sorts/kinds» (*nawʿ*, Pl. *anwāʿ*), some of which are, in turn, divided into subsections called «chapters» (*faṣl*, Pl. *fuṣūl*). The names of the first two sections are borrowed from the Arabic *Almagest* in both Ḥajjāj and Isḥāq-Thābit's translations. Besides, our author introduces the three main parts of his work at the end of I.1 and I.2, each called «general class/sort» (*jins*):[11] «... each main part includes a number of books and chapters. In the first part, we speak of the topics which must be explained before dealing with the motions of the heavenly bodies, which covers five books. The second part is on the motions of the celestial objects, which are called the motion in longitude and the motion in anomaly. The third part is on the issues which are closely related to the motions of the heavenly objects». He does not specify how many books the last two parts comprise; however, it seems that each consists of a single book. In fact, the matters addressed in the second and third parts agree with the contents of books VI and VII respectively.

Book I is on planar trigonometry, and books II–V are, generally speaking, on spherical trigonometry and spherical astronomy. In particular, Abu 'l-Wafā' gives us some information on the instruments he used in his observations and his measurement of the obliquity of the ecliptic, the latitude of Baghdad and the ecliptical and celestial coordinates of certain fixed stars. Book VI is, on the whole, devoted to the basic hypotheses employed for the construction of the Ptolemaic planetary models, as shown in the list of its contents given in detail on ff. 81v–82r. The first five sections (VI.1 to VI.5) deal with the discussion of the various hypotheses used in the Ptolemaic kinematic models in order to account for the planets' motions in longitude and in anomaly (i.e., the eccentric deferent and the epicycle). VI.6.2, which is the last chapter on the Sun, is left unfinished on f. 93v, and the next leaf starts from somewhere towards the end of VI.9.2. So, VI.7 (on the Moon, including nine chapters), VI.8 (on Venus, consisting of three chapters), the entire VI.9.1, and

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[10] An edition of the *Majisṭī* has recently been published (Moussa 2010; since, thanks to the marvels of the digital era, the single copy is now easily accessible online, the references here are made to the manuscript, rather than to its printed transcript), and some parts of it (e.g., on spherical astronomy) have been studied (see, e.g., Delambre 1819, pp. 156–170; King 2004–2005, Vol. 1, p. 113; Moussa 2011). Abu 'l-Wafā''s empirical findings will come under full scrutiny in a forthcoming paper by the present author (cf. Mozaffari 2023).

[11] Abu 'l-Wafā', *Majisṭī*, f. 2r.

apparently a good deal of the initial part of VI.9.2 are absent from the only surviving Paris MS (the first three chapters of VI.9 are on Mercury, but its fourth chapter, which is the last chapter of Book VI, is a brief summary of the types of the planetary orbs, motions, and inequalities, and the sizes of their eccentricities, the radii of epicycles, and orbital inclinations). Book VII discusses at length the ways through which the inequalities in the tropical and anomalistic motions of the Sun, Moon, and five planets were discovered and how they are accounted for by means of the hypotheses explained in book VI.

It comes as a surprise that our author's famous contemporary bio-bibliographer, al-Nadīm, does not mention the *Majisṭī* among his works; rather, he refers to other two works of his, the *Kitāb al-kāmil* (*Complete/Perfect book*) and the *Wāḍiḥ zīj* (*Clarifying zīj*), and, surprisingly, gives the same list of contents for both, which also coincides entirely with the three main parts of the *Majisṭī* mentioned above.[12] Accordingly, it seems that all three titles refer to one and the same work, which is seemingly called a *zīj* for its astronomical tables, named a *complete/perfect* work for its comprehensibility, and entitled a *Majisṭī*,a simple designation indicat- ing that the work in question resembles Ptolemy's *al-Majisṭī* (*Great Syntaxis*) in genre, topics, and volume, among other aspects.

The *Majisṭī*, in the partial form that has come down to us, contains limited information on Abu 'l-Wafā''s empirical findings and novel achievements in the field of observational astronomy. They include the three meridional instruments he used, the results of his measurements of the annual noon-altitudes of the Sun (Max = 80;10° and Min = 33;0°), from which he determined the obliquity of the ecliptic and the latitude of Baghdad respectively as $\varepsilon = 23;35°$ and $\phi = 33;25°$ (II.2.1–3),[13] a passing remark on his solar observations (VI.1),[14] and his stellar

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[12] al-Nadīm, ed. Flügel, Vol. 1, p. 283, ed. Tajaddod, p. 341, En. translation, Vol. 2, p. 667–668.

[13] Abu 'l-Wafā', *Majisṭī*, ff. 19r–20r. It is worth noting that the values for ε and φ were not new in Abu 'l-Wafā''s time. According to Bīrūnī, $\varepsilon = 23;35°$ was the result of the Mumtaḥan observations, carried out by Khālid b. ʿAbd al-Malik al-Marwarūdhī, in Damascus (Bīrūnī 1954–1956, Vol. 1, pp. 363–364; Bīrūnī, *Taḥdīd*, pp. 90–94, English translation, pp. 60–64, E.S. Kennedy's commentary, pp. 32–39), which was also obtained or used later by al-Battānī (E: f. 178r) and others. $\varphi = 33;25°$ for Baghdad was derived by the Banū Mūsā from their 862–863 CE measurements of the extremal altitudes of three circumpolar stars of the Ursa Major, as reported by Bīrūnī (*Taḥdīd*, pp. 66–67, English translation, pp. 37–38, E.S. Kennedy's commentary, pp. 16–18).

[14] Abu 'l-Wafā', *Majisṭī*, f. 97r.

observations: Capella (V.1.2,[15] V.5,[16] and V.3.4[17]), Deneb, Altair (V.4),[18] and Mizar (V.14).[19] His observational data related to the Sun (which he used to determine its orbital elements) and Regulus have been preserved in the works of his outstanding junior contemporary and correspondent, Abū al-Rayḥān al-Bīrūnī (973–1048 CE) in his *al-Qānūn al-masʿūdī* and *Taḥdīd* (see below, Sect. 2.2).[20]

None of Abu 'l-Wafā''s planetary observations and measurements have been preserved in the extant copy of the *Majisṭī* or by Bīrūnī. After his latest recorded stellar observation in 977 CE,[21] he still had two decades ahead of him to continue his observations and extend them to the planets. Of course, for the last five years of his life, he appears to have been quite busy writing his *On the Geometrical Constructions Necessary for the Craftsman*.[22] A thorough reading of books VI and VII, which mainly contain a recapitulation and rephrasing of Ptolemy's statements in the *Almagest*, does not give any impression that he had achieved anything significant in the field of planetary astronomy. Moreover, we know that Bīrūnī deployed Abu 'l-Wafā''s few stellar observations together with his own in the derivation of the rate of precession, and considered Abu 'l-Wafā''s solar observations among others from his Islamic predecessors, but in the case of planetary astronomy he had to modify Ptolemy's theories in a purely artificial way in order to obtain a new set of parameter values.[23] It does not seem unreasonable to assume that if any planetary theory from his remote colleague and correspondent was at his disposal, he would have been the first to use them.

It is almost certain that even though Abu 'l-Wafā' carried out planetary observations/measurements, either they were not intended to measure their structural parameters (eccentricities and epicycles' radii), or he did not obtain any value for them other than those measured by Ptolemy some 850 years before him, because

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[15] Abu 'l-Wafā', *Majisṭī*, f. 68v.

[16] Abu 'l-Wafā', *Majisṭī*, f. 75v.

[17] Abu 'l-Wafā', *Majisṭī*, f. 75r.

[18] Abu 'l-Wafā', *Majisṭī*, ff. 75r–v.

[19] Abu 'l-Wafā', *Majisṭī*, f. 80v.

[20] For Abu 'l-Wafā''s solar and stellar observations, see Mozaffari 2023, sections 2 & 3.

[21] See Mozaffari 2023, section 3.3.

[22] Raynaud 2012, p. 35.

[23] Already explained in Mozaffari 2017, p. 13. Bīrūnī was suffering from a serious eye disease, as he clearly states in the *Taḥdīd*, due to looking at solar eclipses in his youth (Bīrūnī, *Taḥdīd*, p. 168, English translation, pp. 131), which might have been a justified excuse for him not to engage seriously with planetary observations.

the values mentioned for the parameters of this sort in VI.9.4[24] are all Ptolemaic; the sole exception is the double eccentricity of Venus (i.e., the distance between the equant point and the terrestrial centre in the Ptolemaic model), for which, like the majority of the medieval astronomers, he gives his value for the solar eccentricity, i.e., $2;5^p$ (see Sect. 2.2). It is not known whether, like other early Islamic astronomers, he followed the Indian Midnight System in maintaining the apsidal lines of the Sun and Venus coincident on each other. (His contemporary, Ibn al-Aʿlam, d. 985 CE, was seemingly the first Islamic astronomer to reject that erroneous hypothesis, and kept Venus' apogee behind the solar one in longitude, as in the *Almagest*).[25] Of course, the decision to maintain Ptolemy's values for the planetary structural parameters is no surprise at all (cf. Sect. 3.1.2). For the orbital inclinations, the spaces for writing down the values for Mars, Venus, and Mercury are left blank, but for Saturn and Jupiter, we are given, respectively, 2;26° and 1;24°, which are, also, the values determined by Ptolemy in *Almagest* XIII.3; however, «for the sake of what is more appropriate» (*διὰ σύμμετϱον*), the Alexandrian astronomer preferred to work with the round numbers 2;30° and 1;30°.[26]

Nevertheless, there is nothing certain in the case of the mean motions. A list of the mean daily motions in longitude and/or in anomaly of the Sun, Moon, and the planets, as attributed to him, are given in the marginalia of the Berlin copy of a recension of Ḥabash al-Ḥāsib's *zīj* (d. after 869 CE). The value for the solar mean daily motion/the length of the solar year, , which is absent in Abu 'l-Wafā''s *Majisṭī* and Bīrūnī's works, will be discussed in Sect. 2.2 along with the former's observation of the autumnal equinox of 974 CE. The mean motion values for the Moon and planets will be discussed in Sect. 4, where we will see that an unexpected problem emerges because the mean motions in anomaly of the two inner planets are very nearly identical with those adopted by Muḥyī al-Dīn al-Maghribī (d. 1283 CE) in his last *zīj*, *Adwār al-anwār*,which was written on the basis of his extensive,

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[24] Abu 'l-Wafā', *Majisṭī*, f. 95r.

[25] See Mozaffari 2019a.

[26] Toomer [1984] 1998, p. 605, and the latitude tables in *Almagest* XIII.5: p. 632 (opposite the arguments of 90°). I owe the correct translation of the quoted expression to Alan Bowen (private communication). G.J. Toomer translates it as «to achieve greater symmetry». The two extant ninth-century Arabic translations are, also, inaccurate as regards this expression: Ḥajjāj has *min ajl ḥusn al-taqdīr*, «for the sake of good estimation» (LE: f. 202v), and Thābit-Isḥaq: *li-l-tashīl fī al-ʿamal*, «in order to facilitate the procedure/operation» (S: f. 195r, PN: f. 163r, TN: f. 220v, LO1: f. 207r). The translators actually tried to guess the reason for Ptolemy's rounding here.

purposeful observations (certainly, in the case of the Sun, Moon, and superior planets) carried out at the Maragha observatory in the 1260s and the 1270s.

Accordingly, because of the lack of sufficient evidence and in order to avoid subjectivity, we do not address the questions of «whether Abu ’l-Wafā’ conducted a program of any type to cope with planetary observations» or «whether the values attributed to Abu ’l-Wafā’ in the scattered glosses in the Berlin MS of Ḥabash’s *zīj* are his own». Rather, we will address the problem by finding the roots of al-Fahhād’s planetary theories among those established by his outstanding predecessors. If it becomes possible to prove that a theory of al-Fahhād (1) is *independent* of other known ones, (2) is *adopted intact*, with a high degree of certainty, from a known one, or (3) is a *modified* version of a known one, then we can rule out the possibility of his dependence on Abu ’l-Wafā’’s alleged theories. As we will see in Section 3, the problem will be resolved much more easily than can be conceived *prima facie* by means of a technical approach to visualizing and uncovering the correlation and interdependence of the medieval Islamic planetary theories.

The main features of Ibn al-Fahhād’s work and career and its profound influence on medieval Middle Eastern astronomy in the generations that followed him were highlighted earlier in our 2019 study, together with an assessment of his observation report of the 1166 conjunction between Jupiter and Saturn.[27] He was a prolific astronomer who composed four *zījes* on the basis of al-Battānī’s (d. 929 CE) *Ṣābiʾ zīj*, one on the basis of ʿAbd al-Raḥmān al-Khāzinī’s (*fl.* 1100–1130 CE) *Mutabar zīj* (completed about 1121 CE),[28] and his last work, *ʿAlāʾī zīj*, on the basis of his own observations, as he emphasizes in its prolegomenon. Two additional points are worth making here. He was, quite probably, based in Shamākhī, in the northern Azarbāijān state of Iran, the capital of the Shirwānshāhān dynasty at the time.[29] In MS Oxford, Bodleian Library, Thurston 3, which is a highly coherent treasure

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[27] See Mozaffari 2019b, esp. pp. 525–526, 531–542.

[28] See Mozaffari 2022.

[29] Of the coordinates given for Shamākhī in the geographical table in the *ʿAlāʾī zīj* (p. 222), latitude $\phi = 41;0°$ (**40;38°**) N is encountered in the worked examples in the canons (e.g., in I.44&45: pp. 44–45), and longitude $L = 83;55°$ E from the Fortunate Isles (the Canaries) is too close to the adopted base meridian of 84° E. It is worth noting that Shirwān was not a particular locality, but the easternmost region between the Caucasus Mts. and the Kura River to the west of the Caspian Sea, while some *zīj*es (see van Dalen 2021b, pp. 531, 556) give it specific coordinates (maybe confusing it with a city under the same name in Khurāsān).

trove of principal technical treatises and tracts assembled through fixed patterns with a clear intellectual intention and copied by a certain Suhrāb b. Amīr al-Ḥājj, a pupil of Quṭb al-Dīn al-Shīrāzī (d. 1311 CE), during the 1260s–1270s CE,[30] there is a tract (ff. 116r–v) in which we are told, through the medium of Quṭb al-Dīn, that a manuscript of the*ʿAlā'ī zīj*, copied by Ḥusām al-Dīn ʿAlī b. Faḍl-Allāh al-Salār (the author of the *Shāhī zīj*, upon which Sayf al-munajjim based the main solar, lunar, planetary mean motion/position tables in his *Ashrafī zīj*, composed about 1300 CE), was available to Naṣīr al-Dīn al-Ṭūsī (1201–1274 CE). (It is worth noting that the theories of Saturn and Mercury in the *Īlkhānī zīj* are based upon the *ʿAlā'ī zīj*).[31]

2. The Sun

2.1. *Ibn al-Fahhād*

Two passages in the prolegomenon to the *ʿAlā'ī zīj* clearly show that Ibn al-Fahhād was well acquainted with the solar theories established by the Mumtaḥan team, al-Battānī, Ibn al-Aʿlam and al-Khāzinī, as he discusses at some length the differences between them in the prediction of the year's horoscope.[32] Of these theories, he gives preference to the first. As we have shown in our comprehensive 2018 study on the solar theories established in medieval Islam, there is a substantial difference between the solar theory established by Ibn al-Aʿlam and the three others. The first is categorized as a theory of Type II, i.e., independent of Ptolemy's erroneous equinox timings, and hence shows a steady and stable behavior, and also remain tolerably accurate even for some centuries after its introduction (Ibn al-Aʿlam's mean solar motion is so accurate that the error in the mean longitude of the Sun amounts to a single minute of arc in 770 years),[33] while the three

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[30] Some notes on this codex are made in Lorch 1975, p. 99; 1980, pp. 297 and 301; and Morelon 1987, pp. 301–302. A detailed survey is found in J. Bellver's entry in the PAL: ptolemaeus.badw.de/ms/672; see, also,

www.fihrist.org.uk/catalog/manuscript_1807.

[31] The theories of Venus and Jupiter in it are borrowed from Ibn al-Aʿlam; see below, Figures 2, 3, and 5, which will be explained later in Sect. 3.1.

[32] See Mozaffari 2019b, pp. 524 (paragraph **[II]**), 526–527 (paragraph **[VI]**).

[33] Mozaffari 2018a, p. 220.

others are of Type I, i.e., dependent upon Ptolemy’s egregiously erroneous equinox observations, whose errors accumulate to sizable positive amounts within only a few decades.[34] In our 2019 article on Ibn al-Fahhād, the errors in the true longitudes of the Sun as computed on the basis of these four theories in Ibn al-Fahhād’s time (for three decades and a half elapsed after 1150) are plotted, clearly showing their substantial differences.[35] Here, we plot the errors $d\bar{\lambda}$ in the mean longitudes of the Sun on the basis of these four theories (see Figure 1(a)). It can be readily seen that the errors in the case of the Mumtaḥan (Mt/Hb) and al-Khāzinī’s (Kh) theories are so intrinsically entwined that, actually, no distinct and mean- ingful difference can be discerned between them. In fact, Khāzinī’s values for the mean longi- tude of the Sun are almost identical to those in the Mumtaḥan solar theory: his value for the mean daily longitudinal motion of the Sun is only about 0;0,0,0,1,22°/d less than that deployed in the latter (see Appendix), and his value for its radix mean longitude at the Hijra epoch (16–7–622, JDN 1948440), as adapted to the meridian of Marw (L = 96;30° from the Fortunate Isles), is 116;1°, which is less than 0;2° more than the value of 115;59,30° from the Mumtaḥan theory.

Despite the clear evidence (from a modern-day point of view) in the graphs, determining the error in a solar theory and recognizing the substantial difference between two solar theories of the two types were by no means easy problems, as many factors could contribute to making a wrong estimate and blurring the clear-cut distinction between them. As the Sun’s declination (thus, its meridian altitude) changes by only 24’ between two successive days sur- rounding the occurrence of an equinox, the prime requirements for reaching these goals were having access to an accurate meridional instrument (solstice armilla or mural quadrant), grad- uated to every minute of arc, and having a sufficiently accurate value for the latitude of the location. For example, by a mural quadrant of 9 cubits diameter (≈ 450 cm), Bīrūnī measured the solar noon-altitude at the autumnal equinox of 1019 in Ghazni, on the basis of which he both detected the serious error in the Mumtaḥan solar theory and established his solar theory;[36] and it was at the Maragha observatory, equipped with a mural copper quadrant with a radius of over 300 cm,[37] that the high accuracy of Ibn Yūnus’ solar theory was confirmed. Not all of the famous

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[34] See Mozaffari 2018a.

[35] See Mozaffari 2019b, figures 1–4 on pp. 528–529.

[36] Bīrūnī 1954–1956, Vol. 2, p. 647.

[37] See Mozaffari 2018b, pp. 616f.

figures of Islamic astronomy were able to accomplish the same task (e.g., Kushyār b. Labbān adopted al-Battānī's theories in his *Jāmiʿ zīj*,[38] while the errors in the latter's solar theory had already risen to +1/4° in the mean longitude, corresponding to −1/4 of a day in the equinox times, about 1000 CE; cf. the curve Bt in Figure 1(a)). Other conditions had to be fulfilled to detect that something had gone wrong with Ptolemy's equinox observations, which could then pave the way for discovering the difference between the two types of solar theories. When we speak of the two types of solar theories and the gross accumulated longitudinal differences between them over a long period, we should bear in mind that there is a subtle difference in the length of the solar year between the most erroneous solar theories of Type I and the most accurate one of type II, not exceeding ±3 minutes.[39] Bīrūnī detected the blatant errors in the Mumtaḥan solar theory and reported them several times in his works,[40] but did not identify their principal source, as he constructed his own solar theory on the basis of Ptolemy's equinox times.

There might have been a variety of reasons why and how Ibn al-Fahhād reached the false opinion that the old Mumtaḥan solar theory was correct for his time and superior over the three others (especially, Ibn al-Aʿlam's).[41]

[38] See van Dalen 2021a, p. 373 & *passim*.

[39] See Mozaffari 2018a, table 5 on p. 197 and table 11 on p. 217.

[40] The reference to Bīrūnī's *al-Qānūn* in connection with the summer solstice of 988 and the autumnal equinoxes of 1016 and 1019 was made in Mozaffari 2018a, pp. 228–229, table 11 on p. 217, and figure 18 on p. 236; see, also, Bīrūnī, *Taḥdīd*, pp. 129–130, English translation, p. 96, E.S. Kennedy's commentary, p. 67.

[41] A small fragment of the Book IV of the *ʿAlāʾī zīj* (in two pages) is extant in MS. Iran, Parliament Library, no. 184, ff. 243v–244r, in which it is explained that, by making observations in 564 H (4/5 October 1168–23/24 September 1169) / 539 Y (2 February 1170–1 February 1171), our author measured the lengths of the spring and summer, and then derived the same value for the eccentricity as adopted in the Mumtaḥan solar theory and a value for the longitude of the apogee which is in accord with the one employed in the Mumtaḥan theory and the rate of precession of 1°/66^{y} (for details, see Mozaffari 2013, Part 1, table 1, no. 8, on p. 322 and pp. 328–329, 2017, pp. 16–18). Since we are not given the data of the purported equinoxes and summer solstice observations, we cannot say how he measured their times. Generally speaking, in view of the fact that his values of 39;5° and 41° for, respectively, the latitudes of Bākū (his native city, as his *nisba* indicates?) and Shamākhī, as given in the geographical table in the *ʿAlāʾī zīj* on p. 222, are, respectively, more than −1° and about +1/3° in error, it is doubtful that he had assessed the accuracy of the four already-mentioned solar theories by the effective, rigorous method of the direct measurement of the Sun's noon-altitudes.

In our 2019 study just mentioned, we discussed in detail that, insofar as synodic phenomena were the main focus of interest, there was a vast potential for errors in underlying theories employed for the prediction of their circumstances (timings, magnitude, etc.), and it was quite possible that the accidental coincidence between theoretically predicted and actually observed circumstances was nothing other than a quaint illusion produced by a cancelation of errors of the same size and sign in underlying theories of two heavenly bodies involved in a specific phenomenon. These could thus mislead a practitioner into drawing false conclusions about the sources of errors in a specific theory or the level of accuracy achieved in one in comparison with another.[42] Solar and lunar eclipses are synodic phenomena related to the Sun, and we convincingly demonstrated in that study how the times of the solar and lunar eclipses that Ibn al-Fahhād observed in April 1176 as derived from the four solar theories mentioned earlier (and their correlated lunar theories) were so close to each other that, in fact, this blurred the distinct difference between them in the accuracy of the ecliptic positions calculated for the Sun and Moon.[43] Also, we will see in Section 4.1 that an anonymous astronomer predicted the lunar eclipse of 6/7 February 1422 on the basis of the Mumtaḥan solar and lunar theories (some 600 years after the Mumtaḥan period!), and obtained a good agreement both in magnitude and in timings with his observation, and reported them in detail in a comment inserted on the title page of the Berlin MS of Ḥabash's *zīj*.[44]

A second likely reason for Ibn al-Fahhād's adoption of the Mumtaḥan solar theory has emerged in the present study in connection with the assessment of his statement in the prologue of his *zīj* that he evaluated Ibn al-Aʿlam's theory of Venus many times on the occasions of the planet's near appulse to Regulus, and that he had found a good agreement between theory and observations; in fact, he adopted al-Khāzinī's theory of Venus, solely with an improvement of the eccentricity of the planet. As will be shown in Sect. 3.2.2, Ibn al-Aʿlam's values for the

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[42] This does not in itself mean that all fair agreements between theory and observation in the medieval period (either loudly announced in texts, or found by a thorough technical analysis) were acquired accidentally (or, if they were, they were due entirely to the same cause as mentioned); rather, it does not justify us in discarding other possible reasons.

[43] See Mozaffari 2019b, pp. 542–546.

[44] A good number of observational records of this kind can be found here and there in the Islamic astronomical corpus, either preserved in marginal glosses or surviving in worked examples in the canons of *zījes*, which await an in-depth analysis.

longitude of Venus in its close approaches to Regulus in eastern elongations suffer from large negative errors, while the errors in the values extracted from al-Khāzinī's theory on the same occasions are significantly less. This could give Ibn al-Fahhād a strong (though mistaken) rea- son to believe that al-Khāzinī's values for the mean longitudes of Venus (i.e., the mean longi- tudes of the Sun) are correct, which are, in turn (as is apparent in Figure 1(a)), very nearly equal to the mean solar longitudes in the Mumtaḥan theory (see conclusion **(II)** at the end of Sect. 3.2.2).

2.2. *Abu 'l-Wafā'*

No useful data can be found on Abu 'l-Wafā''s solar theory in the surviving copy of his *Majisṭī*. The numerical details in connection to his solar observations and measurements have come down to us through the medium of Bīrūnī. Abu 'l-Wafā' measured the lengths of the seasons spring and summer in 343 Y (363 H/974 CE), respectively, as 93;30,8 and 93;7,10 days, days, on the basis of which he reached the eccentricity $e \approx 2;4\,{}^{1}/_{2}{}^{p}$ and the longitude of the apogee $\lambda_A \approx 84\,{}^{1}/_{2}{}^{\circ}$.[45] He established his final solar theory two years later, in 345 Y (365 H/976 CE), by means of the mid-seasons method: he measured the intervals of time between the mid-winter and mid-spring, and from the latter to mid-summer, which yielded, respectively, 91;34,25 and 94;9,7,30 days, from which he derived $e = 2;5^{p}$ and $\lambda_A \approx 84\,{}^{2}/_{3}{}^{\circ}$.[46] He estimated the time of the occurrence of the autumnal equinox of 974 as 3 hours after the beginning of [*viz.*, sunrise on] Friday, 30 Pachon [9] 1722 Nabonassar[47] (18 September 974; modern: **9:18** Baghdad LT, i.e., with an error of only $-{}^{1}/_{3}{}^{h}$).

His first value for the eccentricity is very close to the Mumtaḥan value ($2;4,35^{p}$), and also, his second value for λ_A is not far from the Mumtaḥan values at the given intervals of time (~ 84;52° ±1'). Nonetheless, the error in the Mumtaḥan solar

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[45] Bīrūnī, *al-Qānūn* VI.7: 1954–1956, Vol. 2, pp. 654–655; Mozaffari 2013, Part 1, p. 322.

[46] Bīrūnī, *al-Qānūn* VI.7: 1954–1956, Vol. 2, p. 658; Mozaffari 2013, Part 1, p. 326.

[47] Bīrūnī, *Taḥdīd*, p. 301, English translation, p. 270, E.S. Kennedy's commentary, p. 228; *al-Qānūn* VI.6: 1954–1956, Vol. 2, p. 640. It is not known why Bīrūnī, in his *Taḥdīd* (p. 100, English translation, p. 69, E.S. Kennedy's commentary, p. 43), only gives the years 365–366 H as the period of Abu 'l-Wafā''s solar observations, which is the only period mentioned by Abu 'l-Wafā''s in his *Majisṭī* in connection with his observations of Capella (V.1.2: f. 68v, V.3.4: f. 75r).

theory at the autumnal equinox of 974 reached about +0;11° in longitude, corresponding to ~ $-4^{1}/_{2}{}^{h}$ in time, principally because of the error in the basic value for the length of the solar year/the Sun’s mean motion. Such a considerable discrepancy between theory and observation was so crucial a problem in the context that it is inconceivable that it escaped Abu ’l-Wafā’’s notice. Accordingly, we can plausibly rule out the possibility that he blindly adopted the Mumtaḥan theory. Rather, it seems reasonable to assume that he both measured a new value for the length of the solar year and fixed his own radix value for the solar mean longitude on the basis of his observation of the autumnal equinox of 974 (a value of about 182;6°, in contrast to the Mumtaḥan 182;17°, at noon on the given date).

Neither in the surviving MS of Abu ’l-Wafā’’s *Majisṭi*, nor in Bīrūnī’s works, are we told anything about the former’s value for the mean solar motion in longitude/the length of the solar year. A value for the Sun’s mean daily motion attributed to him has been preserved in the marginalia of the Berlin MS of Ḥabash’s *zīj*, along with the lunar and planetary mean motions attributed to him (which we will discuss later in Sect 4.2):[48]

0;59,8,20,**43**,17,38,41,42,25,0 °/d [or: 13]

On the one hand, we do not know precisely whether it actually belongs to Abu ’l-Wafā’, but, on the other hand, there is no reason for doubting the attribution. It is unclear whether the fourth sexagesimal fractional digit should be read as 43 or 13; neither value is close to other ones we know from the medieval Islamic period. If the first is correct, the value corresponds to a length of the solar year, T_y, of about 365;14,26 days, and if the latter is the case: $T_y \approx 365;14,29$ days. Regardless of this difference, it is obvious that he established a solar theory of Type I, although, needless to say, the second value is superior to the first one. The substantial difference between the two values is: if we use the first in combination with the only surviving autumnal equinox from him, computing back in time, we reach the mean longitudes, *nearly*, equal to those resulting from the Hipparchian/Ptolemaic solar theory in Ptolemy’s time; that is, Abu ’l-Wafā’’s solar theory is dependent, in one way or another, upon Ptolemy’s solar observations;[49] it is, also

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[48] Ḥabash, *Zīj*, B: f. 29v.

[49] The reason why a medieval Islamic solar theory in whose construction Ptolemy’s equinox observations were certainly used does not *exactly* produce Ptolemy’s mean longitudes for the dates of his observations (as can be seen in Figures 1(a) and 1(b)) simply lies in the difference in the solar

(perhaps, by mere coincidence), in accordance with the erroneous vernal equinox observation come down to us from Yaḥyā b. Abī Manṣūr (17 March 830, 02:00 Baghdad LT, error ~ +4 hours).[50] However, pairing the second value with Abu 'l-Wafā''s preserved autumnal equinox is in accord with the use of one of the relatively accurate equinox observations surviving from other ninth-century astronomers working in Baghdad and Damascus (the one made in Baghdad by an anonymous astronomer on 19 September 831, estimated time: 19:00 LT, error ~ $+1{}^{1}/_{2}{}^{h}$),[51] and the solar theory is independent of Ptolemy's equinox observations. See Figure 1(b).

In the very last sentence of his *Majisṭī* VI.9.2 (which is the second of the three chapters that comprise his lengthy discussions on the planet Mercury), he explicitly states that the motion of its apogee is, «approximately, equal to 1° in 75 years» (the lower boundary of all values measured for the precessional/apogeal motion in the medieval period).[52] We have shown elsewhere a possible way through which he could have obtained this specific value for the rate of precession.[53] In any event, it is different from the Mumtaḥan apogeal/precessional rate of $1^{\circ}/66^{y}$ (which constitutes the upper limit of all values measured for the linear precessional motion in the medieval period) adopted by Ibn al-Fahhād.

To sum up at this stage, it is extremely unlikely that Abu 'l-Wafā' adopted the Mumtaḥan solar theory; in all probability, he made one of his own. So, it is easy to deduce that Ibn al-Fahhād did not depend upon him at all in this respect, simply because, as explained in the preceding section, he used the Mumtaḥan solar theory.

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eccentricity between Hipparchus'/Ptolemy's value of $2;30^{p}$ and the medieval values, mostly, in the range of $2;4^{p}$–$2;19^{p}$.

[50] Yaḥyā's observations of the equinox are more accurate than Ptolemy's, but inferior to those made by other ninth-century astronomers working in Baghdad and Damascus (see Mozaffari 2018a, table 10 on p. 216).

[51] Quoted in the *Fī sanat al-shams* / *De anno solis* attributed to the Banū Mūsā or Thābit b. Qurra (Carmody 1960, p. 68; Neugebauer 1962, p. 269; Morelon 1987, pp. 32–33; see, also, Said and Stephenson 1995, esp. p. 128).

[52] Abu 'l-Wafā', *Majisṭī*, f. 94r.

[53] Mozaffari 2023, section 3.5.

3. PLANETS

In order to inspect whether Ibn al-Fahhād's theories for the longitudinal motions of the planets are his own or not, a reliable basic approach will be to search for their roots in the complicated web of the growth and development of planetary astronomy in the medieval Middle East. If a certain origin for Ibn al-Fahhād's theory for a planet is discovered (i.e., either it is firmly es- tablished that it is his own, or a direct relation between it and another one comes to light), one can reasonably rule out the possibility of any unknown source, and the problem will then be resolved in his favor. But, on the other hand, the absence of any background among the known sources and material in the Islamic astronomical corpus for Ibn al-Fahhād's theory for a planet may strengthen the possibility of the intervention of a lost, unknown, or incompletely preserved origin, and then, we can give more weight to the claim made by the anonymous author of the *Shāmil zīj* that Abu 'l-Wafā''s *Majisṭī* was Ibn al-Fahhād's ultimate source. These matters will be elucidated progressively in what follows.

3.1. The development of medieval Islamic planetary theories

The planetary theories established in the medieval Islamic period can be readily presented and compared with each other in a pictorial form in order to facilitate a thorough evaluation of them by means of plotting the errors in their mean longitudes or mean anomalies for a long interval of time, covering the ancient and medieval periods. This strategy comes in useful in unveiling and clarifying the «interconnection» between them and, sometimes, in detecting their origins. When a good number of supposedly original theories are considered in such a coherent and systematic way, it is possible to get a grasp of the intellectual reality behind the relations between them, and the sophisticated underlying order of their growth and development, and to outline — or, at least, to make plausible conjectures about — how they were laid down one after another.

Figures 2 to 5 show the errors $d\bar{\lambda}$ for the superior planets and $d\bar{a}$ for Venus in the most important theories established in medieval Islam from –300 to 1700. (Mercury has been ignored, because of the large errors in its mean anomalistic motion in the Ptolemaic and medieval theories). The historical values for the mean motions, radix positions, and epochs are summarized in the Appendix. The modern theory employed is the Jet Propulsion Laboratory Development Ephemeris 406 (JPL DE 406).[54] The graphs in each figure, although *prima facie* resembling a confusing spider web, vividly illustrate and visualize a network of development. In order to keep the graphs as simple as possible and to avoid any confusion due to the overcrowded mélange of curves, we take into account only simple polynomial terms in the modern formulae for the computation of the planets' mean positions (which can be considered the linear functions of time, as the coefficients in terms containing higher powers of time are very small), and disregard periodic perturbations (which are sizable in the case of the two giant planets). Accordingly, the graphs are used neither for determining the accuracy of medieval theories nor for dating them, but only for showing the relations between them. The inset figures are to help the reader find the graphs directly pertinent to our discussions on Ibn al-Fahhād, as will be clear in the rest of the paper.

Before proceeding to the problem of Ibn al-Fahhād, we should explain some fundamental points in order to avoid confusion in the subsequent discussions.

3.1.1. Origins

In order to secure the derivation of valid and reliable values for the mean motions, the medieval Islamic astronomers had to make use of one of the two sets of data given in the *Almagest* for the initial values to be sufficiently distant in time from them:

(1) Ptolemy's observations carried out in the 130s (marked as the open circles in the graphs related to his theories in Figures 2 to 5). For instance, Ulugh Beg's Jupiter graph in Figure 3 precisely intersects Ptolemy's at the time of the latter's third opposition of the planet to the mean Sun (on 8 October 137, JDN 1771378, at 5 a.m.), which he used in order to derive his own value for the mean motion in longitude of the planet. So, it is evident that the Samarqand astronomers used that observation for the purpose of deriving the mean motion of Jupiter; in fact, their theory gives the value of 11;36° for the given date and time, as adapted to the meridian of Alexandria, in excellent agreement with Ptolemy's figure.

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[54] Standish 1998.

The ancient observations performed in the third century BCE, which Ptolemy used for the purpose of deriving his own mean motions (indicated as the open circles along Ptolemy's graphs).[55] However, the mean positions associated with this set of observations are none other than what Ptolemy derived from them on the basis of his (in fact, Hipparchus') solar theory and rate of precession. These observations are usually the near appulses of the planets to some fixed stars. Ptolemy calculates the longitudes of the stars at the times of the observations in question from his star catalogue (inherited, at least partly, from Hipparchus) with using the rate of precession of $1^\circ/100^{y}$, which then serve as the true longitudes of the planets at those times. Then, from the solar theory and his values for the structural parameters (eccentricity and longitude of apogee) of the planets, as already determined from his trio analyses, he derives the mean positions in longitude and in anomaly at the times. Therefore, the results can by no means be treated as the «original» ancient data, even though some of the Muslim astronomers considered them in this manner. A prominent example in this regard is Ibn al-Shāṭir's determination of the rate of the motion of planetary apogees, according to our reconstruction in a previous study.[56] Another notable example that can be readily seen is Ibn Yūnus' theory of Jupiter (cf. Figure 3); his graph intersects Ptolemy's very close to the time of the ancient observation (4 September 241 BCE, JDN 1633645, at dawn); actually, computing back in time, Ibn Yūnus' theory gives $\bar{\lambda}$ = 82;58° = 82;58° for the mentioned time, in good agreement with (in fact, less than one day off from) Ptolemy's $\bar{\lambda}$ = 82;58° = 82;54°.

Yet, a third origin to be used as the initial value by a medieval Islamic astronomer from the tenth century onwards is also conceivable:

(2) The abundant ninth-century planetary observations made in Baghdad (and Damascus and Samarra?), only a minor portion of which has come down to us through the medium of Ibn Yūnus' *Hākimī zīj*. An interesting example is related to Ibn al-Aʿlam's theory of Jupiter. Its graph closely approa-

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[55] *Almagest* IX.10 (Mercury), X.4 (Venus), X.9 (Mars), XI.3 (Jupiter), and XI.7 (Saturn): Toomer 1984, pp. 461–467, 474–479, 502–504, 522–525, 541–543.

[56] See Mozaffari 2017, pp. 21–27.

ches that of Ptolemy's corresponding theory and intersects it nearly at the beginning of the Common Era; so, it seems *prima facie* that he used Ptolemy's data, but his theory cannot produce Ptolemy's value at the time. Rather, his theory is in line with an observation made by Ḥabash as preserved in Ibn Yūnus' *zīj*: the conjunction between Jupiter and Regulus on 6 September 864:[57]

> «I observed Jupiter in conjunction with Regulus on Wednesday, the last day of Rajab in the year 250 of al-Hijra, which is the 21st day of the month of Murdādh [i.e., the 5th month] in the year 23*8* [read: 233] Yazdigird, and the sixth day of *Aylūl* [i.e., the 12th month] in the year 1175 Alexander. Jupiter was slightly to the north [of Regulus]». He [*sc.* Ḥabash] said: «I have computed [the longitudes of] the two [celestial objects], and found Jupiter in Leo 14;18°. Thus, it is necessary to subtract 47' from the mean longitude of Jupiter».

We defer a complete analysis of this observational record to a later date. It suffices to say here that Ḥabash arrived at the result that, in order to reconcile theory and observation for this specific event, the mean longitude of Jupiter as calculated from the *Mumtaḥan zīj* must be decreased by 0;47°. The observation would have been made in the morning, between 3 and 5 a.m. (Baghdad Local Time) on the given date, for which the *Mumtaḥan zīj* gives a value of 124;34°, which minus 0;47° reaches 123;47°. The latter figure differs very slightly from the value of 123;50° Ibn al-Aʿlam's theory gives for the same time (the modern value is about **123;44**°). This good agreement does not seem to be due to a mere coincidence; very likely, the result Ḥabash achieved from this observation was among the empirical evidence Ibn al-Aʿlam deployed to establish his theory of Jupiter. Supporting evidence for his use of the ninth-century observations comes from the procedure he used to attain his value of 1°/70^{y} for the rate of precession: namely, by comparing his observation of Regulus carried out in 975/6 CE with the value given in the Mumtaḥan star table dated to 829 CE.[58] It is really a pity that the bulk of the precious collection of early Islamic observation reports have been lost forever.

Therefore, one must be cautious when speculating on the origins and initial data of a specific planetary theory. The mean position error graphs for the three superior

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[57] Ibn Yūnus, *Zīj*, L: p. 108; Caussin 1804, pp. 155–156; Delambre 1819, p. 87.

[58] Mozaffari 2016–2017, pp. 81–82, 84.

planets are clustered around Ptolemy's in the second century CE, which indicate that the Islamic astronomers mostly employed Ptolemy's observations in their derivation of the planetary mean motions. Nevertheless, exact intersections with Ptolemy's graphs can be seen only in the case of Ulugh Beg's theories of Saturn, Jupiter and Mars, Muḥyī al-Dīn al-Maghribī's theories of Saturn and Mars as set forth in his *Adwār* (his third *zīj* written on the basis of his observations made in Maragha in the 1260s and the 1270s), and the *Mumtaḥan*/Ḥabash's and Ibn al-Fahhād's theories of Mars. On the one hand, it is probable that an astronomer actually used Ptolemy's data, but the theory cannot produce its origin, probably because mistakes were made in the calculations. A notable example in this respect is Muḥyī al-Dīn al-Maghribī's theory of Jupiter; he derived his mean motion value by comparing his mean longitude of 330;26,21° for the time of his third observation of the opposition of the planet to the mean Sun (12 August 1274, JDN 2186610, at 13:00 Maragha LT; error ~ –9 hours) with Ptolemy's corresponding observation (the value, date and time have been mentioned earlier); he took the interval of time between the two as $415225^{d}\,8^{h}$; that is, he dropped seven days, and hence, calculated 0;4,59,16,40,55,8° per day,[59] instead of $0;4,59,16,\underline{22},\underline{45},\underline{21}°$. Using the modified value makes his graph (the dashed line in Figure 3) intersect precisely with Ptolemy's at the open circle representative of the latter's observation. The other reason is the use of unknown sources, as we have seen in the case of Ibn al-Aʿlam's Jupiter. The cluster of graphs around Ptolemy's in the second century cannot be seen in the case of Venus, for which only Battānī's graph intersects precisely with Ptolemy's at the marked point, and Khāzinī's/Fahhād's theories come quite close to it (this issue will be investigated in another study).

3.1.2. Development: standard methods versus istidrāk *and* iʿtibār

The consistent methods and necessary conditions that must be used to design observations for establishing the planetary theories are fully explained (except for the construction of latitude models) in the canonical textbook of medieval astronomy, i.e., Ptolemy's *Almagest*. The only preserved account indicating that a

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[59] al-Maghribī, *Talkhīṣ* VIII.9, f. 132r. On al-Maghribī's observations and Mars' measurements, see, respectively, Mozaffari 2018a and Mozaffari 2018–2019, and for his measurements of Jupiter's orbital elements, see Saliba 1986.

medieval astronomer did all things in accord with Ptolemy's proposed strategies is the *Talkhīṣ al-majisṭī* of Muḥyī al-Dīn al-Maghribī.

Some medieval astronomers developed alternative or complementary procedures, methods, and strategies (*ḥīla*, lit. «artifice», as al-Khāzinī puts it)[60] for the standard methods in the *Almagest* for various purposes — usually, in order to facilitate the determination of a specific parameter (e.g., Bīrūnī's four-point method for the derivation of the orbital elements of the Sun and superior planets),[61] or posed some criticisms regarding various aspects (e.g., Jābir b. Aflaḥ).[62]

Apart from these products of lateral thinking, they developed a *general strategy* for dealing with planetary astronomy. As al-Battānī outlines it in the last subsection of chapter 31 of his *Ṣābi' zīj*,[63] a planet is observed in critical positions in anomaly and in its geocentric orbit; theoretical positions computed on the basis of an available «background» theory are compared with the data obtained from observations. Then, an attempt was made to find the source of the deviations detected.

They often attributed the deviations found to the errors in the mean motions (the strongest variables) and the radix mean positions in longitude and in anomaly, but sometimes the orbital elements (i.e., the longitude of apogee/perigee, eccentricity, and radius of epicycle) underwent changes. Of course, the structural parameters (eccentricity and radius of epicycle) were less subject to modification. The latter parameters did not very often need a serious revision, as al-Battānī explicitly remarks in the aforesaid part of his work («on the equations, we found them to be close to those in Ptolemy's word»). For the radii of the epicycles, this was because the Ptolemaic values are near their optimum values,[64] especially for the superior

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[60] al-Khāzinī in *Experimental astronomy* III.2 (*Zīj*, V: f. 11r).

[61] Mozaffari 2013b.

[62] See, e.g., Samsó 2001, 2020, p. 508f; Bellver 2006, 2008.

[63] al-Battānī, *Zīj*, E: ff. 67r–68r; Nallino [1899–1907] 1969, Vol. 1, pp. 65–66, commentary on pp. 239–242, Vol. 3, pp. 98–100.

[64] The structural and motional parameters of a planet are considered «optimum» if they provide the best fit between the Ptolemaic geocentric planetary models based on the eccentric or equant motion in the circular orbs and the modern heliocentric model based on the Keplerian motion in the elliptical orbits, also taking into account the gravitational perturbations, for critical conditions. The critical conditions are demarcated in connection with the historical methods used for the quantification of the underlying models (e.g., in the Ptolemaic context, a good theory of a superior planet is expected to give the best results in the oppositions, while the critical conditions for either inferior planet is its extreme elongations from the Sun). Theoretically, the process of optimization for the Ptolemaic model of a planet consists in the following steps: the heliocentric eccentricates of the Earth and a planet

planets;[65] of the seven non-Ptolemaic values we know for them from the Islamic astronomical corpus, only the value of 6;15^p for Saturn, measured by Jamāl al-Dīn al-Zaydī of Bukhara, a thirteenth-century Persian astronomer in the service of the Yuan dynasty of China, is an improvement on Ptolemy's value.[66] The only modifications made in the radii of the epicycles of the inferior planets are by Ibn Yūnus: he has 22;52^p for Mercury (modern: **23;14**^{p}; Ptolemy: 22;15^p in the *Planetary Hypotheses* and 22;30^p in the *Almagest*, which was widely used by the medieval Islamic astronomers), and 43;28^p for Venus (**43;24**^{p}; Ptolemy: 43;10^p); in a forthcoming paper, we will show that he reached his specific value for the size of the epicycle of Venus through his observations of the near appulses of the planet to Regulus, not by the *standard method* in *Almagest* X.2,[67] according to which two observations in specific orbital positions — one when the center of the planet's epicycle is located at the apogee, and another at the perigee — are necessary for the same purpose. As for the eccentricity, in the case of Mars it changes so slowly with the passage of time that it could be taken as a constant, around 6^p, as affirmed by Muḥyī al-Dīn and, seemingly, by Ibn al-Aʿlam before him; in the case of Saturn and Jupiter, the geo- centric eccentricities of which undergo a sizable change, none of the values measured in the Islamic period are actually more accurate for their times than Ptolemy's![68]

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are combined to obtain the planet's geocentric eccentricity, the ratio of their mean distances (i.e., semi-major axes) is taken and normalized according to the ancient and medieval mean distance of 60^p in order to derive the size of its epicycle, and so on (for a general treatment of this topic, see Carman and Recio 2019; especially for Venus: Mozaffari 2019a; and for the Sun: Maeyama 1998). However, theoretical deviations can be minimized but cannot be removed completely (for an example in the case of Venus, see below, Section 3.2.2).

[65] Saturn: 6;30^p (modern: **6;17**^{p}), Jupiter: 11;30^p (**11;32**^{p}), and Mars: 39;30^p (**39;28**^{p}). The modern values are simply the ratio between the Earth's and the planet's semimajor axes as normalized by the ancient length of 60 arbitrary units ^{p} for the orbit's radius.

[66] The tables of the equation of center and the epicyclic equation at the greatest distance are preserved in MS St. Petersburg, Oriental Institute, C 2460, pp. 15–16 and Sanjufīnī's *zīj*, ff. 47v–48r; the maximum values are, respectively, 6;19° and 5;40°, from which $e = 3;19$ and $r = 6;15$ result; also, see Yabuuti 1997, p. 33. On Jamāl al-Dīn, see van Dalen 2002a, 2002b, Yang 2017, Isahaya 2021.

[67] Toomer 1984, pp. 470–472.

[68] The values under discussion here are given in detail in our previous publications; see, for the eccentricities: Mozaffari 2014, for Ulugh Beg: Mozaffari 2016, for Venus: Mozaffari 2019a, and for Mars: Mozaffari 2018–2019.

In the case of al-Battānī, his background theories under examination were the ones presented by Ptolemy in the *Almagest*, and he found that the observed mean positions were unexceptionally in excess of the corresponding theoretical values; the corrective amounts he explicitly expresses include the mean anomalistic position of Venus (+4 1/2°) and of Mercury (+2 1/2°). He also made a change of –8° in Ptolemy's value of 161;0° for the longitude of Jupiter's apogee as updated for his time by the precessional increment measured by his own stellar observations (11;28°).

This general strategy is called *istidrāk* (*correction*; the term means «correcting mistakes/errors» and «pointing out faults»; in Plato of Tivoli's Latin translation of Battānī's *zīj*: *melioration*),[69] and was applicable when a coherent set of planetary theories was already available. Its use in practice dates back to the ninth century, as straightforwardly inferred from the aforesaid observation report from Ḥabash (Sect. 3.1.1) and other observational records preserved from him and the Banū Amājūr. It seems to have soon become a prevalent strategy widely in use to investigate and to put previous theories to the test.

The Islamic astronomers did not have a guiding manual at their disposal for how to correct the planetary theories, except for what they could indirectly deduce as the *corrective strategies* from the *Almagest* by considering the behavior of the Ptolemaic models and the effect exerted by each parameter in a specific orbital configuration. Thus, it is not known precisely whether quantitative corrections like the ones put forward by al-Battānī were done systematically during a complete tropical rotation of a planet and the effect of errors in all parameters (structural and motional) were isolated from each other, or whether it was confined to a limited period of time, a specific region of the ecliptic, and/or solely for reconciling theory and observation in some special and important synodic phenomena. If any of the latter purposes was the case, the whole method would, at best, reduce to the *manipulation* of data in order to obtain desirable results only at some time, in a specific ecliptic's zone, or for some special observations. The worst situation would be that the changes made in a background theory were not real improvements, but in fact affected it negatively. From al-Battānī's passage, he appears to have done the corrections systematically, but his *correction* of the longitude of Jupiter's apogee represents something different; as we will show elsewhere, an 8° decrease in the longitude of the apogee in Battānī's theory of

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[69] Ed. 1537, p. 42r; ed. 1645, p. 105.

Jupiter results in remarkably accurate longitudes for the planet at its oppositions to the Sun in the years 888–892, 900–904, and/or 912–916, when the planet was located near (on both sides of) its apogee (the first two four-year intervals are most probable, because he observed two solar eclipses in 891 and 901 and a lunar counterpart in 901).[70] As we will see below (Sect. 3.2.3), what Ibn al-Fahhād himself did in the case of al-Khāzinī’s theories of Jupiter and Saturn provides the best examples for a medieval astronomer’s limited evaluation of the planetary theories at his disposal.

In the first half of the 12th century, al-Khāzinī expanded on the *correction* method in a truly significant way and promoted it in his comprehensive *experiment* (*iʿtibār*) method in his treatise on *Experimental astronomy*, where he discusses, in depth, correct and comprehensive strategies for testing and improving various parameters under controlled conditions.[71]

An important point to emphasize is that the new methods — used either individually, or incorporated into the general strategies of *istidrāk* and *iʿtibār* — were not substantially different from the standard ones, but were other variants of, intrinsically related to, and/or inspired by them. We will see in the next section that, in the case of the derivation of mean motions, it is merely another variant of the standard method, and that such a procedure was already used in the *Almagest* for the same purpose (for rectifying the mean motions of the Moon). This could explicitly show to medieval astronomers how the alternative corrective methods work and also give them a helpful clue to expand on it. Some of these alternative methods were attained through a close inspection of the behavior of the Ptolemaic models (as al-Khāzinī puts it: «they were borrowed from the procedures put into effect in, *mustaʿār ʿamal*, the *Almagest*»). In addition, it was thought that Ptolemy himself occasionally used these methods for specific purposes; for example, Muḥyī al-Dīn believed that Ptolemy reached the bisection of the planets’ eccentricities «by means of a strategy (*ḥīla*) rather than by means of proof».[72] It is quite probable that Ptolemy himself applied strategies of this sort for rectifying his *Almagest* theories: as D. Duke hypothesizes, serial improvements of the fundamental parameters based on sequences of independ-

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[70] See Said and Stephenson 1996–1997, Part 2, pp. 43–45; Stephenson 1997, pp. 488–490; Steele 2000, p. 115.

[71] See Mozaffari 2022 and Saliba 2020 for al-Khāzinī and the treatise in question. For more on this interesting work see our forthcoming monograph.

[72] Mozaffari 2018–2019, pp. 170–171.

ent observations formed the basis for Ptolemy's mean motions [at least, for the Moon] in the *Planetary Hypotheses*, as against the elegant standard methods established in the *Almagest*.[73]

All that said, at methodological level the two alternative general strategies for testing and re-measuring the planetary parameters, *istidrāk* and *iʿtibār*, were the results of medieval conceptualization. They were *the only effective methods when a theory is available, but its observational bases and empirical foundations were not preserved or known*. This situation was the case with the majority of the medieval theories, as Bīrūnī complained that his Islamic predecessors did not explain their measurements in the same manner as Ptolemy elucidated his own ones, and did not clarify how they derived the planetary motions and positions through the continuous attempts they made to attain them.[74] Consequently, the individual methods used, although all rooted in the *Almagest*, when integrated into a general strategy found a new dimension in the medieval period.

3.1.3. Correction of the mean motions/positions

In order to determine the mean motions, two values measured from observations made as remote in time from each other as possible had to be compared. Take the initial value $\bar{\lambda}_1$ (for the superior planets) or $\bar{\alpha}_1$ (for the two inner planets) at time t_1 and $\bar{\lambda}_2$ (or $\bar{\alpha}_2$) at time t_2; so, the mean motion is simply computed from $\omega = (n \cdot 360° + \Delta\bar{\lambda}\ \{\text{or } \Delta\bar{\alpha}\})/\Delta t$, in which n is an integer number of the complete revolutions/cycles of the mean planet on the ecliptic/epicycle. An alternative is that if an initial mean motion value ω_1 is available previously, the difference ζ between $\Delta\bar{\lambda}$ (or $\Delta\bar{\alpha}$) and $\omega_1\Delta t$ is taken, and then the mean motion is simply *corrected* as $\omega_2 = \omega_1 + \zeta/\Delta t$. The straightforward comparison does not differ basically from the correction procedure under the condition that the initial observation(s) are available and accessible. Ptolemy applied the first in the case of the planets, but *corrected* the mean motions of the Moon derivable from the Babylonian period relations, as confirmed by Hipparchus (via whom they had come down to him).[75]

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[73] Duke 2009, p. 654.

[74] Mozaffari 2017, pp. 11–12.

[75] *Almagest* IV.2&3: Toomer 1984, pp. 175–176, 179.

When an astronomer wished to check/test an earlier theory against his own observations, he could compare the mean position $\bar{\lambda}_O$ or $\bar{\alpha}_O$ derived from his own observation(s) carried out at a specific time t with the value computed from a theory with the mean motion ω_1 and the radix value $\bar{\lambda}_{01}$ or $\bar{\alpha}_{01}$ for the epoch t_0. The difference ζ between $\bar{\lambda}_O$ or $\bar{\alpha}_O$ and $\bar{\lambda}_1 = \omega_1 \cdot (t - t_0) + \bar{\lambda}_{01}$ or $\bar{\alpha}_1 = \omega_1 \cdot (t - t_0) + \bar{\alpha}_{01}$ is taken. Then, there are two choices:

He (1) could determine a new mean motion as $\omega_2 = \omega_1 + \zeta/\Delta t$ or (2) simply add ζ to the radix value to define a new one: $\bar{\lambda}_{02} = \zeta + \bar{\lambda}_{01}$ or $\bar{\alpha}_{02} = \zeta + \bar{\alpha}_{01}$ for the same t_0 or any other arbitrary epoch. (Note that, naturally, a third mixed solution would be to separate ζ into two parts; one applied to modifying the mean motion, and the other to rectifying the epoch value). In such a manner, *a new theory could emerge on the basis of an already-existing one*.

In the first case (i.e., a change in the mean motion), the graphs of $d\bar{\lambda}$ of the two theories intersect each other in the epoch of the first theory; we call this the «node correlation».

In the latter case (i.e., a change in the epoch value), the two graphs remain parallel to each other: the «parallel correlation».

The first type of correction is explicitly referred to in al-Battānī's aforementioned passage as well as in al-Khāzinī's treatise (e.g., in II.7).[76]

Let us see, as an example, how al-Battānī used these two ways in the case of the two corrective amounts in the mean anomalies of the inner planets. For Venus, it is evident that his graph (see Figure 5) intersects Ptolemy's exactly at the time of the latter's observation of the planet on 16–12–138 CE, in the morning (JDN 1771812). From his observations, he found that the mean anomaly of Venus is $\zeta = 4;30°$ more than that derived from the *Almagest*. The date of his observation is not known, but his tables give this increase for the dates falling in September 886. Dividing the corrective amount into the interval of time results in $\zeta/\Delta t$ = 0;0,0,3,33,32,32°/d which, added to Ptolemy's value of 0;36,59,25,53,11,28°/d, results in Battānī's value of 0;36,59,29,26,44°/d. For Mercury, he preferred to apply the corrective amount to the epoch value rather than to determine a new mean motion value, so that his mean longitude values have a persistent increase of 3;22° over Ptolemy's values throughout the 2000 years under scrutiny here; note that this value differs from the figure of 2;30° he had already announced. The reason for his different choices may have been that, by the correction of the mean

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[76] al-Khāzinī, *Zīj*, V: f. 10r, L: ff. 59r–v.

motion of Venus, the resulting value does not differ radically from the values measured by his ninth-century Muslim predecessors, in the sense that, in fact, it falls midway between the *Mumtaḥan* and Ḥabash's values (see Appendix, A), but by rectifying Mercury's mean motion in this way, the resulting value of 3;6,24,8,58,13°/d will be more than Ptolemy's, *Mumtaḥan*, and Ḥabash's values, all about 3;6,24,6,59,35°/d; perhaps he did not wish to break with this long-standing tradition and thus avoid any undesirable consequences in the future (however, the difference is small, accumulating to a single degree in about 299 Julian years). As we shall see in Sect. 3.2.3, in a similar fashion, Ibn al-Fahhād corrected al-Khāzinī's mean motion in longitude of Jupiter, whereas he applied the corrective amount to the radix value in the latter's theory of Saturn.

To sum up at this stage, we have seen so far that the major, substantial differences between the medieval Islamic planetary theories lie in the mean motions/positions. An astronomer established his theory for a specific planet either (i) independently of another theory made by his predecessors by the direct comparison between Ptolemy's or a medieval predecessor's data and his own observed data, or (ii) by modifying a theory of one of his predecessors; in the second case, a change in the mean longitudes in the earlier theory was made in the two ways explained above (or by a synthesis of them). Now, a look at the graphs in Figures 2 to 5 reveals immediately the possible connections (the *node* and *parallel* correlations) between them. It comes as a surprise to see that the depth of interdependence of the planetary theories of the medieval Islamic period is appreciably greater than could be conceived initially. We can now proceed to the main issue that concerns us.

3.2. The origins of Ibn al-Fahhād's planetary theories

3.2.1. Mars

In the prolegomenon to the *ʿAlāʾī zīj*, Ibn al-Fahhād mentions that he observed Mars for a long period as well as Venus many times with Regulus, and found their observed positions to have been in agreement with Ibn al-Aʿlam's theories.[77]

Figures 6(a) to 6(d) display the graphs of the errors in the true longitudes of Mars in the five theories established in the Islamic period until Ibn al-Fahhād's

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[77] See Mozaffari 2019a, p. 60; 2019b, paragraph **[IV]** in the quotation on p. 526.

time by the *Mumtaḥan* team, al-Battānī, Ibn al-Aʿlam, Ibn Yūnus, al-Khāzinī, together with Ibn al-Fahhād's theory in the two decades: the 970s (the period of activity of Abu 'l-Wafāʾ) and the 1160s (Ibn al-Fahhād's time). As clearly illustrated, they are conspicuously categorized in the two completely separate classes. What makes such a distinct difference is Ibn al-Aʿlam's theory, which justifiably gained a reputation from the eleventh century onwards for having a significantly higher degree of accuracy than other competing theories available at the time.[78]

In the first group are the ones laid down by the Mumtaḥan team, as preserved in Habash's *zīj*, al-Battānī, and Ibn Yūnus. The Mumtaḥan mean longitudes in the ninth century (supposedly derived from the observations performed in Baghdad and/or Damascus) suffer from positive error; hence, when compared with Ptolemy's corresponding values with egregious negative errors in the 130s, this led to the derivation of an appreciably large mean daily longitudinal motion for the red planet (cf. Figure 7, which is solely to provide a schematic diagram of the relative accuracy of the historical values for the mean motion in longitude of Mars). This value was improved by al-Battānī, notwithstanding the fact that his mean longitudes about 900 CE have positive errors of nearly of the same size as in the Mumtaḥan theory in the mid-ninth century. From Figure 4, it is obvious that Ibn Yūnus' theory is independent of Ptolemy's. How he established his theory is unknown to us. It is surprising that his theory gives the value of 183;48° for the time of the ancient observation recorded in *Almagest* X.9 (18–1–272 BCE, at dawn, JDN 1622093),[79] which is different from Ptolemy's 184;12° (though, like his Jupiter's theory, only less than one day off), but highly accurate (modern: **183;42**°). Ibn Yūnus did not acknowledge the remarkable accuracy of Ibn al-Aʿlam's theory of Mars (see below), while his theory of Saturn shows a strong (*quasi*-parallel) relation to Ibn al-Aʿlam's (cf. Figure 2), signaling a dependence on the latter. All these issues deserve further investigation with respect to his observations documented in the *Ḥākimī zīj*.

Towards the end of the tenth century, Ibn al-Aʿlam, Abu 'l-Wafāʾ's elder contemporary, constructed a significantly more accurate theory than those of his predecessors, by removing the large positive systematic error in the *Mumtaḥan*

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[78] Statements in this regard were appended to the canons of one of the two hitherto known recensions of the *Mumtaḥan zīj*, in both of which Ibn al-Aʿlam's theory of Mars has replaced the original theory (van Dalen 2004, pp. 31–33). A similar remark is given in al-Bayhaqī's *Tatimmat* of the 12th century (1932, pp. 82–83; Meyerhof 1948, p. 157).

[79] Toomer 1984, pp. 502–504.

theory of Mars already detected, at least, by the Banū Amajūr, for example, in their periodic observations of Mars with Sirius in the summer of 918 CE.[80] Ibn al-Aʿlam's mean longitudes at his time suffer from a negative error, but, as compared to Ptolemy's values with errors of the same sign, this led to the derivation of a sufficiently accurate value for the mean motion, but less accurate than Ibn Yūnus' (cf. Figure 7). In addition, he significantly improved the longitude of the apogee of the planet in comparison with his predecessors and contemporaries.[81] His stable solar theory was no doubt another contributing factor to the success of his theory of Mars. Ibn al-Aʿlam's theory of Mars exerted a great influence on other theories in the late Islamic period, as is explained in the following lines. Khāzinī seems to have (1) adopted Ibn al-Aʿlam's mean daily motion in longitude, as the value he tabulates in his *zīj* differs slightly from what could be computed from the value of 228;20,51° that the author of the 13th-century *Ashrafī zīj* gives for Mars' mean motion in 20 years of 365¹/₄ days;[82] (2) made a minor decrease of about 0;8° in the radix mean longitude (as is clear in Figure 4, the two curves related to Ibn al-Aʿlam and Khāzinī are almost parallel to each other); and, further, (3) improved Ibn al-Aʿlam's value for the longitude of the apogee to the extent that, as shown elsewhere, his own value is the most accurate for this parameter in the medieval Islamic period.[83] Ibn al-Fahhād worked in this setting. The graph of his mean longitudes conspicuously illustrates how he proceeded. It intersects with that of Ibn al-Aʿlam about 1170 CE, meaning that he made sound and correct observations of Mars in his time, measured its mean longitudes from his observations accurate to within –5', and verified the considerable accuracy of Ibn al-Aʿlam's theory, in line with his explicit assertions in the prologue of his last work. Then, he compared his mean longitudes (which were equal to Ibn al-Aʿlam's) with Ptolemy's corresponding values derived from his observation(s) in the 130s. This procedure inevitably caused his value for the mean daily longitudinal motion to be greater than Ibn al-Aʿlam's. His value for the longitude of the apogee seems to be dependent on Ibn al-Aʿlam's theory.[84]

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[80] Ibn Yūnus, *Zīj*, L: p. 99; Caussin 1804, pp. 106–107; Delambre 1819, p. 83.

[81] See Mozaffari 2018–2019, pp. 215, 220–222, 241.

[82] Kamālī, *Zīj*, F: f. 234r, G: 249r. On Ibn al-Aʿlam's parameter values, see Kennedy 1977, Mercier 1989.

[83] See note 81.

[84] See note 81.

As shown elsewhere, presumably simultaneously with his purposeful and systematic observations at the Maragha observatory, Muḥyī al-Dīn al-Maghribī tested the outcomes of the available theories against his empirical data so as to exploit the best possibilities for his last round of planetary measurements, and recognized that Ibn al-Fahhād's theory of Mars was far superior to Ibn Yūnus' and his previous theory established in his Syrian years as put forward in the *Tāj al-azyāj*. Accordingly, he employed Ibn al-Fahhād's mean motion as a provisional value for the purpose of calculating the planet's motion in mean longitude between his trio of the planet's oppositions to the mean Sun.[85] As Figures 4 and 7 clearly show, Naṣīr al-Dīn al-Ṭūsī used Ibn al-Aʿlam's value for Mars' mean motion in longitude in the *Īlkhānī zīj*, with a correction of about –0;20° in the radix value. The adoption of a new erroneous value for the radius of the epicycle caused the errors in the planet's longitudes to be distributed across a wide amplitude.[86] We should note also that Ibn al-Shāṭir's value is very close to Ibn al-Aʿlam's, although his theory, like Ibn Yūnus', seems, in all likelihood, to have been established on the basis of the ancient observation recorded by Ptolemy, for which time it gives a mean longitude of 183;40°.

The conclusive result of our discussion concerning Mars is that, as the graphs in Figure 4 clearly show, with regard to the mean motion and the radix mean longitude the theory worked out by Ibn al-Fahhād is intrinsically intertwined with that of Ibn al-Aʿlam's, but is, in essence, independent of it, as it is certain that it was constructed on the basis of a direct comparison between Ptolemy's and his own observations. There is nothing hidden, or anything that remains unexplained, that is suggestive of any unknown source.

3.2.2. Venus

As Figure 5 shows, there is no doubt that Ibn al-Fahhād's theory of Venus is based entirely on Khāzinī's. The two hold a clear *parallel* correlation; there is only a small difference of –2' in the epoch mean anomaly, which seems to stem more from an error in the process of adaptation than from a refinement of any sort (actually, the negative error in the background theory increases by the same

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[85] See Mozaffari 2018–2019, pp. 191, 206–211.

[86] See Mozaffari 2018–2019, pp. 222–225.

amount). Also, Khāzinī's value for the mean daily anomalistic motion of Venus is explicitly asserted in *ʿAlā'ī zīj* I.22.[87] In addition, Ibn al-Fahhād took the value for the longitude of the apogee of the planet from Khāzinī's *Muʿtabar zīj* (67;35° for the Hijra epoch) and updated it on the basis of the apogeal/precessional motion of 1°/66^{y} for his epoch (the beginning of 545 Y, 2 February 1272, JDN 2149163) as 75;54,43°.[88]

As noted at the beginning of the previous section, in the prolegomenon to his *zīj*, Ibn al-Fahhād explicitly states that Ibn al-Aʿlam's theory of Venus gave accurate positions «many times» when the planet was observed with Regulus. Then, the questions to answer are: when were these «many times»? If the Baghdadi astronomer's theory gave such satisfactory results on a good number of occasions, then why did our Shirwānī astronomer not adopt it in his work? Were there other times when Ibn al-Alam's theory failed to account for the planet's motion in longitude precisely? If so, when and why? Was Khāzinī's theory preferable to Ibn al-Aʿlam's? And if so, what were its definite advantages?

The first preliminary point to make is that Ibn al-Fahhād added 15;54° to the longitudes of 40 reference stars in the ancient star catalogue incorporated and reworked in the *Almagest* to update them for the beginning of 545 Y;[89] thus, Regulus was assigned a longitude of 138;24°, which, though by mere coincidence, agrees with the correct position of the star with reference to the true equinox of the date; his large value of 1° in 66 Persian years for the rate of precession compensated for the serious error of about −1½° in Ptolemy's value. Consequently, he had at his disposal a secure reference star to be deployed in his evaluation of the theories of Venus.

The second point is that his observation of the Great conjunction of 1166 CE gives the strong impression that he was not able to measure distances in longitude/angular separations with an accuracy of higher than ±¼°. His belief that the conjunction occurred at about 3:45 LT on 10 December 1166 (the two objects were below the horizon at the time) shows that he could not perceive that, in the narrow interval of time between 17:11 Shirwān LT (the end of civil twilight) and 17:26 LT (the last moment when the two planets were at an altitude higher than 10°) on the same day, Jupiter still had to travel a distance in longitude of **9'** to ar-

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[87] Ibn al-Fahhād, *Zīj*, p. 20.

[88] See Mozaffari 2019a, p. 55.

[89] Ibn al-Fahhād, *Zīj*, p. 219.

rive at Saturn's position and the angular separation between them was about **13'**. Even so, the small angular separation between a bright and a relatively faint celestial object (like Saturn, μ = **+0.64**, and Jupiter, μ = **–1.82**, whose brightness fell, respectively, to **+1.8** and **–0.7**, due to the telluric extinction, at the time) or between two bright heavenly bodies is not easily distinguishable by the naked human eye.[90] So, there is no serious question that Ibn al-Fahhād was unable to distinguish a separation of less than 1/4° between the two giant planets at the time. The same optical phenomenon quite probably affected his observations of the near appulse of Venus to Regulus.

The third point is that the observation of the «actual contact or very close approach» of a planet to a fixed star is according to a recommendation given by Ptolemy in *Almagest* IX.2.[91] Some observations of Venus with Regulus have come down to us from the classical period of Islamic astronomy. Examples are the occultation of Regulus observed by the Banū Amājur in Baghdad on 10 September 885 (JDN 2044557) one hour before sunrise,[92] or the eight observations carried out by Ibn Yūnus between 987 and 1003.[93] We are not told about their practical applications, other than that they were used to check the theoretical results of Ḥabash's *Mumtaḥan zīj*, although, as mentioned earlier, it can be shown that Ibn Yūnus' observations were used for the determination of the only non-Ptolemaic value for the size of the planet's epicycle. Muḥyī al-Dīn's observations of four near appulses of the superior planets to Regulus at Mara-

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[90] Under ideal conditions, the unaided human eye can discern the angular separations of as little as a single arc-minute between two dim point sources of illumination. Nevertheless, sort of optical illusion occurs when at least one of the celestial objects involved in a close approach as seen by an earth-bound observer is appreciably brighter than another (e.g., Jupiter and Saturn). This affects both the visibility of the fainter object and the perceived angular separation between them. When both heavenly bodies are particularly bright (e.g., Venus and Jupiter), the apparent distance between them is notably affected to the degree that the close approach between then might look like an occultation. This optical phenomenon is dynamic, depending both on the difference in brightness and on the angular separation. It is encountered in a good number of ancient and medieval observational reports. However, to the best of our knowledge, its effect has not yet been studied in depth or quantified (on this topic, see Włodarczyk *et al*. 2018).

[91] Toomer 1984, p. 423.

[92] Ibn Yūnus, *Zīj*, L: p. 109, F1: f. 10r (the only observation of the Banū Amājūr that has been preserved in MS. F1 of the *Ḥākimī zīj*).

[93] Ibn Yūnus, *Zīj*, L: pp. 113–119.

gha, used to measure the radii of their epicycles, are also extant from the late Islamic period.[94]

Venus is in conjunction with any fixed star once in a solar year. Throughout the 1160s, the near appulse between it and Regulus occurred alternately in the eastern and western elongations, but was not observable in the years 1162 and 1167, because the planet was too close to the Sun (respectively, in a western elongation of about **–7**° from the Sun on 11 August 1162 and in an eastern elongation of about **+8**° from the Sun on 27 July 1167). An astronomer wishing to make careful observations and measurements needed a relatively long time span to watch the phenomenon in the morning sky before sunrise or in the evening sky after sunset at the highest possible altitude of the two objects. Accordingly, we exclude the near appulses that occurred in the two successive years 1164 and 1165 when the planet was in the moderate elongations of about **+23**° and **–21**° from the Sun, respectively, on 12 July 1164 and 26 August 1165. In the remaining six years, Venus was elongated enough (> 30°) from the Sun at its near appulses to Regulus, making it highly probable that Ibn al-Fahhād observed the events. Table 1(A) indicates the dates, the corresponding Julian day numbers, and the rounded modern data, including the elongations, the eccentric anomalies (i.e., the longitudinal separations between the mean Sun and the geocentric apogee), and the mean epicyclic anomalies (i.e., the difference between the longitude of the mean Sun and the mean heliocentric longitude of the planet) at noon on the given dates in Shirwān. These data help the reader visualize the orbital configuration in any of the six conjunctions under scrutiny, which are necessary to understand the subsequent discussions.

The errors in Ibn al-Aʿlam's, Khāzinī's and Ibn al-Fahhād's theories of Venus are presented in the second columns in Tables 1(B)–1(D). Before proceeding further, it is worth noting that, in the recorded observations of the near appulses of a planet to a fixed star from the Islamic period, the planet was not always caught in conjunction with the fixed star; rather, in some, the star was used as a landmark for measuring the longitude of the planet, and the result(s) were compared with theoretical values; e.g., in the observation of Venus with Antares made by the Banū Amājūr on 25 December 918, in which the longitude of Venus was determined as 239;6° (error ~ –0;40°) and that of Antares as 234;31° (error ~ –°11;0), the former being compared with the theoretical value extracted from Ḥabash's *zīj*

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[94] Mozaffari 2018b, pp. 613–616.

(240;46°; error ~ +1°).[95] Moreover, there are some specific conditions to be fulfilled for any star to be discernible by the unaided eye in the twilight sky with respect to its visual magnitude and instantaneous altitude and distance in azimuth from the Sun. Hence, it is not clear whether by «observing Venus with Regulus many times», Ibn al-Fahhād in fact means detecting the conjunction between the two. For these reasons, we also plot the longitudinal errors in the three theories in question along with the theoretical errors in longitude (see below) in Figures 8(a)–8(d) in order to illustrate the errors in the vicinity of the longitude of Regulus marked by a vertical line.

The third columns in Tables 1(B)–1(D) and Figure 8(a) give the theoretical deviations of the Ptolemaic model with the equant motion through the circular orbs from the modern theory based on the Keplerian motion in the elliptical orbits taking into account all noticeable gravitational perturbations. In order to compute these entries, first, the optimal values for the structural parameters are taken into account; for the eccentricity and the longitude of the apogee, the formulae deduced in our 2019 study are used.[96] The eccentricity varied slightly from the mean value of about **0;54**$^{1}/_{2}{}^{p}$ to **0;52**$^{1}/_{2}{}^{p}$ over the past two millennia, and the geocentric apogee progresses in the direction of increasing longitude at a rate of **1**° in **53.2** Julian years (which is distinctly different from the rate of the motion of the heliocentric apogee: **50.5**" per annum);[97] in the 1160s, its longitude increased from **76.5**° to **76.7**° (the longitude of the apogee λ_A and of the perigee λ_Π are marked by the two vertical lines in Figures 8(a)–8(d)); and the epicycle's radius is taken as equal to **41;24**^{p}. Then, the mean Sun and the mean heliocentric longitude of the planet were derived from the JPL DE 406 theory. Finally, the longitudes computed on the basis of these parameter values in the framework of the Ptolemaic model were compared with the corresponding modern values. It is obvious that in the proximity of the longitude of Regulus, the longitudinal errors are less than +1/6° in the western elongation, while in the vicinity of the eastern elongations, they do not exceed –2/6°.

Let us examine, from a geocentric perspective, the impacts of the errors in the basic parameters on the longitudinal errors on the six occasions under considera-

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[95] Ibn Yūnus, *Zīj*, L: p. 99; Caussin 1804, pp. 108–110; Delambre 1819, p. 83.

[96] See Mozaffari 2019a, pp. 49–53.

[97] See Mozaffari 2017, table 2 on p. 8.

tion; on occasions their effects are substantial, and on others minor (cf. Tables 1(B)–1(D)):

As regards the eccentricities, Ibn al-Aʿlam's value is about half his eccentricity of the Sun/Earth, i.e., about $1;2^p$, but Khāzinī keeps Ptolemy's high value of $1;15^p$ (his solar eccentricity is about $2;19^p$),[98] which became obsolete, at least, from the Mumtaḥan observations onward; to the best of our knowledge, besides him, only Bīrūnī used it. Both values have positive errors compared with the aforementioned modern values. Thus, the equations of center would become larger in size. This exerted its undesirable influence on the longitudes in two ways. The first, and more important, was that the equations of center were negative in the six close approaches of Venus to Regulus under scrutiny, when the center of the epicycle was located between the eccentric apogee and the eccentric perigee (see the column headed $\bar{\kappa}$ in Table 1(A)), which naturally causes the *negative* errors in the longitudes. The lower the values of $\bar{\kappa}$, the smaller the resulting negatia ve longitudinal errors; in other words, the sizes of these parts of the longitudinal errors are greater in the western elongations than in the eastern elongations. The second, and less important, was the fact that the absolute values of the equations of center are added to the mean epicyclic anomalies (see the column headed $\bar{\alpha}$ in Table 1(A)) in this region of the orbital configuration. Accordingly, if the planet lies between the epicyclic apogee and the first station in the eastern elongations or the second station in the western elongations, the positive errors in the resulting true anomalies give rise to *positive* errors in the longitudes, but the opposite is true between the first/second station and the perigee. Therefore, the minimum error occurs in events nos. **2** and **6**, and the maximum discrepancy is seen in no. **3**.

Like most of their predecessors, both Ibn al-Aʿlam and Khāzinī adopted the Ptolemaic value of $43;10^p$ for the epicycle's radius (as noted earlier, among the medieval astronomers, only Ibn Yūnus significantly improved on it). This has a negative error compared to the optimal value of $\mathbf{43;24}^p$. Consequently, it causes negative errors in longitude in the eastern elongations, but positive ones whenever the planet is located on the western side of the epicycle. The closer the planet to the extreme elongation, the greater the influence of the error in the epicycle's size; hence, the errors are greater in nos. **3** and **4**.

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[98] Mozaffari 2018a, p. 194, note 7.

The mean anomalies in Ibn al-Aʿlam's theory are accurate, as they have negligible errors of only about –0;7° about two centuries after it was proposed (see Figure 5). How he established his theory is not known, but as the graph in Figure 5 suggests, and as we have already seen in the case of his theory of Jupiter (cf. Sect. 3.1.1), he seems to have used the observations made by his ninth-century predecessors. The evidence comes from an observation preserved from Ḥabash in the evening on 26 October 830 when he measured the longitude of the planet as 262;42° (error ~ –1/4°).[99] Ibn al-Aʿlam's theory produces a close value of 262;45° for the given date and time. In contrast, the errors in Khāzinī's mean anomalies are comparatively large, amounting to about –0;27°. The closer the planet to the extreme elongation, the lesser the effect of the error in the mean anomaly; hence, the errors in nos. **1**, **2**, **5** and **6** are larger than in nos. **3** and **4**.

Khāzinī's epoch value for the longitude of the apogee has been mentioned earlier in this section. Updated for 1120 CE on the basis of the precessional rate of 1°/66^{y}, the result, 75;8°, is one of the most accurate values measured in the medieval period, with an error of –0;37°. The error steadily increases with the passage of time, because the true apogeal motion (see above) is faster than the rate used. Ibn al-Aʿlam's theory gives a value of 71;15° for 975 CE, which is about –1;50° in error. Updated for the 1160s with the use of Ibn al-Aʿlam's rate of precession/apogeal motion of 1°/70^{y}, the error rose, in a similar manner, to –2;40° (if Ibn al-Fahhād used the rate of precession/apogeal motion of 1°/66^{y}, the error would be –2;30°).[100] The longitudinal errors stemming from the discrepancy in the longitude of the apogee are comparatively small. Similar to the eccentricity error, it exerts its influence in the equation of center, which affects the longitudes in the two ways mentioned above: they are greater in the conjunctions in the eastern

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[99] Ibn Yūnus, *Zīj*, L: p. 108. The date in this report is given as «…*sina 199 li-yazdjird* ***dh*** *māy Mihr wa rūz Bahman…*», which means year: 199 Yazdigird, month: *Mihr*, and day: *Bahman*. Caussin (1804, pp. 155–156) have correctly transcripted the phrase, but erroneously translated it as «…du jour de Bahmen (le 2) du mois deïmah, l'an 199 d'Izdjerd». It seems that the additional ***dh*** together with neglecting the term *Mihr* caused Caussin to read the phrase as «… *li-yazdjird* ***d***[***iy***]***-māy*** *wa rūz* ….» He then made another mistake by converting 2 *Diy* (the 10th month) 199 Y to 2***5*** January 831, while the given date is equivalent to 29 January 831 in the early Persian calendar (in its late system: 24 January 831). About this time, Venus was in western elongation, rising before sunrise and setting before sunset, and thus, it was below the horizon in the evening. Caussin's mistaken date is repeated in Delambre 1819, p. 87.

[100] See Mozaffari 2019a, pp. 55, 63.

elongations, when the center of the epicycle is closer to the eccentric apsidal line, and lower on those occasions in the western elongations, when it is near the quadrature.

The errors in the mean solar longitudes have an immediate and substantial impact on the true longitudes of the planet. As said in Sect. 2.1, Ibn al-Aʿlam established a solar theory of Type II, which was very accurate; as shown in our 2019 study, the discrepancy in the mean motion is accumulated to a single minute of arc after passing 770 years. In stark contrast, Kh*ā*zin*ī* has the worst solar theory after Ptolemy, with errors of +0;29° in the 1160s. As noted earlier, the solar mean longitudes in the Mumtaḥan and al-Khāzinī's theories are very close to each other, as their differences do not exceed 0;5° in the two past millennia.

The sum of these five groups of errors is given in the last columns in Tables 1(B)–1(D). They do not agree precisely, but very closely, with the errors in each theory under investigation (the second columns); this is partly because of rounding, and partly because the combined effect of the discrepancies in the underlying parameter values on the longitudinal error is not exactly identical to the algebraic sum of all singular influences exerted by them.

The interaction between these five groups of errors is clear. For Ibn al-Aʿlam, in the western elongations, the large negative errors owing to the eccentricity greatly compensate for the positive errors arising from the epicycle's size and the positive theoretical deviations; the aggregate errors very nearly reduce to zero. In all likelihood, these occasions were the «many times» to which the Shirwan*ī* astronomer refers. In stark contrast, in the eastern elongations, all the errors are negative, which, added to the negative theoretical deviations, makes the total errors of –4/6° and –5/6°, well beyond the tolerance level. Such errors would have been detectable by medieval instruments, and this is probably why Ibn al-Fahhād abandoned the Baghdadi astronomer's theory in favor of Kh*ā*zin*ī*'s. In all likelihood, *he wrongly thought that Ibn al-Aʿlam's solar theory is the main source of these errors*.

In the case of Khāzinī, the negative errors due to the eccentricity are quite large, yet they were minimized to a great degree by the positive errors stemming from the solar theory. In the western elongations, the total errors remain about +1/6°, almost certainly below the critical threshold of detectability by the instruments at Ibn al-Fahhād's disposal. In the eastern elongations, they amount to about –1/2° and –4/6°, but are less than Ibn al-Aʿlam's errors at those times. Ibn al-Fahhād seems to have been convinced that Khāzinī's adoption of the Ptolemaic large eccentricity is the source of these errors, and therefore left it aside.

By doing so, the errors in his *mixed* theory are +1/3° and +1/2° in the western elongations and −1/4° and −1/3° in the eastern elongations. He appears to have been able to reduce them to below the threshold of detectability and, particularly, to eliminate the large errors in his two predecessors' longitudes for the 1166 near appulse of Venus to Regulus occurred about six months before his observation of the Great conjunction, which may have played a major part in shaping his choices.

The case of Venus is of interest for a number of reasons.

(I) A prerequisite in the assessment of the planetary theories for outlining their relationships is to determine to what extent a secondary theory emerging from elements borrowed wholesale from others resembles its backgrounds. Our case of Venus demonstrates that this would not be a simple task, as only a change in the eccentricity made the output of the descendant theory, *viz*. Ibn al-Fahhād's, critically different from its background, *viz*. Khāzinī's, which can be clearly seen in Figures A(c) and A(d) and the boundaries of the errors listed in Table 2. By mere coincidence (let us emphasize that there is no reason to think that he performed a systematic assessment), he significantly reduced the large negative errors in both Ibn al-Aʿlam's and Khāzinī's theories, occurring a few days prior to the last evening visibility phase (under our criterion, elongations > 30°) and also appreciably decreased the serious positive errors in Khāzinī's theory occurring a few days after the first morning visibility phase (*viz*. when the planet is located on the two sides of the epicyclic perigee).

(II) As stated in Sect. 2, in our 2019 study, we adduced Ibn al-Fahhād's material about the solar and lunar eclipses of April 1176 in order to show how a medieval astronomer's concentration on the timings of the synodic phenomena could be misleading for drawing conclusions about the accuracy of theories, and, especially, to explain why he was not able to understand that among the four solar theories he explicitly refers to in his *zīj*, Ibn al-Aʿlam's is the most accurate. Our present case of Venus gives crucial clues to understand why he wrongly thought that Ibn al-Aʿlam's solar theory was incorrect, and that the Mumtaḥan solar theory was preferable to it. It was almost impossible for Ibn al-Fahhād to recognize that the egregious negative errors in Ibn al-Aʿlam's theory of Venus in the eastern elongations in the vicinity of Regulus were the aggregate negative effects of the errors in the values for the eccentricity and the radius of the epicycle and the negative theoretical deviations, simply because he also deployed the same values for the eccentricity and the epicycle's radius, and also, that the latter was undefined for him. So, it appears that he incorrectly attributed those errors to Ibn al-Aʿlam's solar theory. In a like manner, the Shirwānī astronomer was aware of the fact that what reduces the longitudinal errors in Khāzinī's theory of Venus on those occasions is the latter's solar theory. In view of the fact that the Mumtaḥan and Khāzinī's mean longitudes of the Sun remain very close to each other (cf. Figure 1(a)), it might have given Ibn al-Fahhād a false, but convincing, reason to believe that the Mumtaḥan solar theory is correct. All this shows how an erroneous solar theory could contribute, under special conditions, to obtaining a satisfactory outcome for the theory of a planet!

3.2.3. Saturn and Jupiter

Ibn al-Fahhād's theories of Jupiter and Saturn show obvious relations to Khāzinī's.

For Saturn, we see a *parallel* correlation (cf. Figure 2): there is a persistent difference of about +0;42° between the mean longitudes in the two theories; that is, only the radix mean longitude has changed. In turn, Khāzinī's theory depends closely, again in a parallel situation (with the vertical shift amounting to about +0;19°), on Ḥabash's theory: the mean motion in the 30 lunar years in the former is 356;8,33°,[101] close to 356;8,31° in the latter.[102] (It should be noted that there are slight analogous differences in the planet's tabular mean motion entries between the surviving copies of the *Mumtaḥan* and Ḥabash's *zīj*es, conspicuously showing that the underlying mean daily motion value in the former is, clearly, not equal to, but slightly greater than, the one in the latter).[103] Of course, it is also probable that Khāzinī, approximately, derived the same mean motion that had been available since the mid-ninth century by comparing his measured mean longitudes (with fairly small errors) with Ptolemy's observations (for Ptolemy's third mean opposition on 8 July 136, JDN 1771378, Khāzinī's theory gives the mean longitude of 289;37°, not far from Ptolemy's 289;30°). Ibn al-Fahhād's theory of Saturn was later adopted without any changes in the *Īlkhānī zīj*. It is

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[101] Khāzinī, *Zīj*, V: f. 144v.

[102] Ḥabash, *Zīj*, I: f. 103r.

[103] E.g., the *Mumtaḥan zīj* (E: f. 23v) gives 244;33,11° for the mean motion in 20 Persian years of 365 days, fixed.

astonishing to see that the mean motion value determined by Ḥabash was in use for such a long time.

In the case of Jupiter, we are confronted with a *node* correlation: both Khāzinī and Ibn al-Fahhād's theories give the mean longitude of 331;7° at noon in the Hijra epoch (on Friday, 16 July 622, as adopted by Khāzinī). The node can be clearly seen in Figure 3. Nevertheless, Ibn al-Fahhād's graph has a negative slope, showing that his value for the mean motion in longitude of Jupiter is less than Khāzinī's (and the modern mean one as well), so that the difference between the two theories in mean longitude reaches about –0;38° in 1166 CE. Khāzinī's theory, in turn, shows a close *parallel*-relation to Ibn al-Aʿlam; however, his value for the mean motion is noticeably a little lower than Ibn al-Aʿlam's. It is curious that Khāzinī's own value was either used in Ulugh Beg's *Sulṭānī zīj* or independently determined by the Samarqand astronomers.

Both modifications (in fact, «deteriorations», as the graphs clearly show) that Ibn al-Fahhād made in Khāzinī's theories for the two outer giant planets are pertinent to his observation of the Great conjunction between them at 3:45 LT of Shirwān on 14 Ṣafar 562 (10 December 1166; actually, it occurred a day later). Khāzinī's *Muʿtabar zīj* gives for the given date and time:

	$\bar{\lambda}$	λ
Saturn	297;52°	290;35°
Jupiter	301; 9	291;43

These values indicate that the conjunction occurred ten days earlier, as emphasized by Ibn al-Fahhād's harsh criticism of the Georgian/Khurāsānian astronomer. In order to correct the «deficiency» caused by the difference of about 1;10° in true longitude, Ibn al-Fahhād decided to remove 1;20° from the difference in mean longitude, and found that 0;42° of the total amount belonged to Saturn, while the remaining 0;38° should be assigned to Jupiter. He simply modified Khāzinī's radix value in the case of Saturn by $\zeta = +0;42°$, but, for Jupiter, he chose to «rectify» the latter's mean daily motion by dividing the «corrective» amount of $\zeta = 0;38°$ into the time interval passed from the Hijra epoch, and then, subtracting the result from Khāzinī's mean daily motion, which led to the result that his own value for ω was about 0;0,0,0,40° less than Khāzinī's.

The difference in his approach to the same problem has a simple explanation, as we encountered earlier in Battānī's procedure in the case of the inferior planets: Ibn al-Fahhād's modified value for the mean daily motion in longitude of

Jupiter lies between the maximum and minimum values known until his time (respectively, the Mumtaḥan/Ḥabash's and Ptolemy's), but if he did the same in the case of Saturn, the resulting mean daily motion would surpass the highest value known until his time, i.e., Khāzinī's/Mumtaḥan/Ḥabash's value, by about 0;0,0,0,46°. He did not run such a «big» risk, presumably, in order to prevent any undesirable consequences in the future, and changed the epoch value instead.

In our 2019 study, we held up Ibn al-Fahhād's treatment of the Great conjunction of 1166 as an example to show, by means of a simple statistical analysis, how untrustworthy a medieval astronomer's evaluation of the available theories could be with respect to the synodic phenomena, in which at least two celestial bodies are involved.[104] Now, we can confidently add that, with his numerical changes in Khāzinī's theories of Saturn and Jupiter, the Shirwānī astronomer attained his desired result for the Great conjunction in question, but caused considerable deterioration in the background theories.

It is worth noting that, with the change Ibn al-Fahhād made in Khāzinī's theory of Jupiter, the new set of the mean longitudes are very close to Ptolemy's mean longitude at the time of the ancient observation mentioned earlier (82;49° in comparison with Ptolemy's 82;54° for 4–9–241 BCE) (cf. Figure 3), but this seems to have been a mere coincidence, not least because, as we have seen in Sect. 3.2.1, Ibn al-Fahhād used Ptolemy's observation(s) for the derivation of the mean motion of Mars.

4. Berlin MS of Ḥabash's «zīj»: a case of «istidrāk» and Abu 'l-Wafā''s mean motions

MS Staatsbibliothek Preußischer Kulturbesitz zu Berlin, Wetzstein I 90, contains a recension of Ḥabash al-Ḥāsib's Ptolemaic *zīj*, known as the *Mumtaḥan* or *Arabic zīj*. Its provenance seems to be Baghdad, and it dates from around 1300.[105] It has rich marginalia, and some material presented therein is pertinent to our study for two reasons. First, it provides us with some interesting observational records showing how a set of astronomical tables could be treated a long time after their original composition, how their underlying theories could be tested against direct

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[104] See Mozaffari 2019b, Table 1 on p. 522 and pp. 541–542.

[105] On this MS, see Kennedy 1956, p. 151–153; van Dalen 1993, chapter 4.

observations, and how they underwent substantial changes for the sake of reconciling them with the observations. Second, a set of the mean daily motions of the Sun, Moon, and planets attributed to Abu ʾl-Wafāʾ is given in the marginal glosses.

4.1. The Mumtaḥan/Ḥabash’s theories in the fifteenth century

In the ninth/fifteenth century, the codex was in the hands of some competent anonymous astronomers, who left some interesting accounts of the predictions and the records of observations of some heavenly phenomena in the marginalia, which were even used later for serious modifications of the Mumtaḥan/Ḥabash’s planetary theories from 871/1466 onwards.

The title page presents in detail theoretical estimations and correlated numerical values of the predicted circumstances of two eclipses, one lunar and another solar.

At the top, we read: «the examination (*imtiḥān*) of the lunar eclipse that happened on the night of Saturday, 14 Ṣafar 825» (civil Hijra calendar, 6/7 February 1422, JDN 2240480/81, 5MCLE #08255). The time of the maximum phase was computed as 6;7 hours, apparently after sunset (at **17:43** Baghdad LT). The longitudes of the Moon and its ascending node at mid-eclipse are given respectively as 147;35° and 142;11°; then, the argument of latitude = 5;24° and the latitude = +0;27°. The magnitude was determined as 10;25 digits. The eclipse was entirely observable in Baghdad, and at the maximum phase, occurring at **22:50** LT, the Moon was located at an altitude of about **62°**. The magnitude was **0.922 ≈ 11** digits, not so far from the theoretical derivation. On the left side of the leaf, next to the quantitative data, we are told: «correct, happened at nighttime». Below it, in bold letters we read: «the intensity of its color was extremely low» (في لونه أضعف الشدّ[ة]).[106] According to the medieval theory of the lunar eclipse colors originating in India, the lunar disk was expected to appear as reddish black in the aforementioned ecliptical latitude.[107] This seems to have been in clear contrast to our anonymous observer’s

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[106] The remark is followed by a three-line gloss heavily crossed out and almost illegible, but the existence of the word «dust» (غبر) and the year number of «827» in it fuels speculation that it might have pertained to our anonymous astronomer’s probable experience with the very low-magnitude lunar eclipse of 13 Muḥarram 827 (17 December 1423, JDN 2241159, 5MCLE #08260, mag = **0.036**).

[107] For an overview, see, e.g., Goldstein 2005.

experience. In fact, in the partial lunar eclipses, the contrast between that portion of the lunar limb immersed in the umbral shadow and its part remaining in the penumbra is so sharp that the first appears very dark or black, without any perception of a hue.

At the bottom of the page, we are told of the circumstances of the solar eclipse on Thursday, 28 Shaʿbān 824[108] (28 August 1421, JDN 2240318, 5MCSE: #08137). The time of the true conjunction was computed as 6;3 hours after sunrise or 0;17 hours before noon (thus, sunrise was seemingly taken to have occurred at 05:40 LT; in Baghdad: **05:32** LT), and the midpoint of the eclipse, as 5;51 hours after sunrise, *viz.* 11:31 LT. The solar longitude at sunrise is given as 163;42°, and the true (geocentric) lunar longitude in the maximum phase as 163;56°. The longitude of the lunar ascending node was calculated as 150;47°, and hence, the true argument of latitude = 13;9°, and the true (geocentric) latitude = +1;5°. The apparent (topocentric) latitude was derived as –0;26° and the apparent distance in latitude between the two Luminaries as 0;19°. The magnitude is not provided, but it is evident that the eclipse was expected to be partial. Our anonymous astronomer emphasizes that it «did not occur». In reality, it did take place, but could be observed only in the Arctic. The principal reason for the mistaken prediction was the erroneous Ptolemaic solar and lunar parallax theories. In Baghdad, the true conjunction occurred at **11:45** LT, but the apparent conjunction at **11:04** LT, when the apparent distance in latitude of about **46**' between the Sun and Moon was greater than the sum of the halves their apparent angular diameters (Sun: **32**', Moon: **30**').

A report of the *conjunction* (*qirān*) between (*sic*! near appulse of) Venus and Jupiter in the morning on Monday, 24 [Dhi] 'l-hijja 86[3][109] [al-Hijra], 22 Tishrin I [1771 Alexander] (22 October 1459, JDN 2254252), i.e., about 38 years after the two mentioned eclipses, is given on f. 45r, just at the top of the table of Jupiter's mean annual and monthly motions in longitude (in the *expanded* years from 1 to 30, in intervals of one year) and yearly mean longitudes (in the *collected* years from 511 H to 691 H, in steps of 30 years). Our anonymous commentator derives the mean longitude of Jupiter at midday as 195;12,2° (NB. this is for two days before

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[108] The equivalent date in the Julian calendar in the Eastern Christian tradition, in which the years were counted according to the Seleucid era, mistakenly called the Alexandrian calendar, is also given as «3 Kānūn II», without indicating the year number, which is incorrect; it should be 28 Āb 1732 Alexander.

[109] The digit in the first place in the year number is illegible, but only this date in the 860s Hijra agrees with the report.

the date given), and states that it must be decreased by 1;30° to 193;42,2°, so that the resulting true longitudes of Jupiter, 196;14° (modern: **196;19**°), and of Venus, 196;39° (**195;57**°), indicate that at noon, «Venus surpassed Jupiter by the amount of true motion [after morning]. So, know that this *zīj* is obviously correct».

The two planets were *not*, in reality, in conjunction with each other in the morning. As the modern values just given denote, it took place on the same day, after sunset, when the two heavenly objects were below the local horizon. Between 05:26 Baghdad LT (the two planets rising to an altitude higher than 10°) and 05:48 LT (the start of civil twilight at Baghdad), Venus (λ = **195;37**°, β = **+1;35**°) still had to travel a distance of **39**' in longitude to join Jupiter (λ = **196;16**°, β = **+1;8**°).

From the longitude of Venus at noon, it is deduced that its morning longitude was taken as equal to about 196 1/3° on account of the fact that its true daily motion in longitude was about 1;15° at the time. According to our anonymous observer, it was also the longitude of Jupiter at that moment. However, his final correction implies that the latter should have been approximately equal to 196;14° – 1/4 × 0;12° ≈ 196 1/6° (NB. Jupiter's true daily longitudinal motion was about 0;12° at the time). Both figures, surprisingly, come close to the modern value (**196;16**°). It is probable that he did not notice this inconsistency or ignored it. Nevertheless, he might have had a strong reason to believe that the longitude of Venus was correct, and, instead, it was the mean longitude of Jupiter that required a substantial modification to achieve the same morning longitude as the first figure. Jupiter (and, in his estimation, also, Venus) was, in fact, nearly in conjunction with Spica (λ = **196;18**°) in the morning. In an updated Mumtaḥan star table extant in the Berlin copy (f. 62r),[110] the star is given the longitude of 188;5° for 304/916–917, and hence, applying the rate of precession of 1°/66^{y} yields, by *mere coincidence*, a longitude of about 196 1/3° for the date of the observation. Note that the higher precessional rate compensates for the error of about –2/3° in the initial value. This situation is similar to what we already saw in the case of Ibn al-Fahhād's value for the longitude of Regulus, updated from the *Almagest* catalogue (cf. Sect. 3.2.2).

It is curious that the same decrease of 1;30° in Jupiter's mean longitude as suggested by our observer seems to have been initially applied, in practice, to the original mean annual motions/positions table. However, it was, presumably, re-

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[110] See Mozaffari 2016–2017.

placed later by a decrease of –2;30°. It is not known whether the same observer changed his mind later, or whether another person was responsible for this change. In the two sub-tables appended in the left margin to the main table, the mean longitudes in the collected years were extended to 1111 H (in the Istanbul copy, closer to the original, they are given up to 691 H). The first has the values for 721 H to 991 H (10 rows), and the latter (below the first), for 871 H to 1111 H (nine rows). The tabular values up to 841 H are in excellent agreement with Ḥabash's theory. In the first table, the integer digits in the entries for 871 H to 991 H are scratched away by a sharp tool. Nonetheless, from the remaining traces (see Figure 9) as well as from the arc-minutes and arc-seconds digits, it is obvious that, at least, for the two rows (for 901 H and 931 H), the entries decreased by 1;30° with respect to the values derived from Ḥabash's theory (the arc-seconds in all five entries are in excellent agreement with them), which is in accord with the corrective amount our commentator announced previously. The latter sub-table has the entries for 871 H to 1111 H, which are on average 2;30° less than the values computed from Ḥabash's theory (the decreases are not consistent, running from –2;21° to –2;37°). Both corrective values are, indeed, astronomically relevant: Ḥabash's theory gives the value of 42;7° for the mean longitude of Jupiter for 1–1–871 H, as compared to the corresponding modern value of **40;14**° (cf. Figure 3).

Several points deserve comment here. **(I)** The observer of the 1459 near appulse of Venus to Jupiter seems, also, to have been responsible for the longitudes of the apogees registered at the top of the planetary mean motions/positions tables for 878 H and also listed in a separate table for 872 H (on f. 17v). **(II)** All quantities given in the text, including the noon longitude of Venus on 22 October 1459, are in fair agreement with the values calculated on the basis of Ḥabash's *zīj* for the meridian of Baghdad around the times given on the same dates, except for the initial value for the mean longitude of Jupiter, as noted earlier.

The material presented above provides us with more concrete evidence to ascertain what we have attempted to elucidate and clarify in the previous sections concerning the assessment of planetary theories, the application of the *correction* strategy, and its prevalent and widespread use in various and diverse ways by the medieval Islamic astronomers.

(I) Ibn al-Fahhād was not alone in thinking that the Mumtaḥan/Ḥabash's solar and lunar theories were satisfactory for the prediction of the circumstances of the eclipses. Insofar as the timings of such synodic phenomena, within the limits of the accuracy of medieval timekeeping methods, and their magnitudes were the focus of attention, it was quite possible for an astronomer even further distant in time from the Mumtaḥan observations than Ibn al-Fahhād to achieve the same (false) conclusion.

(II) The post-871/1466 *correction* of Ḥabash's theory of Jupiter in the Berlin copy of his *zīj* is merely another example exhibiting how background theories might undergo substantial changes (either improvements or deteriorations) on the basis of a minimum number of pieces of empirical evidence (say, a single alleged conjunction between the two auspicious bright planets, comparable with our reconstruction of the way in which Ibn al-Fahhād modified Khāzinī's theories of Jupiter and Saturn).

4.2. The mean motion values attributed to Abu 'l-Wafāʾ

In Sects. 2 and 3.2, we have explained in detail the sources of Ibn al-Fahhād's solar, lunar and planetary theories and reconstructed the ways through which he arrived at them. So, he has already been cleared of the allegation of plagiarism. Here, we present the mean daily motions attributed to Abu 'l-Wafā', as found in the marginalia of the Berlin MS of Ḥabash's *zīj*, in order to see whether any relation between them and Ibn al-Fahhād's parameter values can be found. They are given at the bottom of the tables for the hourly and daily mean motions of the planets:[111]

Saturn:	0; 2, 0,36, 4,27,58,33,41,41,42 °/d
Jupiter:	0; 4,59,16,58,50,44,30,49,53,17
Venus:	0;36,59,29, 7,49, 1,36, 9,21,59
Mercury:	3; 6,24, 6,59 45,22, 0,37,26,24
Moon:	13;10,35, 1,55,37,39, 6,16,45,43
	13; 3,53,56,17,50,25, 7,59,17,31
Node:	0; 3,10,37,35,10, 1,51,42,13,28

We have already quoted the value for the Sun in Sect. 2. The value for Mars is missing. Note that all of them are tropical.

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[111] Ḥabash, *Zīj*, B: ff. 32v (lunar node), 33v (Moon's mean longitude), 34v (Moon's mean anomaly), 41v (Saturn), 45v (Jupiter), 53v (Venus), and 57v (Mercury).

These differ, very slightly, from the basic values underlying Ḥabash's tabular entries: Saturn: ~ +0;0,0,0,0,4°, Jupiter: ~ –0;0,0,0,0,6°, Mercury: ~ +0;0,0,0,0,10°, Moon's mean motion in longitude: ~ +0;0,0,0,0,5°, Moon's mean motion in anomaly: ~ –0;0,0,0,0,2°, and the lunar node: ~ –0;0,0,0,0,14°, except for Venus, in which case the difference amounts to about +0;0,0,0,7°.

The degree of accuracy to which the mean motions are presented in the marginal glosses gives a strong impression that a mathematician, meticulous in mathematical rigor, was responsible for them (someone like Abu 'l-Wafā'; recall that, for example, in his *Majisṭī*, he wrote down Ptolemy's original values for the planetary inclinations, instead of the rounded values declared by Ptolemy and actually used in the *Almagest* latitude tables). Quite probably, there were a good number of ways for a competent mathematician/astronomer to extract/deduce the mean daily motions from the numerical tables in Ḥabash's *zīj* (e.g., simply by means of dividing the mean motion in a sufficiently long interval of time given in the tables into the number of days comprised in it) if he did not have access to them somewhere else. If these are, in reality, the values Abu 'l-Wafā' deployed in his works, then he appears to have adhered to the Mumtaḥan/Ḥabash's theories, except for the Sun and Venus. From this assumption, it immediately and directly results, again, that Abu 'l-Wafā' and Ibn al-Fahhād used different mean motion values.

Nevertheless, here we are confronted with a more serious problem, concerning the values of the mean daily motion in anomaly of the inferior planets. It comes as a great surprise that they are identical to the values Muḥyī al-Dīn adopts ed in his last *zīj*, the *Adwār*,[112] although for Mercury, he has 42 instead of 45 in the fifth sexagesimal fractional place. From his *Talkhīṣ*, we know that he measured all his parameter values for the Sun, Moon, and superior planets on the basis of his independent observations carried out at Maragha from the beginning of 1262 until the end of 1274. Nevertheless, since the sole surviving copy of this work is incomplete, ending somewhere in the middle of Book VIII (on the theories of planetary motion in longitude) just before beginning the presentation of the material related to the inferior planets,[113] we do not know how he derived his values for the parameters of these two planets.

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[112] al-Maghribī, *Adwār*, M: f. 75v, CB: f. 73v.

[113] On the preserved manuscript and contents of the treatise, see Mozaffari 2014, pp. 70, 75; 2018, pp. 595–596; 2018–2019, pp. 157–162.

On the one hand, an important point is that Muḥyī al-Dīn never gives his final numerical results in the case of the solar, lunar, and planetary mean motions to the same degree of precision as is available in the aforesaid values attributed to Abu ʾl-Wafāʾ, but up to the sixth sexagesimal fractional digit (for Moon, Saturn, Jupiter, Mars, and Mercury) or the seventh sexagesimal fractional digit (in the case of Sun and Venus), in both the *Talkhīṣ* and the *Adwār*. Accordingly, it seems more reasonable to conclude that it was Muḥyī al-Dīn that borrowed them from his Baghdadi predecessor (either directly from a work of the latter, or through an intermediary source, like a manuscript of Ḥabash's *zīj* commented upon by Abu ʾl-Wafāʾ, of which the Berlin MS may be a faithful apograph). Otherwise, it is clear that the assumption that these values are originally due to Muḥyī al-Dīn implies that **(a)** he had presented the way through which he reached these figures in the *Talkhīṣ* (i.e., in fact, he had finished this voluminous treatise), or in any other treatise of his own, and **(b)** someone continued his calculations (the final divisions in which the mean motions plus the complete cycles/revolutions were divided by the time intervals in terms of days and fractions of a day) in order to derive his values up to the tenth sexagesimal fractional place. If so, then a serious difficulty arises in explaining why the same assumed person would have attributed them to Abu ʾl-Wafāʾ and registered them along with a different set of the values for the mean daily motions of the Sun, Moon, and superior planets in the marginal commentaries on Ḥabash's *zīj*. Such a scenario is difficult to conceive, and in fact seems barely plausible at all.

On the other hand, we cannot be sure whether Muḥyī al-Dīn adopted the values for the mean motion in anomaly of the inner planets from an external source, rather than measuring them on the basis of his own observations; and if he did so, we do not know why. Did he deal with the inferior planets in the same manner as he accomplished the enormous, arduous task of constructing the fresh theories for the Sun, Moon, superior planets, eclipses, and fixed stars on the basis of his extensive, purposeful observations? And if the answer is affirmative (putting aside how probable it might have been), what were his reasons? We know that he also ran a program for testing the previous planetary theories available to him against his observations made at the Maragha observatory.[114] Did he obtain the result that the aforesaid values for the mean daily anomalistic motions of the inner planets were compatible with his observations, in which case

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[114] In the case of Mars, see Mozaffari 2018–2019, pp. 206–211.

there was no need for new derivations or for making any further modifications? Once again, we do not know.

Let us mention that his theory of Venus in the *Tāj al-azyāj* shows a close connection to Ḥabash's (cf. Figure 5). Moreover, he maintained (maybe under the influence of Ibn Yūnus) the erroneous Indian-originated hypothesis of the equality of the orbital elements of the Sun and Venus. Although he put it aside after moving to Maragha, his theory of Venus in the *Adwār* (as the related graphs in Figure 5 suggest) bears a conspicuous *node* relation with Ibn Yūnus' corresponding theory *exactly* in the Hijra epoch (the node falls below the lower limit of the abscissa in Figure 5), so that both theories give nearly the same value of 43;10° for the planet's mean epicyclic anomaly at the time (noon in the meridian of Maragha). Consequently, Muḥyī al-Dīn appears to have rectified Ibn Yūnus' theory, which gives a highly erroneous result in his time, by combining Abu 'l-Wafā''s (?) mean daily epicyclic motion with Ibn Yūnus' radix value for the mean anomaly. How probable it is that such a synthesis could yield satisfactory results with regard to reconciling theory and observation awaits further investigation. Also, his epoch value of 71;35° for the longitude of the apogee for 17 January 1232 (error ~ –6;20°) seems to have been updated from Ptolemy's value of 55;0° for the mid-130s by the precessional rate of 1°/66^{y}. Accordingly, generally speaking, he does not appear to have made any meaningful progress over the course of his career with regard to the theory of Venus' motion in longitude. This finding seems to be in stark contrast to the significant improvements he made in the orbital inclinations of the inferior planets, and hence in their latitude theories.[115] As can be seen, there is a high degree of uncertainty surrounding all these issues, which needs a profound study; hopefully, it will be resolved in the future.

5. CONCLUSION

In the above sections, we have seriously considered the bewildering dilemma over the alleged planetary theories of Abu 'l-Wafā' in relation to the ones set forth in the 12th-century *ʿAlāʾī zīj* composed by Ibn al-Fahhād, according to a wild claim by the anonymous author of the 13th-century *Shāmil zīj* that the latter are not original, but are taken from Abu 'l-Wafā''s. This proclaimed case of fraud,

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[115] See Mozaffari 2016, pp. 520–521, 525, 529–535.

crucial not least because the *ʿAlāʾī zīj* was a highly influential *zīj* in the late Islamic period, had remained unexplored for 66 years since its formulation and the classification of the historical sources involved by the late E.S. Kennedy in 1956. We have clarified the situation and rejected the fraud with the following results.

(I) For the Sun and Moon, Ibn al-Fahhād borrowed all motional and structural parameter values (i.e., eccentricity and epicycle's radius) from the Mumtaḥan theories, together with the precessional rate of 1°/66^y. In contrast, Abu 'l-Wafā''s accurate measurement of the autumnal equinox of 974 CE, together with his specific rate of precession of 1°/75^y, leave no doubt that he did not adopt the Mumtaḥan solar theory. So, almost certainly, our Baghdadī and Shirwānī astronomers had nothing in common in this respect.

(II) In the case of the planetary theories that constitute the core of the puzzle and, indeed, its most difficult part from a technical perspective, a close analysis of Ibn al-Fahhād's planetary parameters convincingly demonstrates that they were:

(A) derived from his observations, independent of, or bearing no significant relation to, any other available theory, which is the case with Mars;

(B) the results of his modifications/deteriorations of the parameters of other theories on the basis of the results he achieved from his observations, which is the case with Saturn and Jupiter: he changed Khāzinī's mean motions in longitude and/or radix mean longitudes on the basis of his observation of the conjunction between the two planets in 1166 CE; or

(C) taken directly and faithfully from other sources known and available today, which is the case with Venus; in this case his source theory was, again, Khāzinī's.

(III) We have seen that the difference between the mean motion values attributed to Abu 'l-Wafā' in the marginalia of the Berlin MS of Ḥabash's *zīj* and those belonging to the Mumtaḥan tradition are of the order of the fifth sexagesimal fractional place in the case of the Moon, Saturn, Jupiter, and Mercury, which is insignificant, to the extent that, under the condition that they were really what Abu 'l-Wafā' adopted in his work, it can be safely

> assumed that he derived them from the tables in the Mumtaḥan and/or Ḥabash's *zīj*es. The degree of accuracy to which the mean motions are presented in the marginal glosses gives — to the tenth sexagesimal fractional digit, not usual in the Islamic astronomical corpus — weighs heavily in favor of the idea that a mathematician, meticulous in mathematical rigor, was behind them. This supports the hypothesis that they might have been a work done by Abu 'l-Wafā'. Nevertheless, we are confronted with the dilemma that, in the case of Venus, the difference is of the order of the fourth sexagesimal fractional fourths, and, surprisingly, the values for both interior planets are equal to the values Muḥyī al-Dīn al-Maghribī gives in his *Adwār al-anwār*. Nothing is clear in this regard, and a further detailed investigation is required in order to throw light on this new problem.

The origins of the planetary theories in the *ʿAlā'ī zīj* are revealed by a simple method we have used in order to visualize and clarify the interrelation and interdependence of the planetary theories established in the medieval Islamic period: namely, by plotting the errors in their mean positions in the ancient and medieval periods. In doing so, we have presented part of our comprehensive study on this general topic in order to show its applicability and the original results that it can achieve.

The types of the relations between the planetary theories in different *zīj*es assumed to have a solid observational basis have been brought to light by considering the fundamental ideas and strategies set forth in the handbooks and treatises penned in medieval Islam which have not been studied in depth; these strategies are classified under the general, frequently-used terms *istidrāk* («correction») and *iʿtibār* («experiment»). A detailed account of the development of planetary astronomy in medieval Islam in both theoretical and practical aspects has yet to be written.

In our present case study, we conclude that in the case of Saturn, Jupiter, and Venus, Ibn al-Fahhād did not apply the *experimental* method established by Khāzinī. The effects of the defect in the fundamental parameter values were not isolated from each other; this would have needed a great deal of time and energy to conduct a large number of previously-planned observations, in each of which several specific requirements and conditions had to be fulfilled, and to overcome various kinds of probable or unexpected difficulties involved in their performance. Rather, what he did in the case of the three planets, as examined in this study, is typical of what the medieval astronomers performed more or less routinely: that is, he carried out an analysis of his predecessors' theories in some specific situations in a limited region of the ecliptic. For Mars, however, he seems to have used the *standard* method, i.e., making a direct comparison between his observations (which were, as he explicitly stated, in agreement with Ibn al-Aʿlam's theory) and those documented in the authoritative *Almagest*.

The last point to emphasize is that the only clear *breach of ethics* Ibn al-Fahhād committed in his *ʿAlāʾī zīj* was his failure to explicitly mention the fact that the source of his theories for Saturn, Jupiter, and Venus was al-Khāzinī, while at the same time criticizing his Georgian/Khurāsānian predecessor's legacy.

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Mīrzā I (1384–1415 ce), the ruler of central Iran from 1409 ce, who had a strong interest in knowledge and culture and was Jamshīd Ghiyāth al-Dīn al-Kāshī's (d. 22 June 1429 ce) patron (cf. Kennedy 1998, p. 2; Mozaffari 2020–2021, p. 70, note 2), finished on 3 Ramaḍān 814/19 December 1411); L: Leiden, Universiteitsbibliotheek, Or. 75; F: Paris, Bibliothèque nationale de France, Persan 163 (copied by Aṣīl al-Dīn Ḥasan, the second son of Naṣīr al-Dīn al-Ṭūsī); B: Berlin, Staatsbibliothek Preußischer Kulturbesitz zu Berlin, Sprenger, no. 1853 (completed in 689 h/1290 ce).

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Table 1: Isolation of the sources of the longitudinal errors in the theories of Venus established by Ibn al-Aʿlam, Khāzinī and Ibn al-Fahhād at its near appulses to Regulus during the 1160s.

(A) General data:

		Date	JDN	Elongations	$\bar{\kappa}$	$\bar{a}$
1	**W**	1160–09–09	2145000	**−35°**	**99°**	**270°**
2	**E**	1161–06–28	2145292	**+36**	**26**	**90**
3	**W**	1163–09–20	2146106	**−46**	**109**	**232**
4	**E**	1166–06–18	2147108	**+45**	**16**	**130**
5	**W**	1168–09–09	2147922	**−35**	**98**	**271**
6	**E**	1169–06–28	2148214	**+36**	**26**	**91**

W = Morning star in Western elongation | **E** = Evening star in Eastern elongation.

(B) Ibn al-Aʿlam:

	IA	Th. Dev.	e	r	$\bar{a}$	λ_A	$\bar{\lambda}$	Sum
1	**− 2.2’**	+ 6.4’	−11.2’	+ 8.6’	−2.4’	−0.6’	−2.0’	− 1.2’
2	**−41.2**	−16.4	− 8.3	− 8.9	−2.4	−2.3	−2.0	−40.3
3	**+ 0.5**	+ 5.1	−15.7	+16.3	−1.0	−0.7	−2.0	+ 2.0
4	**−53.7**	−17.6	−11.5	−17.1	−0.6	−3.5	−2.0	−52.3
5	**− 2.7**	+ 6.2	−11.1	+ 8.4	−2.5	−0.6	−2.0	− 1.6
6	**−42.1**	−16.9	− 8.5	− 9.1	−2.3	−2.3	−2.0	−41.1

▫ The underlying values/errors: $e = 1;2^p$ | $r = 43;10^p$ (= Pt) | $d\bar{a} = -0;7°$ | $d\lambda_A = -2;40°$ ($\lambda_A \approx$ **76.6**°) | $d\bar{\lambda} = -0;2°$.

(C) Khāzinī:

	Kh	Th. Dev.	e	r	$\bar{a}$	λ_A	$\bar{\lambda}$	Sum
1	**+ 5.8**	+ 6.4’	−27.8’	+ 8.6’	−9.4’	−0.2’	+28.9’	+ 6.5’
2	**−28.0**	−16.4	−20.8	− 8.9	−9.1	−0.7	+28.6	−27.3
3	**+ 6.8**	+ 5.1	−38.9	+16.3	−3.8	−0.2	+28.9	+ 7.4
4	**−38.5**	−17.6	−28.4	−17.1	−2.4	−1.0	+28.4	−38.1
5	**+ 6.1**	+ 6.2	−27.6	+ 8.4	−9.5	−0.2	+28.9	+ 6.2
6	**−28.2**	−16.9	−21.0	− 9.1	−9.0	−0.7	+28.6	−28.1

▫ The underlying values/errors: $e = 1;15^p$ | $r = 43;10^p$ (= Pt) | $d\bar{a} = -0;27°$ | $d\lambda_A = -0;47°$ ($\lambda_A \approx$ **76.6**°) | $d\bar{\lambda} = +0;29°$

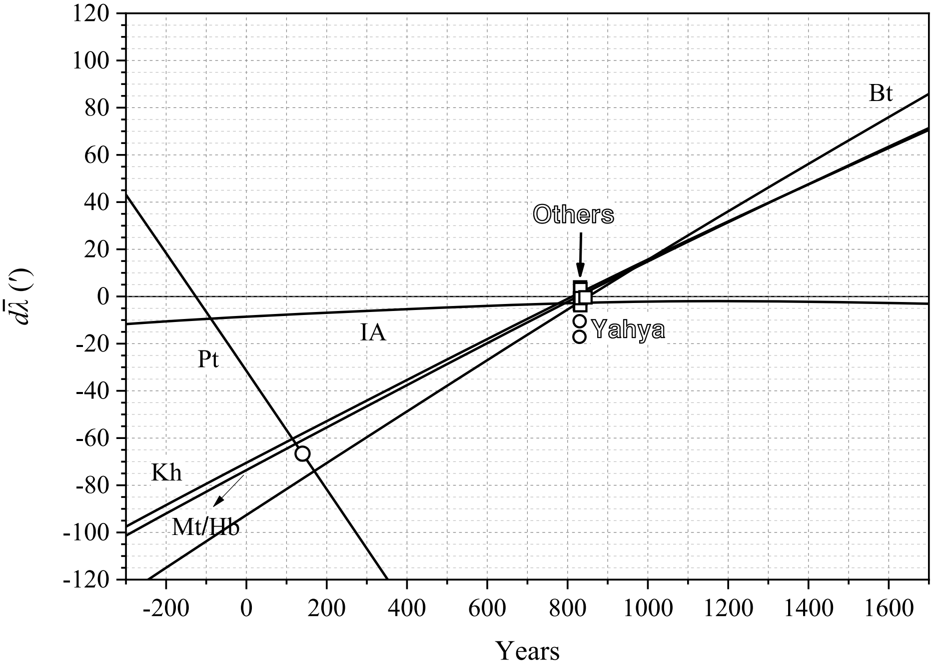
(D) Ibn al-Fahhād:

	F	Th. Dev.	e	r	$\bar{a}$	λ_A	$\bar{\lambda}$	Sum
1	**+21.4'**	+ 6.4'	−11.2'	+ 8.6'	−10.1'	−0.2'	+28.9'	+22.4'
2	**−16.4**	−16.4	− 8.3	− 8.9	− 9.8	−0.7	+28.6	−15.5
3	**+29.4**	+ 5.1	−15.7	+16.3	− 4.1	−0.2	+28.9	+30.3
4	**−21.8**	−17.6	−11.5	−17.1	− 2.6	−1.0	+28.4	−21.4
5	**+21.6**	+ 6.2	−11.1	+ 8.4	−10.2	−0.2	+28.9	+22.0
6	**−16.4**	−16.9	− 8.5	− 9.1	− 9.6	−0.7	+28.6	−16.2

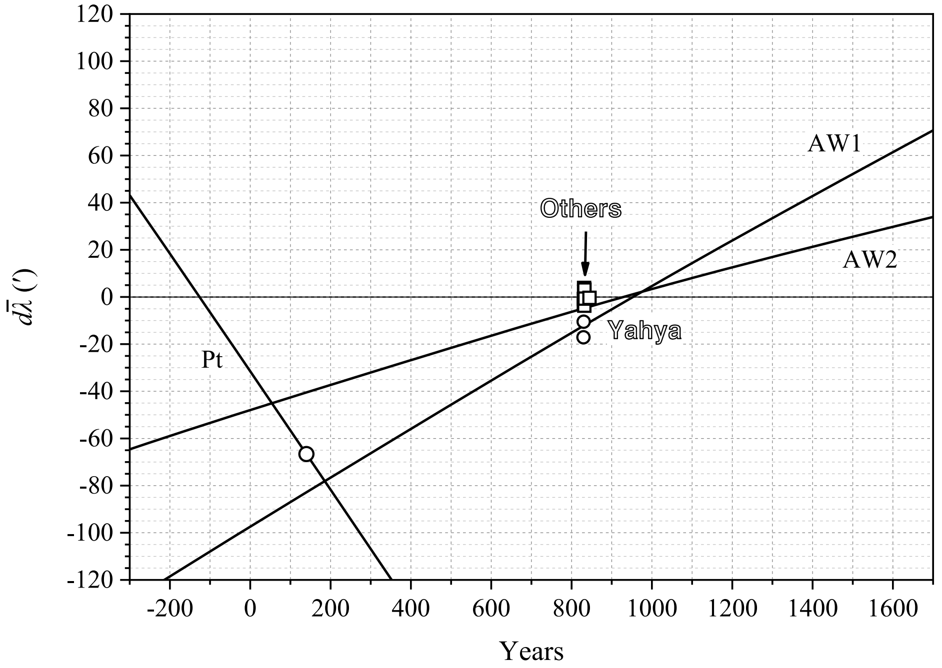
□ The underlying values/errors: $e = 1;2^p$ | $r = 43;10^p$ (= Pt) | $d\bar{a} = -0;29°$ | $d\lambda_A = -0;47°$ ($\lambda_A \approx$ **76.6°**) | $d\bar{\lambda} = +0;29°$.

TABLE 2: Boundaries of the longitudinal errors in the theories of Venus laid down by Ibn al-Aʿlam, Khāzinī and Ibn al-Fahhād in the 1160s (elongations > 30°).

	IA		**Kh**		**F**	
W	1165–02–28	+1;49°	1165–03–01	+3;41°	1165–02–28	+2;48°
E	1166–08–12	−1;24	1166–08–12	−1;22	1166–07–09	−0;24



(a)



(b)

Figure 1: (a) Errors in the mean longitudes of the Sun in the solar theories of the Mumtaḥan team (Mt), al-Battānī (Bt), Ibn al-Aʿlam (IA), and al-Khāzinī (Kh), in comparison with those in the Hipparchian-Ptolemaic solar theory. (b) Errors in the mean longitudes in Abu 'l-Wafā''s (AW) solar theory on the basis of the two values for the mean daily motion given in Sect. 2. The open circles in both figures stand for the errors $d\bar{\lambda}$ corresponding to the errors in the equinox times measured by Ptolemy and Yaḥyā b. Abī Manṣūr, and the open squares, clustered around 830–844, display the errors committed by other ninth-century astronomers observing in Baghdad and Damascus (Khālid b. ʿAbd al-Malik al-Marwarūdhī, etc.), which are significantly less than Ptolemy's and Yaḥyā's ones.

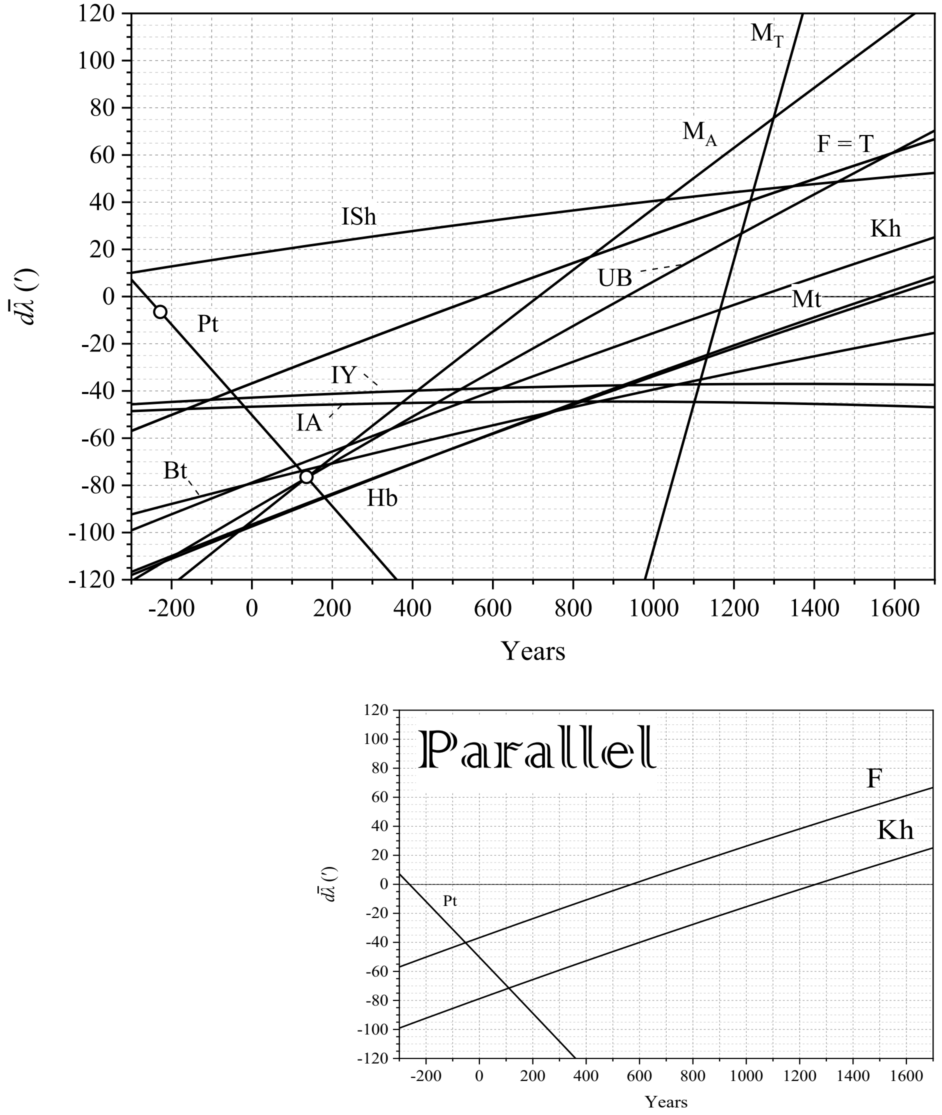


Figure 2: Errors in Saturn's mean longitude in the medieval Islamic theories. Abbreviation in Figures: Pt = Ptolemy (*Almagest*), Mt = Mumtaḥan, Hb = Ḥabash, Bt = Battānī, IA = Ibn al-Aʿlam, IY = Ibn Yūnus, Kh = Khāzinī, F = Ibn al-Fahhād, T = Naṣīr al-Dīn al-Ṭūsī (the *Īlkhānī zīj*), M_A = Muḥyī al-Dīn al-Maghribī in the *Adwār al-anwār*, M_T = Muḥyī al-Dīn in the *Tāj al-azyāj*, ISh = Ibn al-Shāṭir, UB = Ulugh Beg.

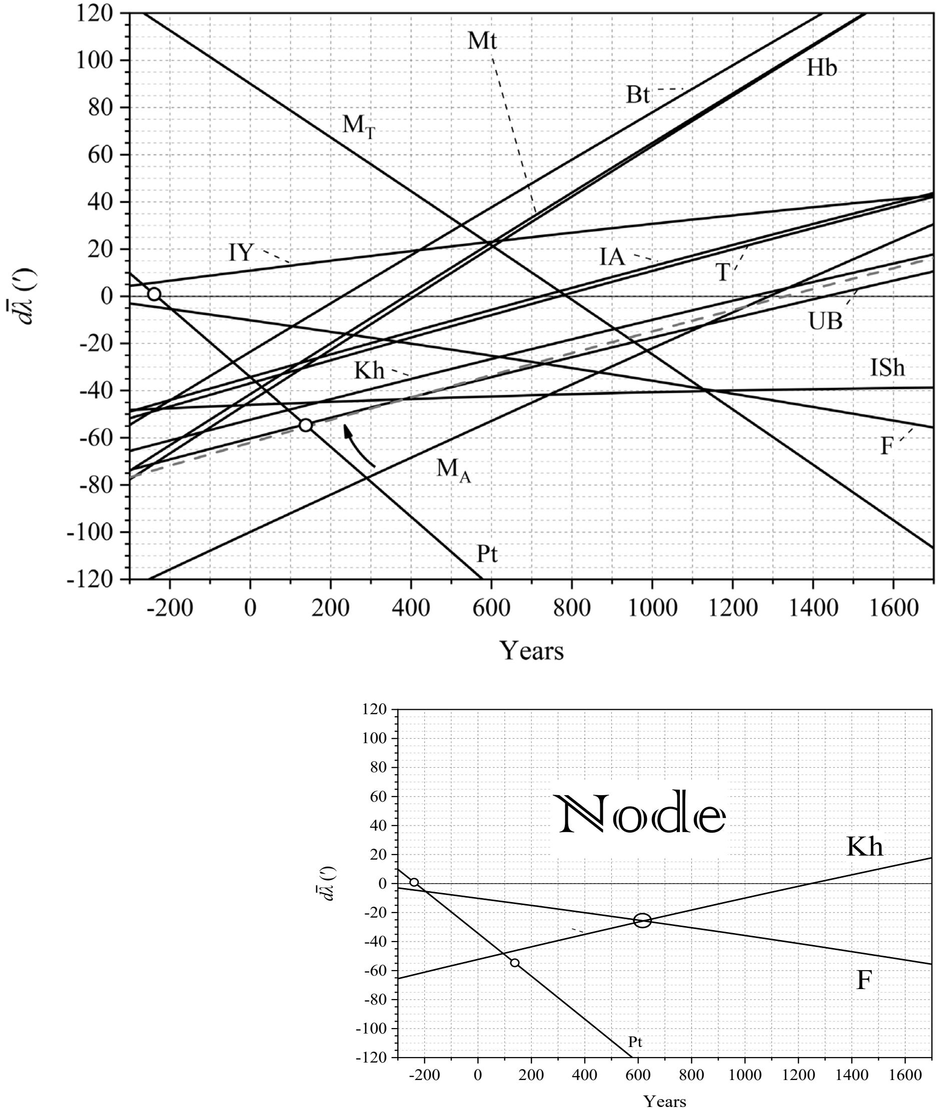


FIGURE 3: Errors in Jupiter's mean longitude in the medieval Islamic theories.

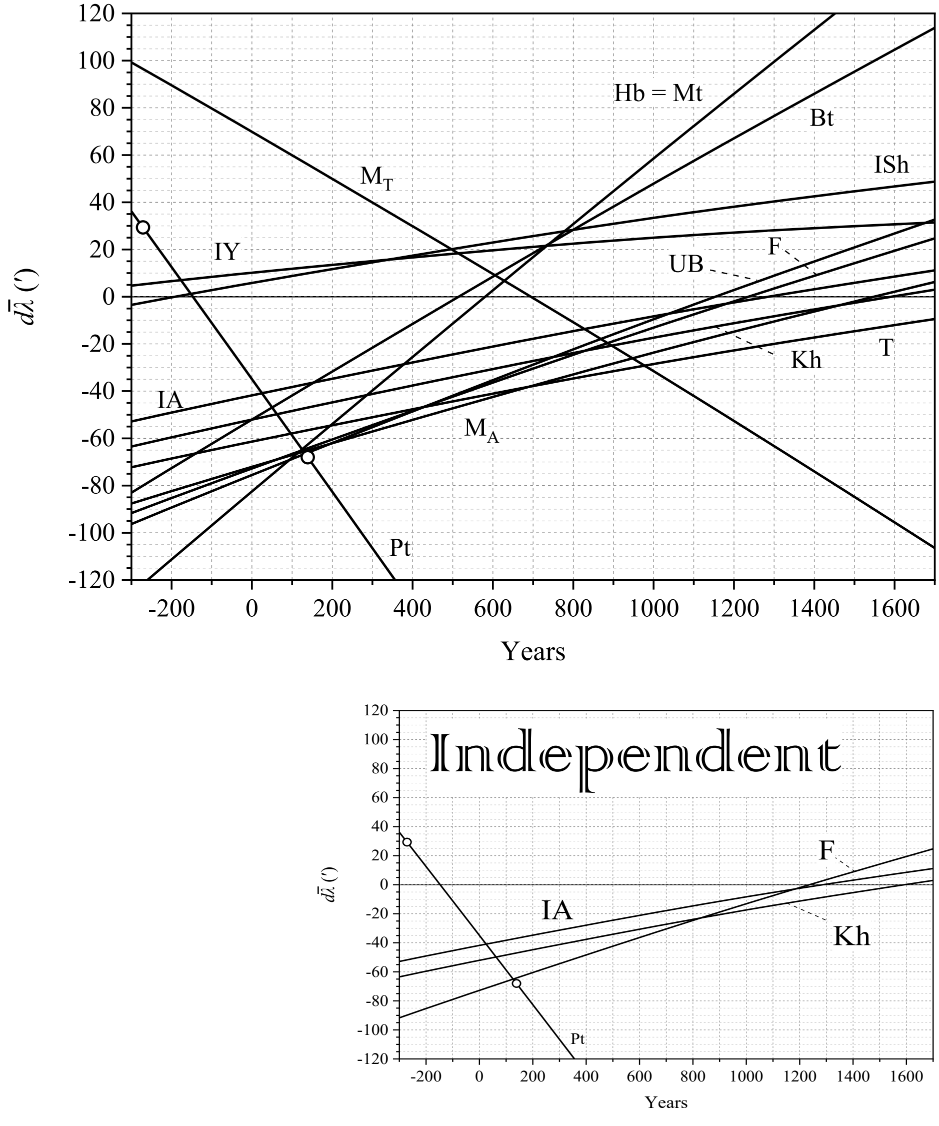


Figure 4: Errors in Mars's mean longitude in the medieval Islamic theories.

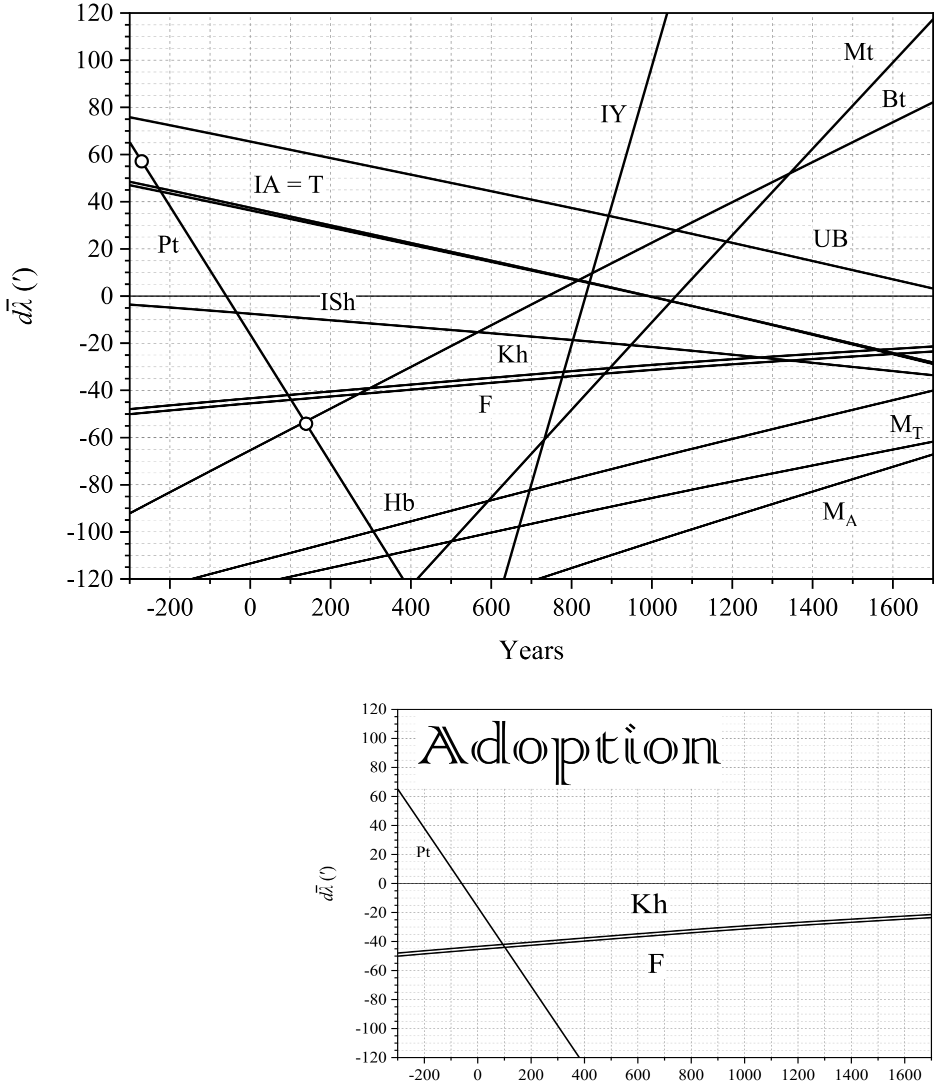
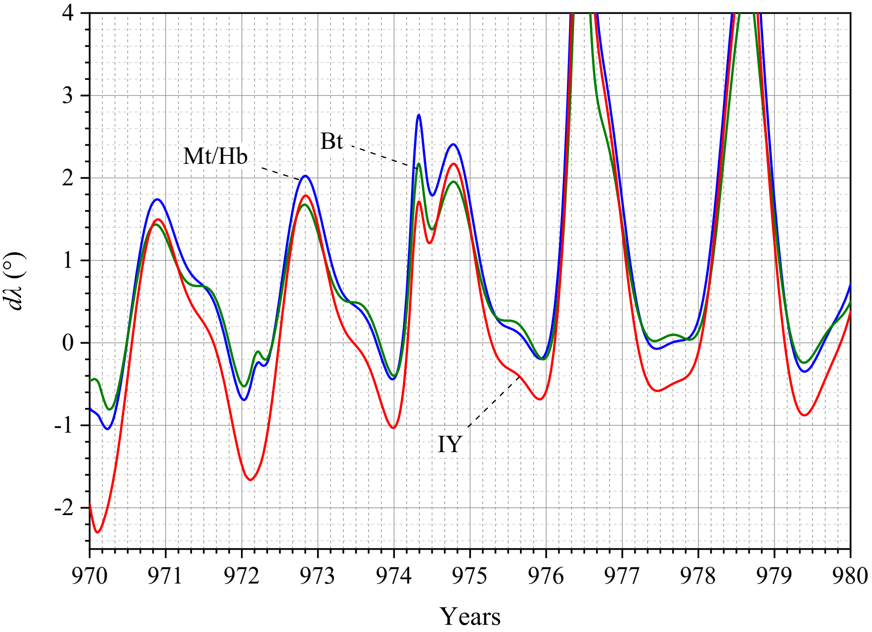
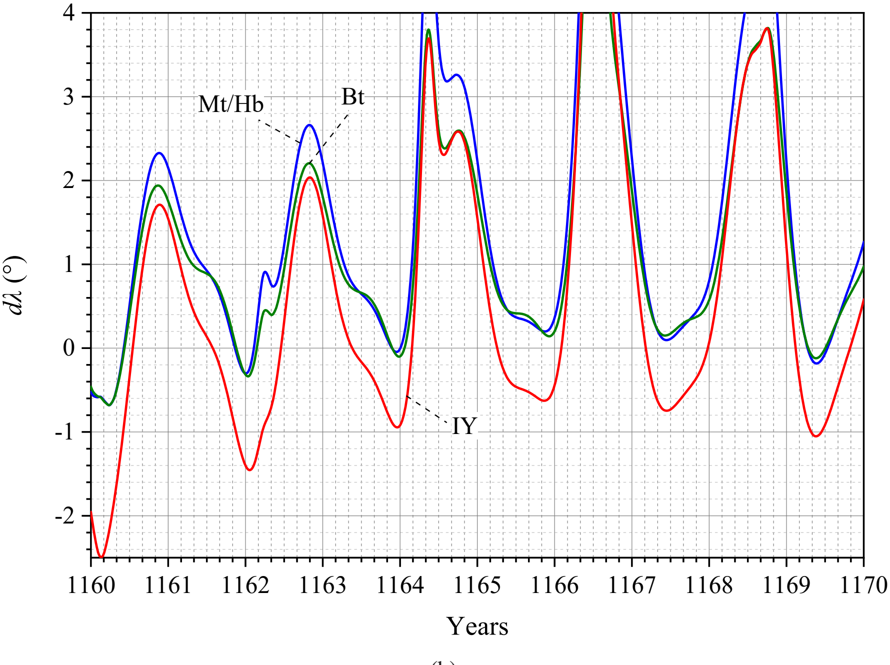


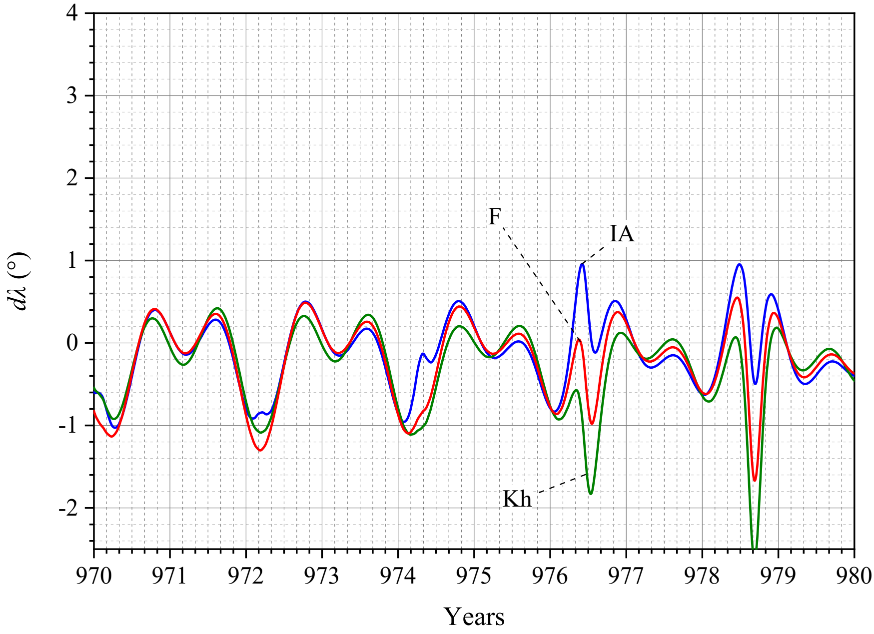
FIGURE 5: Errors in Venus's mean longitude in the medieval Islamic theories.


4
3
2
1
0
-1
-2
dλ (°)
970
971
972
973
974
975
976
977
978
979
980
Years
Mt/Hb
Bt
IY

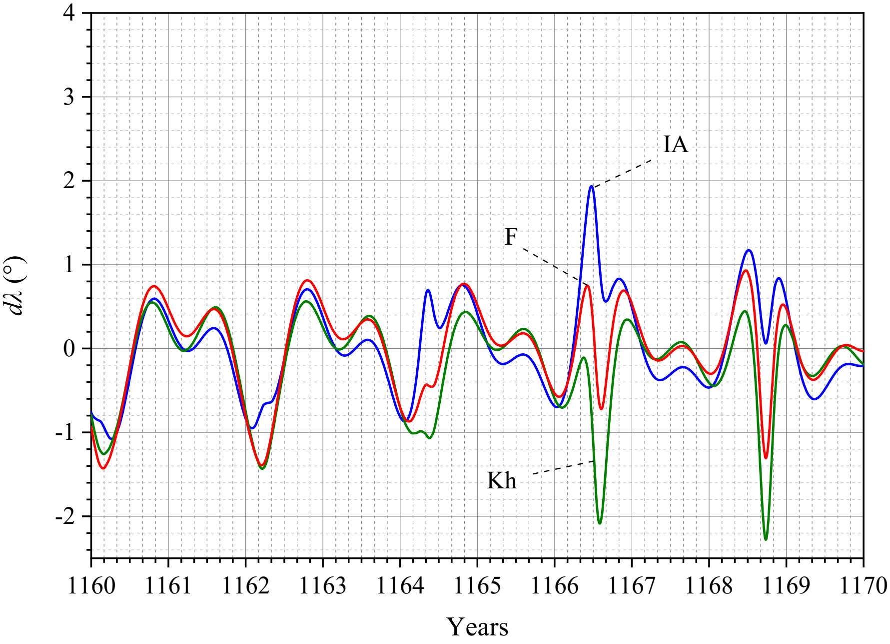
(a)


4
3
2
1
0
-1
-2
dλ (°)
1160
1161
1162
1163
1164
1165
1166
1167
1168
1169
1170
Years
Mt/Hb
Bt
IY

(b)


4
3
2
1
0
-1
-2
dλ (°)
F
IA
Kh
970
971
972
973
974
975
976
977
978
979
980
Years

(c)


4
3
2
1
0
-1
-2
dλ (°)
IA
F
Kh
1160
1161
1162
1163
1164
1165
1166
1167
1168
1169
1170
Years

(d)

Figure 6: Errors in the longitudes of Mars as computed on the basis of the Mumtaḥan/Ḥabash's, Battānī's, and Ibn Yūnus' theories displayed in (a) for the 970s and in (b) for the 1160s, in comparison with those as derived from Ibn al-Aʿlam's, Khāzinī's, and Ibn al-Fahhād's theories in the two mentioned decades exhibited, respectively, in (c) and (d). In the online version of the present paper: Hb and IA (blue), Bt and Kh (green), and IY and F (red).

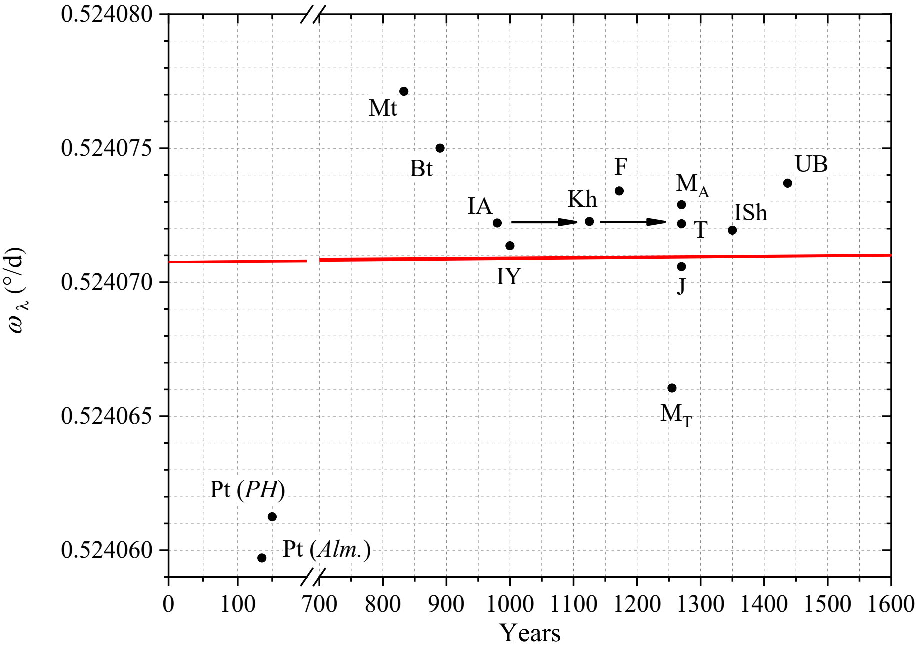


Figure 7: Mars' mean daily motion in longitude in the medieval Islamic theories (*Alm.* = *Almagest*, *PH* = *Planetary Hypotheses*).

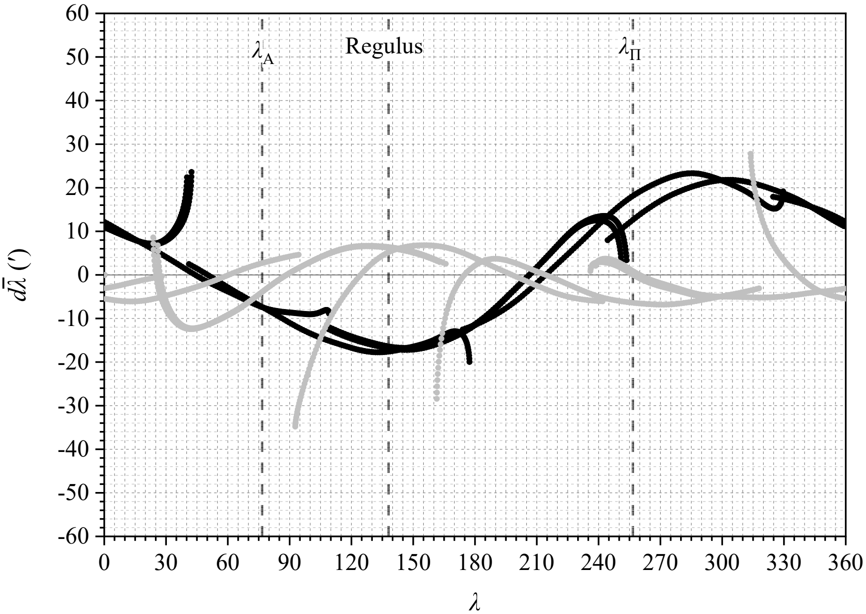


FIGURE 8(a): Theoretical deviations of the Ptolemaic model from the modern theory in the longitudes of Venus in the 1160s. The curves in light grey indicate the western elongations, and the black curves stand for the eastern ones.

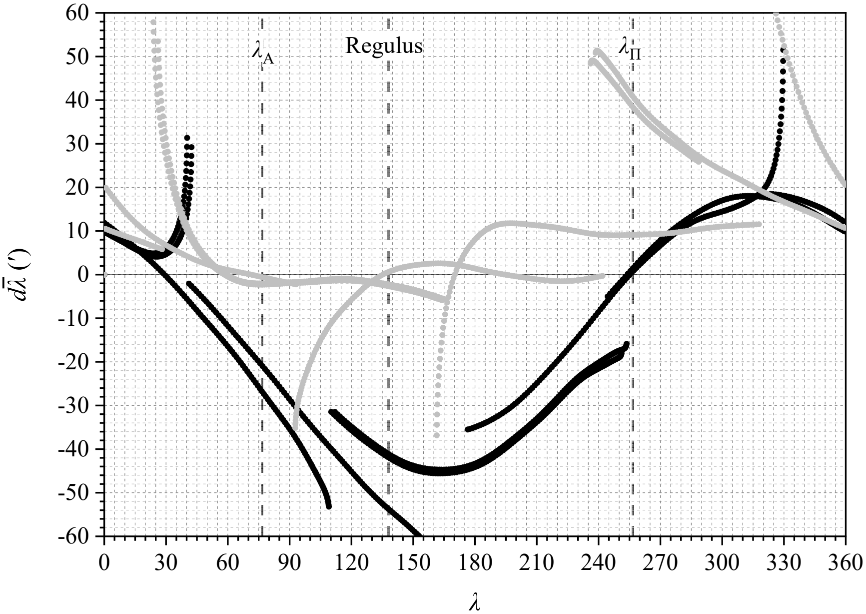


FIGURE 8(b): Errors in the longitude of Venus in Ibn al-Aʿlam's theory in the 1160s.

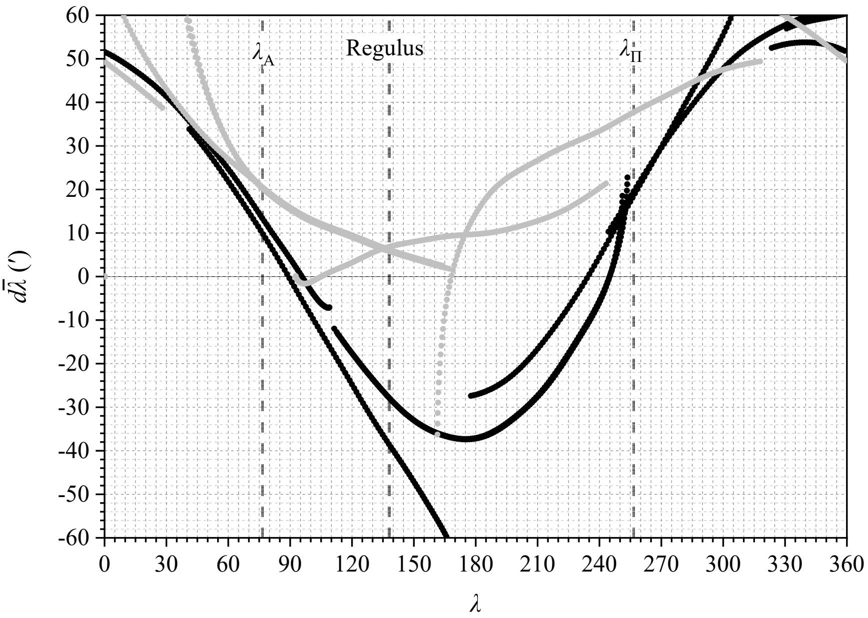


Figure 8(c): Errors in the longitude of Venus in Khāzinī's theory in the 1160s.

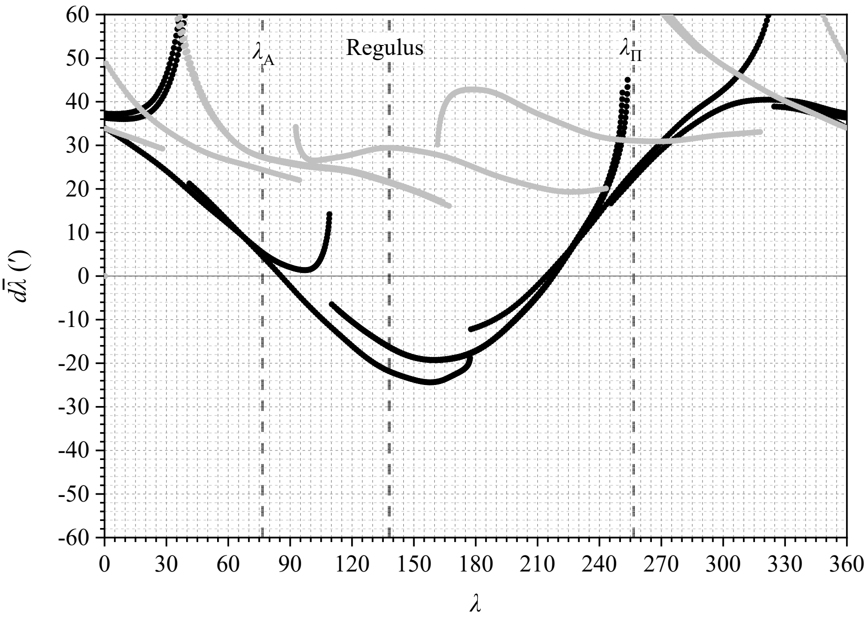
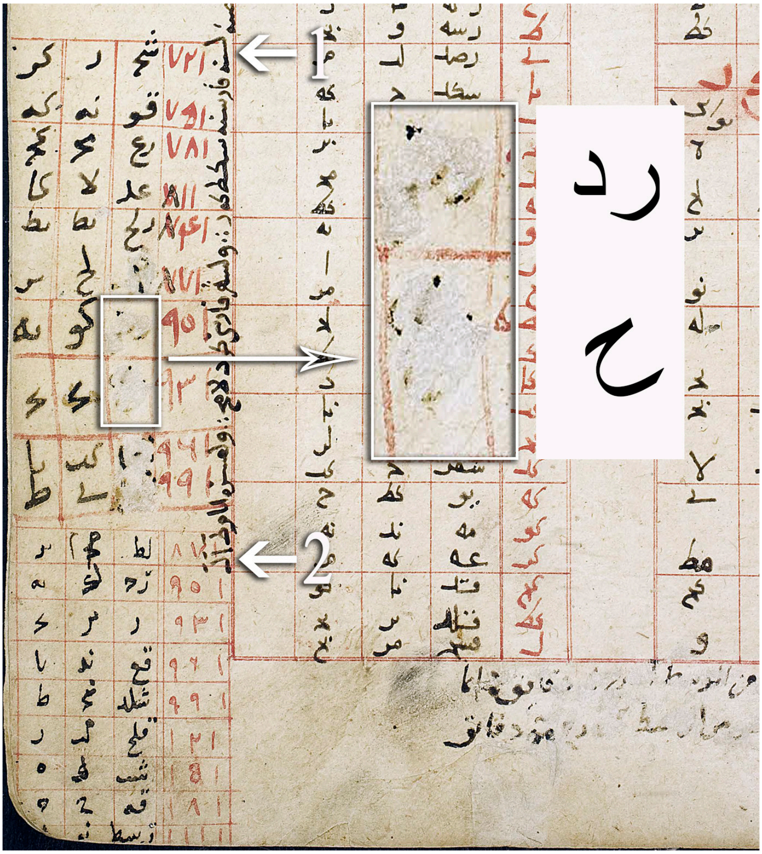


Figure 8(d): Errors in the longitude of Venus in Ibn al-Fahhād's theory in the 1160s.



Years	Computed from Ḥabash	First sub-table	Difference	Second sub-table	Difference
871	42; 7,17°	[?];38,17°		39;43,17°	−2;24°
901	205;55,15	204;26,15	−1;29°	203;34,15	−2;21
931	9;43,13	8;13,13	−1;30	7;17,13	−2;26
961	173;31,11	[?];22,11		170;54,11	−2;37
991	337;19, 9	[?];10, 9		334;53, 9	−2;26
1021	141; 7, 7			138;32, 7	−2;35
1051	304;55, 5			302;30, 5	−2;25
1081	108;43, 3			105; 8, 3 [106;…?]	−3;35! [−2;35?]
1111	272;31, 1			269;56, 1	−2;35

FIGURE 9: Two sub-tables appended to Jupiter's mean annual longitudes in the Berlin MS of Ḥabash's *zīj*.

Appendix: Mean motions and mean radix positions

In what follows, we summarize the mean daily motions in longitude (Sun and superior planets) and in anomaly (Venus) underlying the solar and planetary mean motion/position tables, together with the correlated mean epoch positions adopted, in some important and influential *zīj*es compiled in the medieval Near and Middle East. Most of the mean daily motions were derived by Benno van Dalen from the numerical tables in the works under investigation here (private communication), except for ʿAbd al-Raḥmān al-Khāzinī and Muḥyī al-Dīn al-Maghribī, who explicitly present their values for this sort of the fundamental parameters. Further close inspections by the present author, running since 2016, in both forms of occasional spot checks and the entire reconstruction of a specific table in a set of astronomical tables on the basis of a calculated value, proved that they are remarkably accurate and trustworthy.

A crucial note to emphasize is that as regards the comparison between two historical values for the mean daily motion of a planet, a difference of the order of at least the fourth sexagesimal fractional place may be significant. Otherwise, there would remain a little doubt that the two values should be taken as equal to each other (i.e., a source is depended upon another, or both share a common ancestor).

A. The *Mumtaḥan zīj*/Ḥabash's Ptolemaic *zīj* (Baghdad)

(E: ff. 13v–14r (Sun), 23v–25r (Saturn), 28v–30r (Jupiter), 35v–37r (Mars from Ibn al-Aʿlam), 39r–v, 43r–v, L: ff. 66r–v, 75v–77r, 80v–82r, 85v–87r (Mars from Ibn al-Aʿlam), 90v–91r, 94v–95r. See van Dalen 2004 for MS L. Ḥabash, B: ff. 29r–v, 41r–v, 45r–v, 49r–v, 53r–v, 57r–v, I: ff. 89r–v, 103r–v, 107r–v, 111r–v, 115r–v, 119r–v. See Debarnot 1987 for MS. I).

Sun	0;59, 8,20,35,14,37,48 °/d	87; 3, 0,56°
Saturn	0; 2, 0,36, 5,25,28,46	237;38,26,54
Jupiter	0; 4,59,16,57,51,52, 4	273; 9,56,14
Mars	Non-existent, = Hb?	
Venus	0;36,59,30,25, 0, 1,58	118;49,27, 0
Mercury	3; 6,24, 6,59,35,20,33	170;40,52,51
	Date:	16– 6– 632
	JDN	1952063

Sun	0;59, 8,20,35,14,45 °/d	116; 2,14°
Saturn	0; 2, 0,36, 4,24	116;15,52
Jupiter	0; 4,59,16,58,56,41	331;56,15
Mars	0;31,26,40,39,30,26	212;46,11
Venus	0;36,59,29, 0,54	45; 3,53
Mercury	3; 6,24, 6,59,34	75; 7,38
	Date:	16– 7– 622
	JDN	1948440

B. al-Battānī (d. 929 CE, Raqqa & Antakya): *Ṣābiʾ zīj*

(E: ff. 164v–170r, Nallino [1899–1907] 1969, Vol. 2, *passim*.)

Sun	0;59, 8,20,46,55, 0 °/d	340;23,37,24°
Saturn	0; 2, 0,35,50,44	87;14,19
Jupiter	0; 4,59,16,54,54,57	260; 3,34
Mars	0;31,26,40,15,11,13	117;57,40
Venus	0;36,59,29,26,44	231;11
Mercury	3; 6,24, 7,45, 8,35	262;59,23
	Date:	29– 2– 620
	JDN:	1947572

C. Ibn al-Aʿlam (d. 985 CE, Baghdad): *Aḍudī zīj* (lost)

(preserved in Sayf al-munajjim's *Ashrafī zīj* (*ca*. 1300 CE, Shiraz), F: ff. 231v–234r, G: 248v–249v. See, also, Kennedy 1977; Mercier 1989).

Sun	0;59, 8,19,46,41,38,33,45 °/d	357;58,41°
Saturn	0; 2, 0,35,28,28	164;17,41
Jupiter	0; 4,59,16,22,40	118;45,42
Mars	0;31,26,39,35,51	181;54, 1
Venus	0;36,59,28,12,19	319;37, 5
Mercury	3; 6,24, 6,59,23	243;41,19
	Date:	13– 3–1303
	JDN:	2197050

The epoch values for the meridian of Shiraz ($L = 88°$ from the Fortunate Islands).

D. Ibn Yūnus (d. 1009 CE, Cairo): *Ḥākimī zīj*

(L: pp. 137–172).

Sun	0;59, 8,19,44,10,31,14 °/d		87;23,17, 0°
Saturn	0; 2, 0,35,30,24,39,27	= IA?	237;55,41, 3
Jupiter	0; 4,59,16, 6,34,31,14		273; 7,37,56
Mars	0;31,26,39,24,49,38,38		311;44, 0, 0
Venus	0;36,59,34,24,39,27, 7		118;10,44,10
Mercury	3; 6,24, 8,13, 9, 2,28		170;32,31,49
		Date:	16– 6– 632
		JDN	1952063

E. Khāzinī (fl. 1100–1130 CE, Marw): *Muʿtabar zīj*

(V: ff. 163r–v, L: ff. 102r–v).

Sun	0;59, 8,20,33,53, 4,29,40, 0 °/d		116; 0,25°
Saturn	0; 2, 0,36, 4,43, 2, 8, 0	= Mt/Hb?	116;33,52
Jupiter	0; 4,59,16,19,53,47,11,20		331; 7, 9
Mars	0;31,26,39,36,34, 5,16,50	= IA?	212; 8,31
Venus	0;36,59,28,43, 1,37,38,20		45;52,55
Mercury	3; 6,24, 7, 9,39,35,45,50		76; 2, 5
		Date:	16– 7– 622
		JDN	1948440

The epoch values for the meridian of Qubba ($L = 90°$ from the western shore of the Encompassing Ocean).

F. Ibn al-Fahhād (fl. 1160–1180 CE): *ʿAlāʾī zīj*

(pp. 70–72 (Sun), 94–101 (Saturn), 110–114, 132, 117 (Jupiter), 126–131, 136–137, 141 (Mars), 143–144, 148–153 (Venus), 164–171 (Mercury)).

Sun	0;59, 8,20,35,14,37,48 °/d	= Mt	318;49,41,19°
Saturn	0; 2, 0,36, 4,33,33	= (Mt/Hb?)/Kh	1;33,19
Jupiter	0; 4,59,15,39,41, 0		96;50, 2
Mars	0;31,26,39,51,21, 0		285;40, 1

Venus	0;36,59,28,43, 1,38	= Kh	315;59,17, 4
Mercury	3; 6,24,22, 7,59, 0		177;35,12,11
		Date:	2– 2–1172
		JDN	2149163

G. Naṣīr al-Dīn al-Ṭūsī et al. (the last part of the 13th ct., Maragha): *Īlkhānī zīj*

(C: pp. 56–59, 95–98, 102–105, 111–114, 120–123, 129–132; T: ff. 26r–27v, 48r–49v, 55r–56v, T: 62v–Suppl. P: p.21 (Mars, partially preserved in MS T and partially in Suppl. P), Suppl. P: pp. 32–34–T: 71r (Venus, scattered in the two codices, just like the tables of Mars), 76v–78r; P: ff. 19v–21v (Sun), 33r–34r (Saturn), 35v–36v (Jupiter), 40r–v & 38r (Mars), 41r–42r (Venus), 44v–45v; IT: ff. 19r–20v, 41r–42v, 44v–46r, 49r–50v, 53v–55r, 58r–59v; L: ff. 31r–32v, 51v–53r, 55r–56v, 59v–61r, 64r–65v, 68v–70r; F: in this early manuscript in the hand of al-Ṭūsī’s second son, Aṣīl al-Dīn Ḥasan, the daily and yearly motions along with the motions per decade, per century, per millennium are listed in a separate table on f. 25v; they show trivial differences from the values extracted from the mean motion/position tables: ff. 26r–27r, 40v–42r, 43v–45r, 47r–48v, 50v–51v, 54r–55r; B: ff. 29v–31r, 48v–50r, 52r–53v, 56v–58r, 61r–62v, 65v–67r).

Sun	0;59, 8,19,44,10,31,14 °/d	= IY	304; 1, 6
Saturn	0; 2, 0,36, 4,33, 8	= (Mt/Hb?)/Kh/F	15;12,47
Jupiter	0; 4,59,16,23, 1,57	= IA	118;23,57
Mars	0;31,26,39,35,29,27	= IA/Kh?	242;31, 3
Venus	0;36,59,28,12,55,39	= IA	138; 8,22
Mercury	3; 6,24,22, 7,54,22		160;50, 0
		Date:	18– 1–1232
		JDN	2171063

H. Muḥyī al-Dīn al-Maghribī (d. 1283 CE, Damascus): *Tāj al-azyāj*

(Dorce 2002–2003, pp. 197–198, 210–212, Dorce 2003, *passim*.)

Sun	0;59, 8,20, 8, 4,37 °/d	114; 0,31°
Saturn	0; 2, 0,41,30,59,54	111;30,48
Jupiter	0; 4,59,14,46,58,13	331;42, 9
Mars	0;31,26,38,16, 2,26	211;45,20
Venus	0;36,59,28,56,37, 0	43;36,52

Mercury	3; 6,24, 8,11, 4, 1	78;12,52,51
	Date:	14– 7– 622
	JDN	1948438

I. Muḥyī al-Dīn al-Maghribī (Maragha): *Adwār al-anwār* & *Talkhīṣ al-majisṭī*

(*Adwār*, M: ff. 75v–82v, CB: f. 73v–80v; *Talkhīṣ*, IV.4: ff. 57v–58v (Sun), VIII.5: ff. 127v–128r (Saturn), VIII.9: f. 132r (Jupiter), VIII.13: ff. 135v–137r (Mars). See, also, Mozaffari 2018a, pp. 197, 204 (Sun), Mozaffari 2018–2019, pp. 202–203 (Mars)).

Sun	0;59, 8,20, 8, 4,36,38 °/d	303; 0,14°
Saturn	0; 2, 0,36,45,35,41	15;37,53
Jupiter	0; 4,59,16,40,55, 8	117;53,13
Mars	0;31,26,39,44,40,48	242; 8, 3
Venus	0;36,59,29, 7,49, 1,36	136; 9, 4
Mercury	3; 6,24, 6,59,42,22	153; 0,51
	Date:	17– 1–1232
	JDN:	2171062

J. Ibn al-Shāṭir (1306–1375/1376 CE, Damascus): *Jadīd zīj*

(K: ff. 50v–51r, 58v, 61r, 63v, 66r, 68v, O: ff. 28v–30r, 41v–42r, 44v–45r, 47v–48r, 50r, 53r, L1: ff. 51r–v, 58v, 60v, 62v, 64v, 66v, L2: ff. 63r–64v, 75r–v, 78r–v, 81r–v, 84r–v, 87r–v, PR: ——).

Sun	0;59, 8,19,43,33,45,58 °/d	115;23,17°
Saturn	0; 2, 0,35,40,29,41	117;43,23
Jupiter	0; 4,59,15,58, 3	330;45,50
Mars	0;31,26,39,32,21,42	212;33, 5
Venus	0;36,59,28,26,18	45;37,11
Mercury	3; 6,24,14,52, 2	83;19,20
	Date:	15– 7– 622
	JDN:	1948439

K. Ulugh Beg & the Samarqand observers (the first part of the 15th ct.): *Sulṭānī zīj*

(P1: ff. 117r–v, 134r–v, 137r–v, 140r–v, 143r–v, 146r–v, P2: ff. 133r–v, 151r–v, 154r–v, 157r–v, 160v–161r, 164r–v, P3: pp. 208–209, 255–256, 267–268, 279–280, 296–297, 309–310, O: ff. 102v–103r, 120r–v, 123r–v, 126r–v, 129v–130r, 133r–v).

Sun	0;59, 8,19,37,42,41,32 °/d		110;51,58°
Saturn	0; 2, 0,36,24,28,19		9; 7,31
Jupiter	0; 4,59,16,20, 9,36	= Kh?	236;24, 5
Mars	0;31,26,39,55, 4,45		331;31, 5
Venus	0;36,59,28,13,40,56		324;42,41
Mercury	3; 6,24,15,24,51,13		22; 1, 2
		Date:	4– 7–1437
		JDN:	2246107