Ptolemy and Jābir b. Aflaḥ on Solar Eclipses

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Abstract: Abū Muḥammad Jābir b. Aflaḥ, a 12th-century Andalusian mathematician and astronomer, is recognized for his influential work *al-Kitāb fī l-Hay'a*, a work today better known as *Iṣlāḥ al-Majisṭī*. Jābir b. Aflaḥ's *al-Kitāb fī l-Hay'a* is a reedition of Ptolemy's *Almagest* in which he also included a number of criticisms of Ptolemy's work. The present study focuses on Jābir b. Aflaḥ's criticisms of Ptolemy's theory of solar eclipses, highlighting three key objections: two on Ptolemy's choice of the mid-heaven instead of the mid-heaven of the ascendant to obtain the parallax in longitude and its effect on solar eclipses; and an additional one on Ptolemy's treatment of the lunar parallax in latitude. While two of these criticisms appear unjustified, they offer insight into Jābir's methodology and his reliance on defective or abridged manuscripts. Jābir's novel approach, particularly in his use of the new trigonometry, together with his disregard for certain celestial motions, reveal both his mathematical strength and his limitations in practical astronomy. His failure to account for the Sun's additional motion in the computations of solar eclipses further underscores his inexperience. Nevertheless, Jābir b. Aflaḥ emerges as a creative astronomer, whose work demonstrates a deep engagement with and unfrequent understanding of Ptolemy's *Almagest*, albeit with notable oversights in the practical aspects of astronomy.

Keywords: Astronomy, Greek Astronomy, Medieval Astronomy, al-Andalus, Ptolemy, Jābir b. Aflaḥ, *Almagest*, *Iṣlāḥ al-Majisṭī*, Eclipse Theory, Solar Eclipses, Criticisms of Ptolemy's *Almagest*.

Resum: Abū Muḥammad Ǧābir b. Aflaḥ, matemàtic i astrònom andalusí del segle xii, és autor de *al-Kitāb fī l-Hay'a*, una obra avui més coneguda com *Iṣlāḥ al-Maǧisṭī*. El *al-Kitāb fī l-Hay'a* de Ǧābir b. Aflaḥ és una reedició abreujada de l'*Almagest* de Ptolemeu, en la qual Ǧābir també va incloure de critiques matemàtiques a l'astronomia ptolemaica. El present estudi se centra a les crítiques que Ǧābir b. Aflaḥ fa a la teoria dels eclipsis solars

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de Ptolemeu, destacant tres objeccions principals: dues sobre l'elecció de Ptolemeu del mig cel en lloc del mig cel de l'ascendent per obtenir la paral·laxi en longitud i el seu efecte sobre els eclipsis solars; i un altre sobre el tractament de Ptolemeu de la paral·laxi lunar en latitud. Tot i que dues d'aquestes crítiques semblen injustificades, ofereixen una visió de la metodologia de treball de Ǧābir b. Aflaḥ, així com del fet que moltes de les seves crítiques de l'astronomia ptolemaica estiguin motivades pel manuscrits defectuosos amb els quals treballava. El nou enfocament de Ǧābir b. Aflaḥ a l'astronomia de Ptolemeu, especialment en el seu ús de la nova trigonometria, juntament amb el fet de no tenir en compte certs moviments celestes, revelen tant la seva capacitat matemàtica com les seves limitacions pel que fa a l'astronomia pràctica. El fet que no tingués en compte el moviment addicional del Sol en els càlculs dels eclipsis solars subratlla la seva inexperiència. No obstant això, Ǧābir b. Aflaḥ emergeix com un astrònom creatiu, el treball del qual demostra una profunda familiaritat i poc habitual comprensió de l'*Almagest* de Ptolemeu, tot i que amb oblits notables en els aspectes pràctics de l'astronomia.

Paraules clau: astronomia, astronomia grega, astronomia medieval, al-Àndalus, Ptolemeu, Jābir b. Aflaḥ, *Almagest*, *Iṣlāḥ al-Majisṭī*, teoria d'eclipsis, eclipsis solars, crítiques a l'*Almagest* de Ptolemeu.

1. Introduction

Abū Muḥammad Jābir b. Aflaḥ, commonly referred to in Latin as Geber, was a mathematician and theoretical astronomer from al-Andalus, likely active in Seville during the early 12th century (6th century AH). He is best known for his work *al-Kitāb fī l-Hay'a* (*The Book on Astronomy*),[1] a reworking of Ptolemy's *Almagest*, which is currently widely known as *Iṣlāḥ al-Majisṭī* (*Correction of the*

1. For general introductions to Jābir b. Aflaḥ, see Richard P. Lorch, «The Astronomy of Jābir b. Aflaḥ», *Centaurus* 19 (1975), pp. 85-107; José Bellver, «On Jābir b. Aflaḥ's Criticisms of Ptolemy's *Almagest*», in Emilia Calvo *et al.* (2008), *A Shared Legacy: Islamic Science East and West. Homage to professor J.M. Millàs Vallicrosa*, Barcelona: Publicacions i edicions de la Universitat de Barcelona, 2008, pp. 230-238; and José Bellver, «El lugar del *Iṣlāḥ al-Maŷisṭī* de Ŷābir b. Aflaḥ en la llamada "rebelión andalusí contra la astronomía ptolemaica"», *al-Qanṭara* 30.1 (2009), pp. 83-136. See also José Bellver, «The Arabic Versions of Jābir b. Aflaḥ's *al-Kitāb fī l-Hay'a*», in Dag N. Hasse *et al.* (eds.), *Ptolemy's Science of the Stars in the Middle Ages*, Turnhout: Brepols, 2020, pp. 181-199.

Almagest).[2] Today, there are four extant Arabic manuscripts in Arabic characters and two partial Arabic manuscripts in Hebrew characters.[3] Additionally, Jābir's *al-Kitāb fī l-Hay'a* was translated into Latin, and twice into Hebrew.

In the *al-Kitāb fī l-Hay'a*, Jābir revised and adapted Ptolemy's *Almagest* for his contemporaries, omitting practical elements such as calculations and tables, while streamlining its trigonometric proofs by utilizing the Rule of Four Quantities. He also introduced several mathematical corrections of «mistakes» that, in his view, Ptolemy committed, which he listed in his Introduction to the *al-Kitāb fī l-Hay'a*, the most notable being his criticism of Ptolemy's cosmology. Contrary to Ptolemy's arrangement, Jābir argued that the spheres of Mercury and Venus should be positioned above the sphere of the Sun, rather than below it. Additionally, *al-Kitāb fī l-Hay'a*'s Book I —an introduction to plane and spherical trigonometry—proved to be highly influential in Medieval Europe. An additional unique aspect of Jābir's *al-Kitāb fī l-Hay'a*, found in Book V, was the introduction of a new instrument, similar to the *torquetum*, which Jābir claimed could replace the four instruments described by Ptolemy in the *Almagest*.

The aim of this article is to study a number of criticisms that Jābir b. Aflaḥ levels at Ptolemy in his theory of solar eclipses.[4]

Jābir b. Aflaḥ lists three «mistakes» that in his view Ptolemy made in his study of solar eclipses: two on the election of the mid-heaven instead of the mid-heaven of the ascendant to obtain the parallax in longitude and its effect on the solar eclipse; and one more on the way in which the lunar parallax in latitude should be considered in the apparent conjunction to determine the value of its argument in apparent latitude.

2. For the reasons supporting that Jābir b. Aflaḥ's treatise was known in his own time as *al-Kitāb fī l-Ḥay'a*, see Bellver, «The Arabic Versions».

3. In this article, three Arabic manuscripts written in Arabic characters have been taken into account. These are: MS Escorial, Real Biblioteca del Monasterio de San Lorenzo, ár. 910 (henceforth referred to as Ea); MS Escorial, Real Biblioteca del Monasterio de San Lorenzo, ár. 913 (henceforth referred to as Eb); and MS Berlin, Staatsbibliothek Preußischer Kulturbesitz, Landberg 132 (henceforth referred to as B).

4. See G.J. Toomer, *Ptolemy's Almagest*, London: Duckworth, 1984, pp. 310-313 (henceforth referred to as Toomer); O. Pedersen, *A Survey of the Almagest*, Odense: Odense University Press, 1974, pp. 232-234 (henceforth referred to as Pedersen); and O. Neugebauer, *A History of Ancient Mathematical Astronomy*, 3 vols., Berlin: Springer-Verlag, 1975, pp. 134-139 (henceforth referred to as HAMA).

As with lunar eclipses, Jābir b. Aflaḥ avoids to resort to the tables that Ptolemy used to determine the magnitude and duration of the phases of the eclipse. The main part of Ptolemy and Jābir b. Aflaḥ's discussions on the topic are devoted to addressing the effect of parallax on the magnitude and duration of the phases of the eclipse. Once the effect of the parallax is accounted for, Jābir b. Aflaḥ follows his method to obtain the magnitude and phases of lunar eclipses to compute solar eclipses.[5]

Since neither Neugebauer nor Pedersen study the effect of the parallax in the solar eclipse, it is thus important to study how Ptolemy tackles this effect before addressing Jābir b. Aflaḥ's criticisms of Ptolemy on the topic. Thus, the present article will focus on both Ptolemy and Jābir b. Aflaḥ's studies of solar eclipses.

2. Ptolemy on the magnitude and duration of the phases of solar eclipses

2.1. Terminology and notation

We define «apparent course of the Sun» and «apparent course of the Moon» as the courses described by the Sun and the Moon during their motion as they appear to a particular observer.

Regarding the notation, apparent values are noted with an apostrophe. Subscripts «cv» and «ca» make reference to the *conjunctio vera* and the *conjunctio apparens*, i.e., the true and apparent conjunctions. Thus, for instance, ☽$_{ca}$ refers to the true Moon in the apparent conjunction, and ⊙'$_{cv}$ refers to the apparent Sun in the true conjunction. p refers to parallax. Thus, for instance, p(⊙)$_{ca}$ refers to the solar parallax in the apparent conjunction. In turn, p_λ and p_β refer to the components in longitude and latitude of the parallax. Lastly, ☊$_{ap}$ and ☋$_{ap}$ refer to the intersection —or node— of the apparent course of the Sun and Moon when the Moon goes, in the first case, from the south of the apparent course of the Sun to the

5. For a study of Jābir b. Aflaḥ's treatment of lunar eclipses, see José Bellver, «Jābir b. Aflaḥ on Lunar Eclipses», *Suhayl. Journal for the History of the Exact and Natural Sciences in Islamic Civilization* 8 (2008), pp. 47-92.

north, and, in the second case, from the north of the apparent course to the south. The following table summarizes the notation used in this article.

Notation	
☉	True Sun
☽	True Moon
☉’	Apparent Sun
☽’	Apparent Moon
☊	Ascending lunar node
☋	Descending lunar node
I	Angle of inclination of the lunar inclined orbit relative to the ecliptic
cv	Time of the *conjunctio vera*, i.e., the true conjunction
ca	Time of the *conjunctio apparens*, i.e., the apparent conjunction
ca-cv	Time interval between the apparent and true conjunctions
Subscript 1	Time of the true conjunction
Subscript 2	Time after the first correction in longitude equivalent to $\Delta p_{\lambda 1}$ (cf. *infra*)
Subscript 3	Time after the second correction in longitude equivalent to $\Delta p_{\lambda i}$ with i = 2 (cf. *infra*)
Subscript 4	Time after the third correction in longitude
Subscript i	Eclipse initial time
Subscript m	Eclipse mid-time
Subscript f	Eclipse end time
p	Parallax
p’	Variation (derivate) of the parallax over time
γ	Angle between the ecliptic and the altitude circle containing the parallax vector
p(☽)	Lunar parallax
p(☉)	Solar parallax

λ(☽)	Longitude of the true Moon
λ(☽')	Longitude of the apparent Moon
β	True latitude
β'	Apparent latitude
$\Delta p_{\lambda 1} = p_{\lambda}$(☽)$_1 - p_{\lambda}(\odot)_1$	Difference between lunar and solar parallaxes in longitude in the true conjunction
$\Delta p_{\lambda i} = [p_{\lambda}$(☽)$_i - p_{\lambda}(\odot)_i] -$ $- [p_{\lambda}$(☽)$_{i-1} - p_{\lambda}(\odot)_{i-1}]$	Difference between the lunar and solar epiparallaxes in longitude between two successive times
$\Delta p_{\lambda i}^{(P)}$	*i*-th difference between the lunar and solar epiparallaxes in longitude according to Ptolemy. When there is no superscript, it implicitly refers to Ptolemy's method
$\Delta p_{\lambda i}^{(J\text{-}Es)}$	*i*-th difference between the lunar and solar epiparallaxes in longitude according to Jābir b. Aflaḥ as from the Escorial manuscripts
$\Delta p_{\lambda i}^{(J\text{-}B)}$	*i*-th difference between the lunar and solar epiparallaxes in longitude according to Jābir b. Aflaḥ as from the Berlin manuscript
Δp_{β}	Difference between the lunar and solar parallaxes in latitude in the apparent conjunction
$e_p = \Delta p_{\lambda 2} + l$	Total epiparallax as the sum of the epiparallax resulting from the first correction in longitude ($\Delta p_{\lambda 2}$) and the one obtained from interpolation (l)
$l = \Delta p_{\lambda 3} = m\,\Delta p_{\lambda 2}$	Epiparallax in longitude from the interpolation of a previous parallax
$m = \Delta p_{\lambda 2} / \Delta p_{\lambda 1}$	Pendant to obtain the interpolated epiparallax
$e_p = e_p$(☽)$ - e_p(\odot)$	Total epiparallax as the difference between the total epiparallaxes of the Moon and Sun
$w_{\bar{\odot}}$	Motion in longitude of the mean Sun
$w_{\bar{☽}}$	Motion in longitude of the mean Moon
$\Delta\lambda$(☽)$_{ca\text{-}cv}$	Difference in longitude of the true Moon between the apparent and true conjunctions
Δt_+	Additional time interval in which the Moon traverses with its motion in the true conjunction the longitude traversed by the Sun during the time the Moon has traversed the difference in apparent longitude between the Sun and the Moon in the true conjunction
ω	Argument in latitude

β	Angle between the lunar inclined orbit and the parallax
β'	Angle between the apparent course of the Moon and the parallax
μ	Immersion
m	Magnitude of the eclipse
$r_{☽}$	Radius of the Moon
$d_{☽}$	Diameter of the Moon
$r_{\odot}$	Radius of the Sun
$d_{\odot}$	Diameter of the Sun
r_{T}	Earth radius
$d_{☽\odot}$	Distance between the centers of the Moon and the Sun at the eclipse mid-time
h	Geocentric altitude
h_{ob}	Observed altitude
e	Eccentricity
q	Angle of the solar anomaly
R_{ex}	Radius of the eccentricity
$d_{\mathrm{T}☽}$	Distance of the Moon to the Earth's center

2.2. Preliminary knowledge

In solar eclipses, since the obscuration of the solar disk depends on the interposition of the lunar disk between the Sun and the observer's position, and the position of the apparent Moon depends on the position of the observer, the magnitude and duration of the phases of the solar eclipse are affected by the solar and lunar parallax. To understand how the parallax evolves over time as a function of the solar and lunar motions in longitude, we will study how the solar and lunar parallaxes change in relation to the motion of the celestial sphere.[6]

6. For the concept of parallax in Islamic astronomy, see E.S. Kennedy, «Parallax Theory in Islamic Astronomy», *Isis*, Vol. 47, no. I. (1956), pp. 33-53 (reprinted in Kennedy *et al.*, *Studies in the Islamic Exact Sciences*, Beirut: American University of Beirut, 1983, pp. 164-184).

To do so, firstly, it should be pointed out that the zodiac houses in the ecliptic, as seen by an observer on the Earth's surface, are arranged from west to east. Therefore, at any specific time, the more an observed position in the ecliptic falls to the east, the greater its longitude is. In addition, the celestial sphere, together with the ecliptic, rotates with its universal motion from east to west. The different planets seen from the Earth's surface move from east to west with a motion —in general— slower than the rotation of the celestial sphere, so that their longitude —except during their periods of retrogradation— increases over time. Nevertheless, the Sun and the Moon do not experience periods of retrogradation. And lastly, since the Sun is always on the ecliptic, its course is always parallel to the celestial equator, and since the ecliptic is inclined relative to the celestial equator, the azimuth of the points of intersection of the ecliptic with the horizon changes continuously. The «ascendant» is defined as the point of intersection of the ecliptic with the eastern horizon, and the «descendant» as the point of intersection of the ecliptic with the western horizon. Likewise, the «mid-heaven of the ascendant» is the point of the ecliptic at 90º from the ascendant. It is, thus, the point with higher altitude of the ecliptic. The mid-heaven does not need to coincide with the meridian. The mid-heaven of the ascendant divides the visible arc of the ecliptic above the horizon into two quadrants. We will call «first quadrant» to the one delimited by the ascendant and the mid-heaven of the ascendant, and «second quadrant» to the one delimited by the mid-heaven of the ascendant and the descendant.

Figure 1 represents these relations as seen from the north of the ecliptic in the direction of the mid-heaven of the ascendant. Likewise, it also represents the universal motion of the celestial sphere and the zodiac houses. Figure 2, in turn, represents these relations as seen from the south of the ecliptic in the direction of the mid-heaven of the ascendant.

In addition, a node is an intersection of the lunar inclined orbit with the ecliptic. The ascending node is the intersection in which the Moon transits from negative to positive latitude, and the descending node is the one in which the Moon transits from positive to negative latitude. To the south of the ecliptic, nodal transits are inverted since the latitude of the observer is negative (see Figure 3).

Next, the effect of parallaxes makes the observed altitude of a body to be smaller than the true one. Since altitudes are involved, the easier way to represent the effect of parallaxes would be using horizontal coordinates. Nevertheless, it is important to know how solar and lunar parallaxes change over time. Since celestial motions are traditionally represented in ecliptic coordinates, we will describe parallaxes using ecliptic coordinates. Thus, since parallaxes involve a decrement

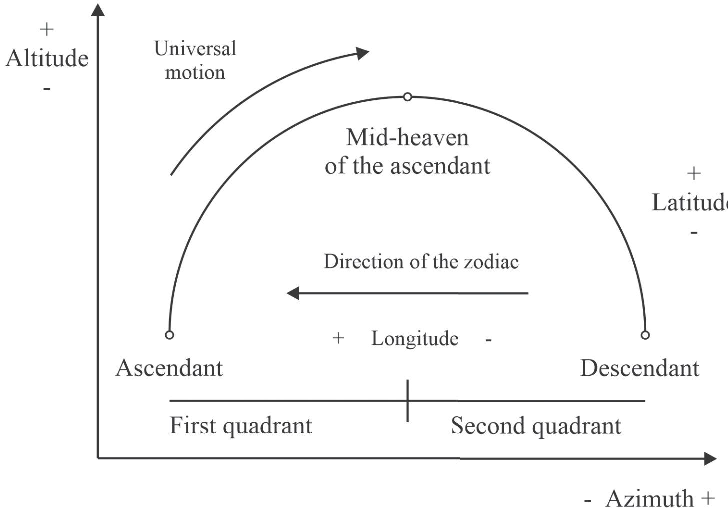


Figure 1. Ecliptic in horizontal coordinates as seen from its north

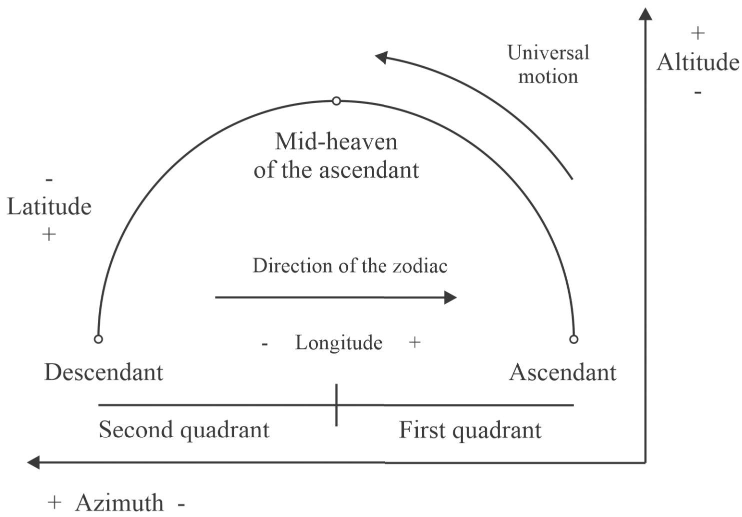


Figure 2. Ecliptic in horizontal coordinates as seen from its south.

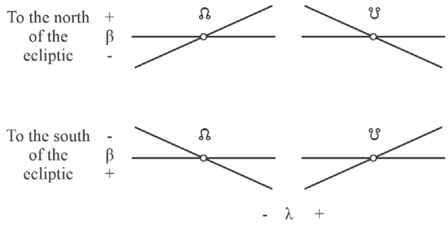


Figure 3. Ascending and descending nodes to the north and south of the ecliptic.

of the observed altitude and, additionally, a given observer can be placed at any point on the surface of the Earth, the components in longitude and latitude of a parallax can either be positive or negative; that is, the angle subtended by the altitude circle in which lays the vector of the parallax and the ecliptic can have any value. This angle is henceforth referred to as γ.[7]

If, in Figure 1 describing the ecliptic seen from its north, we take a point of the first quadrant of the ecliptic and we trace a vertical through it indicating a decrement in altitude to account for the effect of the parallax, this vertical would have a positive component in longitude. In turn, if we take a point of the second quadrant and, again, we trace a vertical indicating a decrement in altitude, the vertical would have a negative component in longitude. Consequently, the quadrant in which the eclipse takes place affects the components in longitude, either positive or negative, of the solar and lunar parallaxes. In addition, the position of the observer to the north or to the south of the ecliptic affects the component in latitude of the parallax. Thus, the angle γ is affected by two factors: (i) the quadrant in which the eclipse takes place; and (ii) the hemisphere of the observer relative to the ecliptic. Figure 4 represents the ranges of angle γ according to both factors.

To show these relations, we present a number of figures in which the true conjunction takes place before the transit of the Moon through the descending node. Figure 5 shows this case when it takes place in the first quadrant to the north of the ecliptic, while Figure 6 shows this case when this takes place in the second quadrant to the north of the ecliptic. The angle of inclination of the lunar inclined orbit relative to the ecliptic is exaggerated.

In both cases, the parallax decreases with the altitude. The previous figures show the apparent node of the lunar inclined orbit located on its intersection

7. See HAMA, pp. 115-116.

with the ecliptic, for both quadrants, as if the celestial sphere would not experience any motion.

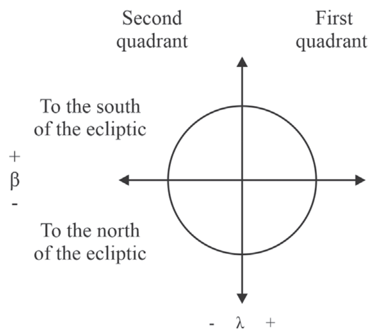


Figure 4. Angular intervals of γ according to the quadrant and the hemisphere in relation to the ecliptic in which the eclipse takes place.

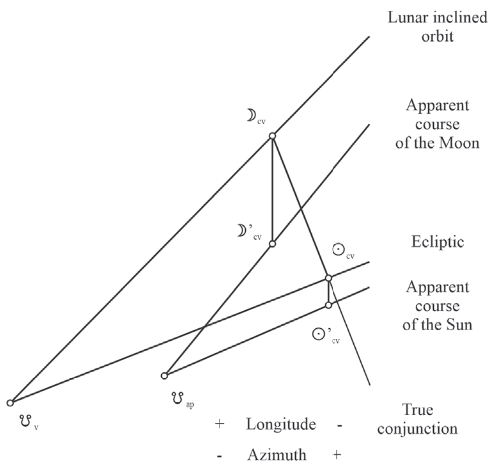


Figure 5. True conjunction in the first quadrant to the north of the ecliptic in horizontal coordinates.

To further our understanding, the next step is to show these relations in ecliptic coordinates instead of horizontal ones. Figure 7 shows the true conjunction in the first quadrant (as of Figure 5) in ecliptic coordinates. Likewise, Figure 8 shows the true conjunction in the second quadrant (as of Figure 6) in ecliptic coordinates. Both figures show that the parallax in longitude is positive when the conjunction takes place in the first quadrant and is negative when it takes place in

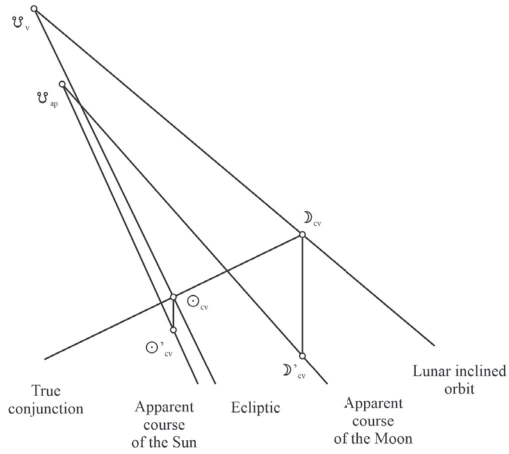


Figure 6. True conjunction in the second quadrant
to the north of the ecliptic in horizontal coordinates.

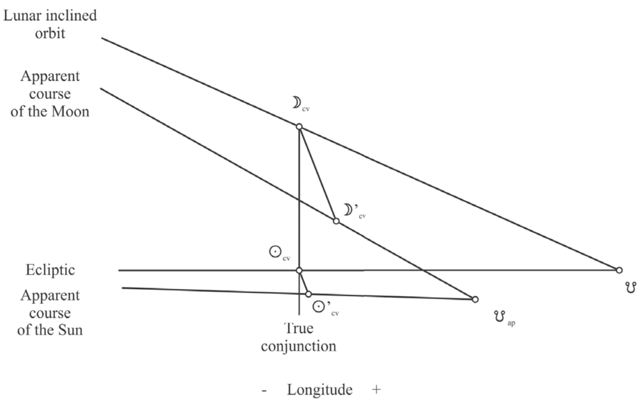


Figure 7. True conjunction in the first quadrant
to the north of the ecliptic in ecliptic coordinates.

the second quadrant. When the parallax in longitude in the true conjunction is positive —as in the first quadrant—, since the Moon and the Sun move forward in longitude, the apparent conjunction takes place before the true one. Likewise, when the parallax in longitude in the true conjunction is negative —as in the second quadrant—, the apparent conjunction takes place after the true one. Thus, we should introduce a correction in time contrary to the direction of the parallax to obtain the apparent conjunction. That is,

- if the parallax in longitude is positive, the increment in longitude is positive, and thus the correction in time to obtain the apparent conjunction should be negative; and
- if the parallax in longitude is negative, the increment in longitude is negative, and thus the correction in time to obtain the apparent conjunction should be positive.

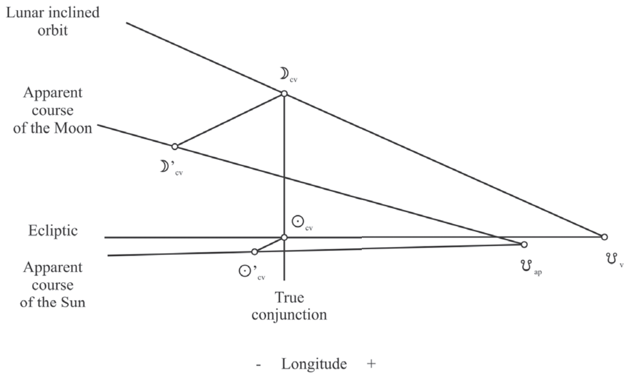


Figure 8. True conjunction in the second quadrant
to the north of the ecliptic in ecliptic coordinates.

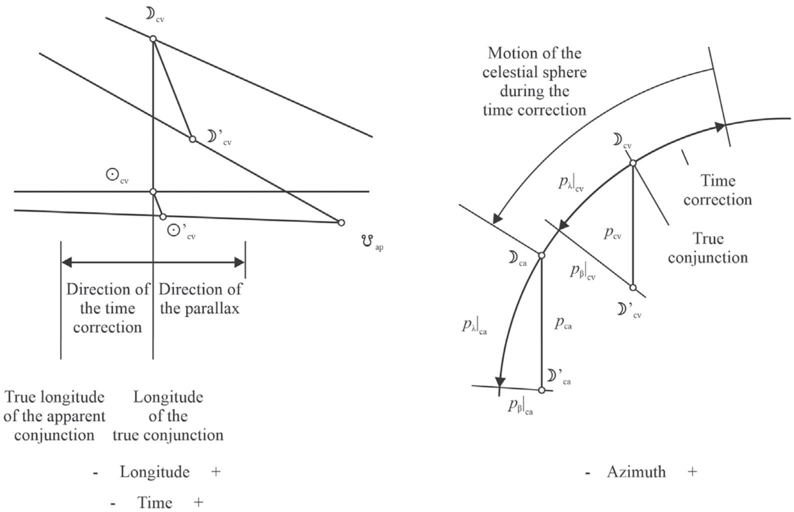


Figure 9. Effect of the motion of the celestial sphere
on parallaxes in the first quadrant to the north of the ecliptic.

Figure 9a shows this situation for the first quadrant with positive parallax in longitude, and Figure 10a shows it for the second quadrant.

In turn, Figure 9b shows, in horizontal coordinates, the effect of the correction in time to obtain the apparent conjunction when it takes place in the first quadrant. Since the correction in time is negative, the longitudes of the Sun and Moon during this correction decrease and their azimuths increase. However, during the same time interval relative to the negative correction in time introduced to obtain the apparent conjunction, the celestial sphere experiences a negative motion in azimuth greater than the one experienced by the Sun and the Moon. Hence, the altitude of the Sun and the Moon in the apparent conjunction is smaller than in the true one, and, consequently, their parallaxes should be greater than before the introduction of the correction.

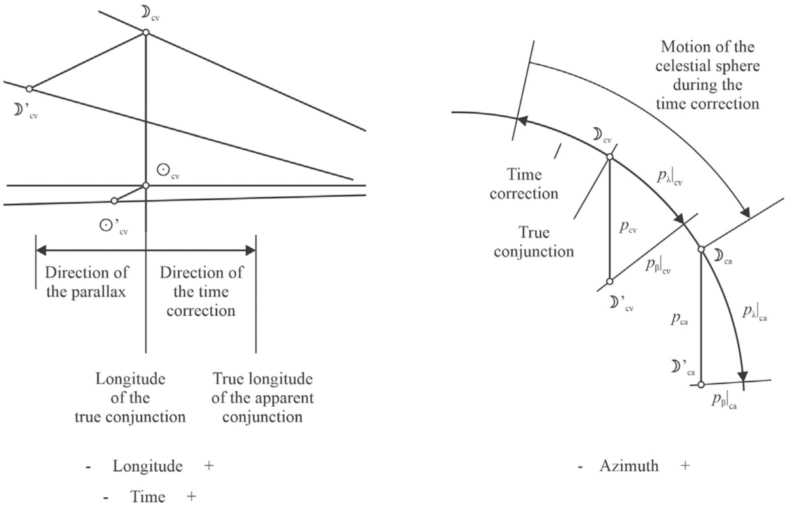


Figure 10. Effect of the motion of the celestial sphere on parallaxes in the second quadrant to the north of the ecliptic.

In turn, Figure 10b shows, in horizontal coordinates, the correction in time introduced to obtain the apparent conjunction when it takes place in the second quadrant. Since the correction in time is positive, the longitudes of the Sun and Moon during this correction increase and their azimuths decrease. However, during the interval in time relative to the positive correction introduced to obtain the apparent conjunction, the celestial sphere experiences a positive motion in azimuth greater than the one experienced by the Sun and Moon. Hence, the altitude of the Sun and the Moon in the apparent conjunction is smaller than in the true one and, consequently, their parallaxes, as in the previous case, are greater than before the introduction of the correction.

Thus, in both quadrants, the parallax in longitude in the apparent conjunction is always greater than in the true conjunction. That is, it is always true that

$$p_{\lambda}|_{ca} > p_{\lambda}|_{cv} \qquad (1).$$

Thus, changes introduced by the motion of the celestial sphere in Figure 7 and Figure 8 result in figures in which, for a given observer on the Earth's surface, parallaxes change over time in significantly different ways. Figure 11 shows how the parallaxes change over time when the conjunction takes place in the first quadrant. In turn, Figure 12 shows how parallaxes change over time when the conjunction takes place in the second quadrant.

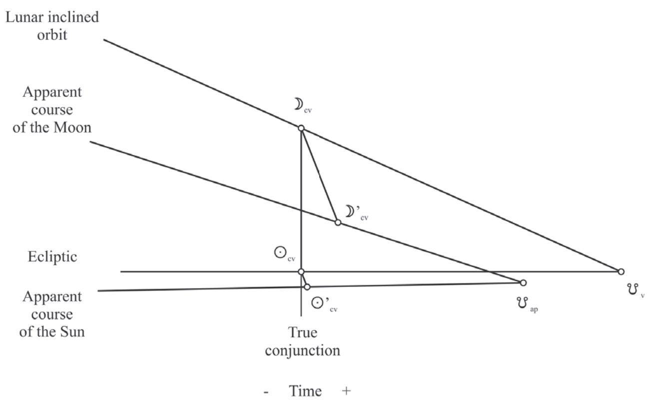


Figure 11. Change of parallaxes over time for conjunctions in the first quadrant to the north of the ecliptic.

Consequently, from comparing Figure 11 with Figure 7, for any positive parallax in longitude —as in the first quadrant—, the absolute value of the parallaxes decreases over time. In turn, from comparing Figure 12 with Figure 8, for any negative parallax in longitude —as in the second quadrant—, the absolute value of the parallaxes increases over time.

Thus, Figure 11 and Figure 12 show the two main situations needed to study Ptolemy and Jābir b. Aflaḥ's methods to compute the magnitude of a solar eclipse and the duration of its phases.

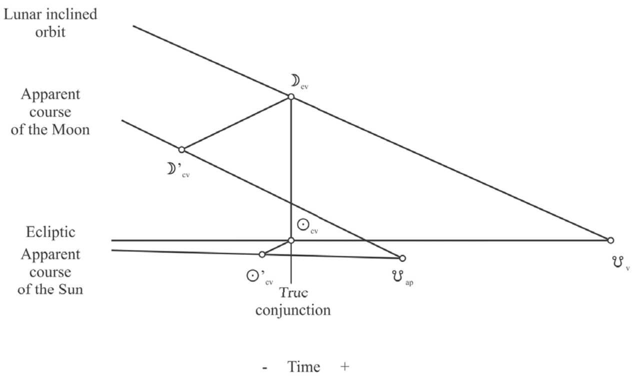
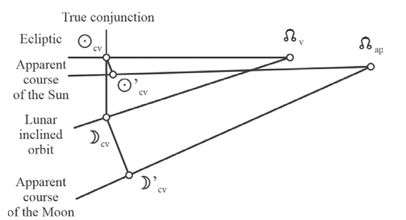
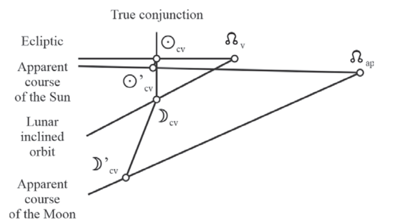
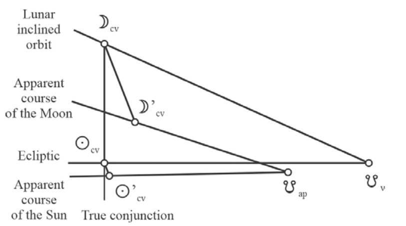
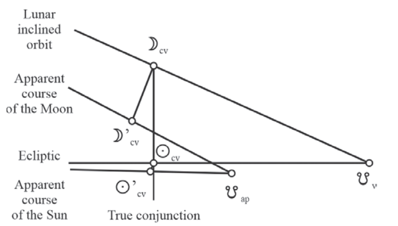


Figure 12. Changes of parallaxes over time in the second quadrant to the north of the ecliptic.

And last, the following table shows the possible configurations taking into account the different factors at play. These are: (i) The quadrant in which the eclipse takes place; (2) the hemisphere delimited by the plane of the ecliptic, either to its north or to its south, in which the eclipse takes place; and (iii) lastly, the type of the node, either ascending or descending, in whose close area the eclipse takes place. The quadrant, which is independent of the hemisphere, in which the eclipse takes place results in a positive or negative parallax in longitude. In turn, the hemisphere affects whether the apparent courses are to be found above or below the true ones, as shown in Figure 4.

<table>
<tr><td></td><td></td><td>First quadrant</td><td>Second quadrant</td></tr>
<tr><td rowspan="2">Northern hemisphere</td><td>Ascending node</td><td>
</td><td>
</td></tr>
<tr><td>Descending node</td><td>
</td><td>
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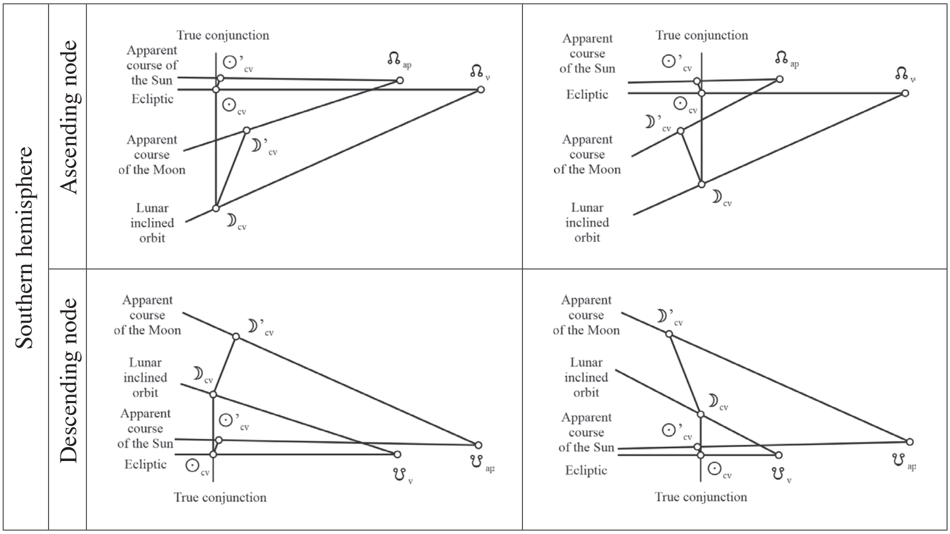


2.3. *Phases of solar eclipses*

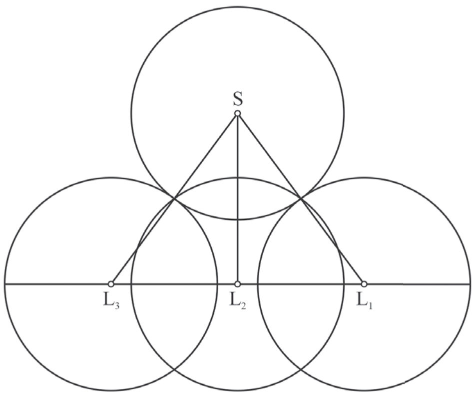


Figure 13. Phases of the solar eclipse.

Lunar eclipses are divided into four phases, since the lunar radius is significantly smaller than the radius of the Earth shadow cone. In solar eclipses, since the radii of the Sun and the Moon are almost identical —although with small variations owing to their position in relation to their perigees and apogees—, Ptolemy took only two phases into account defined by three specific times: the initial, middle and end times of the eclipse, which define the phases of immersion and emersion.

Thus, considering Figure 13, where S is the center of the Sun and L_i is the position of the Moon at time *i*, we can define two phases (*zamān*, pl. *azmina*) in a solar eclipse defined by three times:

(1) Beginning of eclipse (*awwal al-kusūf*); the Moon is on L_1.
(2) Eclipse mid-time (*wasaṭ zamān al-kusūf*); the Moon is on L_2.
(3) End of eclipse (*ākhir al-kusūf*); the Moon is on L_3.

2.4. The Almagest on the magnitudes of solar eclipses

For the computation of the magnitude of solar eclipses,[8] Ptolemy used tables for solar eclipses —tables I and II—, whose computation and use are very similar to the tables of lunar eclipses —tables III and IV.[9] For the computation of the magnitude of eclipses, Ptolemy needs to know the position of the apparent syzygy, which for the lunar eclipse is equal to the true syzygy. For the computation of the magnitude and the phases of the solar eclipse, Ptolemy cannot deem the apparent conjunction and the true conjunction to be equal because of the effect of the parallax. Thus, from his initial data —that is, (i) the longitude of the true conjunction, which can be obtained through computation; and (ii) the position of the observer—, he should obtain the position of the apparent conjunction relative to the apparent node, so that he can then follow the same steps as for the lunar eclipse. Consequently, the computation of the magnitude of the solar eclipse is divided into two steps: (i) reduction of the effect of parallax; and (ii) obtention of the magnitude.

8. On solar eclipses, see J. Mogenet *et al.*, *Nicéphore Grégoras; calcul de l'éclipse de soleil du 16 juliet* 1330, Amsterdan, 1983; J. Mogenet *et al.*, *Barlaam de Seminara. Traités sur les éclipses de Soleil de* 1333 *et* 1337. *Histoire des textes, éditions critiques, traductions et commentaires*, Leuven, 1977. For the computation of solar eclipses by Yaḥyā ibn Abī Manṣūr, see E.S. Kennedy and N. Fares, «The Solar Eclipse Technique of Yaḥyà ibn Abī Manṣūr», *Journal for the History of Astronomy* 1 (1970), pp. 20-38 (reprinted in Kennedy *et al.*, *Studies*, pp. 185-203). For a list of pre-modern observations of lunar and solar eclipses, see Bernard R. Goldstein, «Medieval Observations of Solar and Lunar Eclipses», *Archives Internationales d'Histoire des Sciences* 29 (1979), pp. 101-156.

9. For the computation of tables I to IV and VI.8, see *Almagest* VI.7 (Toomer pp. 306-308).

Effect of the parallax

The initial data are the positions of the Sun and Moon in the true conjunction. For the computation of the solar eclipse, we need to know their positions in the apparent conjunction. We have to account for the effect of the parallax to obtain the apparent conjunction from the true one. To obtain the magnitude of an eclipse with the aforementioned tables, Ptolemy needs the argument in apparent latitude in the apparent conjunction. To obtain this value from a true conjunction, he should follow three steps: (i) firstly, he should obtain the position of the true conjunction in the horizon of the observer; (ii) secondly, he should obtain the longitude of the apparent conjunction; and (iii) lastly, he should obtain the argument in apparent latitude for the apparent conjunction.

(i) Position of the true conjunction in the horizon of the observer.

Firstly, Ptolemy should find the true conjunction in the horizon of the observer. To do this, he computes the time before or after midday in the horizon of the observer in which the true conjunction[10] takes place following these steps:

- Firstly, he finds the difference in equinoctial hours between the true conjunction and the midday of Alexandria.
- He, then, computes the difference in geographic longitude between the meridian of Alexandria and that of the observer in equinoctial hours.
- He adds or subtracts this difference to the equinoctial hour of the true conjunction according to the geographic longitude of the observer to obtain the difference in equinoctial hours between the true conjunction and the midday of the observer.

(ii) Longitude of the apparent conjunction

In the second step, Ptolemy aims to obtain the apparent conjunction from the true one. Broadly speaking, he computes the time the Moon needs to traverse a longi-

10. See *Almagest* V.19 (Toomer pp. 264ff).

tude approximately equal to the difference in longitude between the true conjunction and the apparent one with its true motion in the true conjunction. This difference in longitude is caused by two factors: (i) the effect of the parallax; and (ii) the distance traversed by the Sun and the Moon during the time between the true conjunction and the apparent one. Ptolemy discusses both factors separately. He, firstly, addresses the effect of the parallax; and he, then, addresses the effect of the additional distance traversed by the Sun and the Moon.

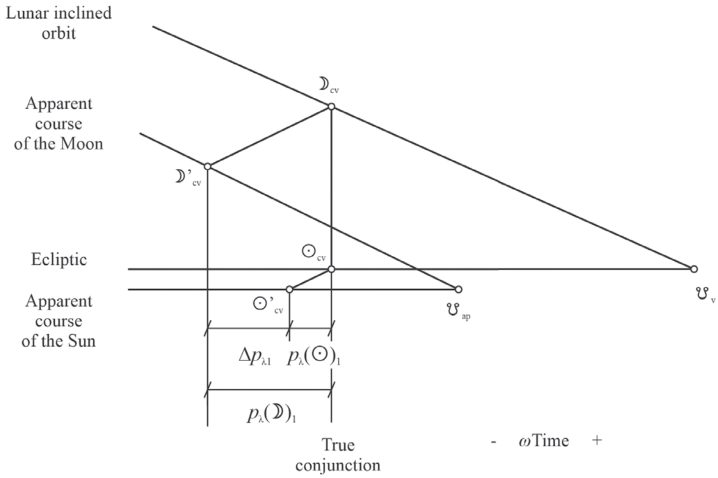


Figure 14. True conjunction in the second quadrant in ecliptic coordinates.

To obtain the time of the apparent conjunction for the geographical position of the observer, Ptolemy, first, finds the difference in longitude between the lunar and solar parallaxes. To do so,

- he introduces (i) the distance in hours from the meridian, (ii) the point of the ecliptic where the conjunction takes place, and (iii) the distance of the Moon, using the local latitude, in the Table of Angles (*Almagest* II.13) and the Table of Parallaxes (*Almagest* V.18);
- he finds the lunar parallax within the great circle passing through the zenith and the center of the Moon, that is, $p(☽)_1$ —where p indicates the parallax and the subscript '1' refers to the initial time, the time of the true conjunction;
- he obtains the difference between the lunar and solar parallaxes—that is, Δp_1— by subtracting the solar parallax —that is, $p(\odot)_1$— from the lunar parallax —that is, $p(☽)_1$ —. Thus, $\Delta p_1 = p(☽)_1 - p(\odot)_1$;

- and lastly, he finds the component in longitude of the difference between the lunar and solar parallaxes in the true conjunction, $\Delta p_{\lambda 1}$.

In fact, Ptolemy should have obtained, instead, the difference in longitude from the difference of the parallaxes in longitude of the Moon and the Sun —that is, $\Delta p_{\lambda 1} = p_{\lambda}(☽)_1 - p_{\lambda}(\odot)_1$—. Thus, Ptolemy grossly deems that the arcs of great circle passing between the true and apparent positions of the Moon, on the one side, and those of the Sun, on the other, are parallel.

Once this difference in longitude is obtained, Ptolemy considers the situation in which the Moon has traversed in its inclined orbit an argument in latitude equal in value to the difference in longitude between the parallaxes of the Sun and the Moon. That is, he examines the situation in the degree in longitude $\lambda(☽_2) = \lambda(☽_1) \pm \Delta p_{\lambda 1}$, as shown by Figure 15, where the sign depends on whether the true conjunction takes place in the first or second quadrants. If the lunar parallax was constant for any altitude of the Moon —and this is impossible—, the longitude of the apparent position of the Moon for the new found longitude, $\lambda(☽_2)$, —that is $\lambda(☽'_2)$— would be equal to the longitude of the apparent position of the Sun in the true conjunction, $\lambda(\odot'_1)$. To obtain the apparent conjunction, we would only need to deal with the correction needed to account for the traversed distance of the Sun during $\Delta p_{\lambda 1}$. However, parallaxes change with altitude. Thus, during the course of the Moon in its inclined orbit from $\lambda(☽_1)$ to $\lambda(☽_2)$, since the lunar altitude changes, there is also a variation in its parallax. This change is called «epiparallax».[11] For this reason, once the component in longitude of the difference between the parallaxes of the Sun and the Moon is obtained, Ptolemy:

- adds the epiparallax resulting from the number of equinoctial hours corresponding to the parallax in longitude.[12]

Next, Ptolemy divides the procedure to find the epiparallax into three steps:

11. See Toomer pp. 310-311. For an example of the computation of the epiparallax according to the *Almagest*, see Toomer p. 656.

12. «We always add to this [longitudinal parallax] the increment of «epiparallax» corresponding to the number of equinoctial hours represented by the longitudinal parallax». See Toomer p. 310.

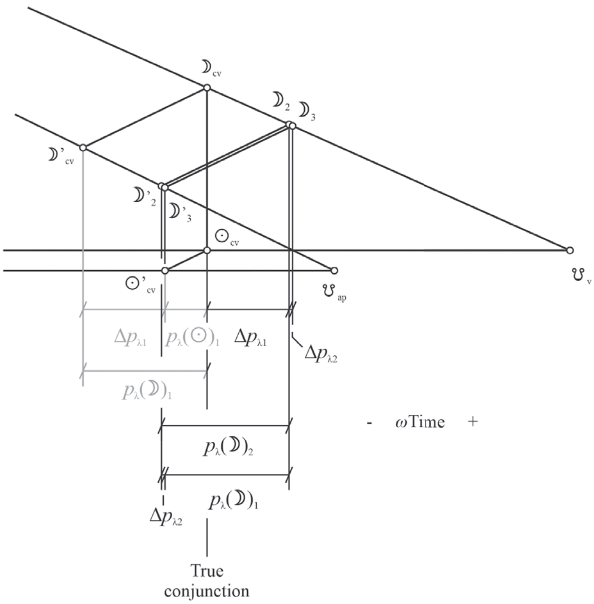


Figure 15. Intermediate steps to solve the apparent conjunction from the true conjunction in the second quadrant.[13]

- firstly, using the same table, he obtains the difference between the parallax caused by the zenithal distance at the initial time and the parallax caused by the zenithal distance after the equinoctial hours have passed;[14]
- then, he takes the component in longitude of the difference obtained before;[15]
- and lastly, he adds an additional longitude, if it is significant, corresponding to the fraction of the difference obtained before, as the latter is of the original longitudinal parallax.[16]

13. In this figure, the solar parallax in the interval between the second situation and the first one has been deemed as constant.

14. «We take the difference (as determined from the same table) between the parallax corresponding to the original zenith distance and the parallax corresponding to the zenith distance after the passage of the number of equinoctial hours [represented by the longitudinal parallax]» (Toomer pp. 310-311).

15. «We take the longitudinal component of this by itself» (Toomer p. 311).

16. «Plus an additional amount (if it is significant) which is the same fraction of the latter as the latter is of the original [longitudinal] parallax» (Toomer p. 311).

With these first two steps, he finds the difference between the parallax at the initial time —the time of the true conjunction, referred to with subscript '1' in Figure 15— and the intermediate step—referred to with subscript '2'— after traversing the distance $\Delta p_{\lambda 1}$ in longitude—i.e., the difference in longitude between the lunar and solar parallax in the true conjunction. Ptolemy does not point out the type of parallax that plays a role in this difference. Nevertheless, taking into account the previous procedure to find the difference between parallaxes, for parallax he most likely refers to the difference between the lunar and solar parallaxes at a specific time —that is, $p(☽)_i - p(\odot)_i$. Thus, the difference that he seeks to obtain should refer to the variation between times 1 and 2 of the difference between the lunar and solar parallaxes —that is, $[p(☽)_2 - p(\odot)_2] - [p(☽)_1 - p(\odot)_1]$. Next, in the second step, he finds the component in longitude; that is, $\Delta p_{\lambda 2}$. Again, this procedure only makes sense if Ptolemy deems that the meridians passing through $\lambda(☽_1)$ and $\lambda(☽_2)$ are grossly parallel in the interval. In Figure 15, to show the procedure in a clearer way, we obtain $\Delta p_{\lambda 2}$ from the components in longitude of the lunar parallax at the initial time and after traversing $\Delta p_{\lambda 1}$, and we deem the solar parallax as constant. In this case, the difference becomes $\Delta p_{\lambda 2} = [p_\lambda(☽)_2 - p_\lambda(\odot)_2] - [p_\lambda(☽)_1 - p_\lambda(\odot)_1] = p_\lambda(☽)_2 - p_\lambda(☽)_1$.

Since, as pointed out, the parallax in the true conjunction is always smaller than in the apparent conjunction, the parallactic difference at the initial time, $p_\lambda(☽)_1 - p_\lambda(\odot)_1$, is always smaller than the one that takes place in situation 2, $p_\lambda(☽)_2 - p_\lambda(\odot)_2$, so that $\Delta p_{\lambda 2}$ is always positive. Even though Ptolemy does not point out that the parallax in the true conjunction is always smaller than in the apparent one, he does not consider either the case in which $\Delta p_{\lambda 2}$ could be negative.

Once he finds this difference, Ptolemy finds the epiparallax, e_p, as

$$e_p = \Delta p_{\lambda 2} + l \qquad (2),$$

where l is the longitude added in the previous step, which we will study with more detail in what follows.

After the true conjunction, Ptolemy deems a second situation, referred to with the subscript '2', in which the Moon is located at longitude $\Delta p_{\lambda 1}$ from the true conjunction. In case the parallax would be constant in altitude, the apparent position of the Moon for this true longitude would indicate the apparent conjunction between the Sun and the Moon. However, since the parallax is not constant, we should take the lunar epiparallax into account. Once the epiparallax of this second situation —$\Delta p_{\lambda 2}$— is obtained, we can consider a third situation in which the Moon is located in the argument in latitude corresponding to the longitude $\lambda(☽_3)$

$= \lambda(☽_1) \pm (\Delta p_{\lambda 1} + \Delta p_{\lambda 2})$, as shown in Figure 15. We will refer to this new situation with subscript '3'.

We see that the apparent longitude of the Moon for this longitude, $\lambda(☽'_3)$, converges with the apparent longitude of the Sun in the true conjunction, $\lambda(\odot'_1)$. Similarly to the epiparallax Δp_2 resulting from the distance in longitude $\Delta p_{\lambda 1}$ traversed by the Moon, the distance in longitude traversed by the Moon, equal to the component in longitude of the previous epiparallax, $\Delta p_{\lambda 2}$, results in an additional epiparallax, Δp_3, whose component in longitude, $\Delta p_{\lambda 3}$, can be added —or subtracted depending on the quadrant— to $\lambda(☽_3)$; that is, $\lambda(☽_4) = \lambda(☽_1) \pm (\Delta p_{\lambda 1} + \Delta p_{\lambda 2} + \Delta p_{\lambda 3})$—, so that the longitude of its apparent position converges with $\lambda(\odot'_1)$. This is an iterative procedure. Ptolemy only takes the increment $\Delta p_{\lambda 3}$ into consideration, and only if it is significant.

As for Ptolemy's method to compute the increment $\Delta p_{\lambda 3}$, Ptolemy does not resort to the tables of parallaxes. Instead, he uses an interpolation. Ptolemy points out:

> Plus an additional amount (if it is significant) which is the same fraction of the latter as the latter is of the original [longitudinal] parallax.[17]

Ptolemy, thus, plays with two differences: the «latter» and the «original». With the «latter», Ptolemy seems to be making reference to the difference obtained in the first two steps during the computation of the epiparallax, that is $\Delta p_{\lambda 2}$. And with the «original», he seems to be making reference to the difference in longitude between the lunar and solar parallaxes in the true conjunction, that is, $\Delta p_{\lambda 1}$.[18] Thus, by «an additional amount (if it is significant) which is the same fraction of the latter as the latter is of the original [longitudinal] parallax», he refers to a function the like of

$$l = \Delta p_{\lambda 3} = m \, \Delta p_{\lambda 2} \qquad (3)$$

where *m* is the fraction pointed out by Ptolemy with «which is the same fraction of the latter as the latter is of the original [longitudinal] parallax». Therefore

$$m = \Delta p_{\lambda 2} / \Delta p_{\lambda 1} \qquad (4)$$

17. See Toomer p. 311.

18. See Toomer p. 311 n. 71. The equivalences between Toomer's notation and the one used here are the following: $l_1 \equiv \Delta p_{\lambda 1}$, $l_2 \equiv \Delta p_{\lambda 2} + \Delta p_{\lambda 1}$ and $e \equiv \Delta p_{\lambda 2}$.

and thus

$$l = \Delta p_{\lambda 3} = \Delta p_{\lambda 2}{}^2 / \Delta p_{\lambda 1} \qquad (5)$$

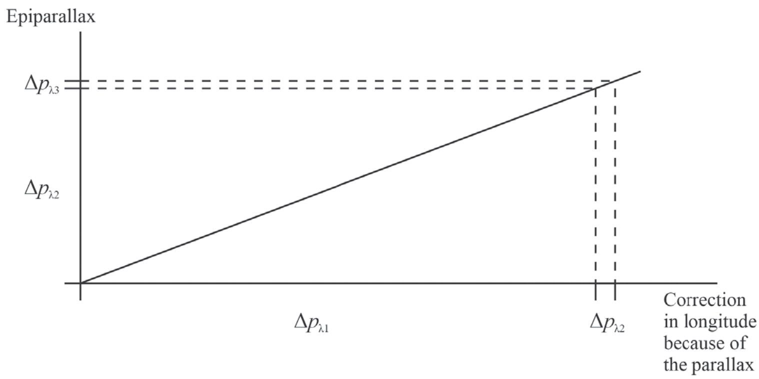


Figure 16. Epiparallax as function of the correction in longitude because of the parallax.[19]

Figure 16 graphically illustrates the procedure followed by Ptolemy. It also shows that the epiparallax is lineal with the correction in longitude due to the parallax.

Once the value of $\Delta p_{\lambda 3}$ is obtained and, depending on the quadrant, added to or subtracted from the longitude $\lambda(☽_3)$, we obtain a fourth situation. The resulting epiparallax is

$$e_p = \Delta p_{\lambda 2} + \Delta p_{\lambda 3} = \Delta p_{\lambda 2} + \Delta p_{\lambda 2}{}^2 / \Delta p_{\lambda 1} \qquad (6)$$

and the longitude, $\lambda(☽_4)$, is

$$\lambda(☽_4) = \lambda(☽_1) \pm (\Delta p_{\lambda 1} + e_p) = \lambda(☽_1) \pm (\Delta p_{\lambda 1} + \Delta p_{\lambda 2} + \Delta p_{\lambda 2}{}^2 / \Delta p_{\lambda 1}) \qquad (7).$$

At this point, for a better understanding of Ptolemy's method, we should examine what does the longitude $\lambda(☽_4)$ refer to. We have seen that, in case the parallax would not change with the altitude, or with the zenithal distance, the increment $\Delta p_{\lambda 1}$ would have been enough to obtain the longitude needed to account for the effect of the lunar parallaxes. In this case, $\lambda(☽'_2) = \lambda(\odot'_1)$. However, since the

19. The axis of the epiparallax has been exaggerated for clarification.

parallax changes with the altitude, Ptolemy uses $\Delta p_{\lambda 2} + \Delta p_{\lambda 3}$ to account for the epiparallax. However, in $\Delta p_{\lambda 2} + \Delta p_{\lambda 3}$, both effects of the lunar and solar epiparallax are taken into account. Thus, the resulting epiparallax can be expressed as

$$e_p = e_p(☽) - e_p(☉) \qquad (8),$$

where $e_p(☽)$ is the lunar epiparallax and $e_p(☉)$ the solar epiparallax. The fact that the solar epiparallax is included in the epiparallax means that we would not be able to deem that the apparent longitude of the Moon found, $\lambda(☽'_4)$, agrees with the apparent longitude of the Sun in the true conjunction, $\lambda(☉'_1)$; but that

$$\lambda(☽'_4) = \lambda(☉'_1) \mp e_p(☉) \qquad (9).$$

That is, it agrees with the apparent longitude of the Sun in the true conjunction, once the effect of the solar epiparallax in the time between the situation 4 and the true conjunction, that is $t_4 - t_{cv}$, is accounted for.

In the previous iterative procedure, Ptolemy refers to the sum of these differences, would they either be differences between parallaxes —that is $\Delta p_{\lambda 1}$— or between epiparallaxes —that is $\Delta p_{\lambda i}$ with $i > 1$—with the term «total parallax». This total parallax is the correction in longitude to account for the parallax that should be introduced in the longitude of the true conjunction to obtain the apparent one later on. In the following steps, we will refer to the total parallax with Δp_λ, so that, after generalizing the previous equation in case it converges, the total parallax is

$$\Delta p_\lambda = \sum_i \Delta p_{\lambda i} \qquad \text{with } i \in \mathbb{N}, i \geq 1$$

and

$$\Delta p_{\lambda i} = \Delta p_{\lambda 2}{}^{i-1} / \Delta p_{\lambda 1}{}^{i-2} \qquad (10).$$

In any case, Ptolemy only consider cases up to $i = 3$. Thus, as shown by Figure 17, $\lambda(☽_4)$ is

$$\lambda(☽_4) = \lambda(☽_1) \pm \Delta p_\lambda \qquad (11).$$

We should now examine if Ptolemy's method to obtain the true lunar longitude, whose apparent longitude equals the apparent longitude of the Sun in the true conjunction, is correct.

Broadly speaking, Ptolemy's method is based on correcting the longitude of the Moon with the longitudinal distance traversed by the Moon corresponding, first, to the difference between parallaxes of the Moon and the Sun and, then, between their epiparallaxes. Firstly, the increments that are not obtained through interpolation —that is, $\Delta p_{\lambda 1}$ and $\Delta p_{\lambda 2}$—, since the solar and lunar parallaxes increase when the Sun and the Moon tend to their true position in the apparent conjunction, the correction resulting from a previous parallax or epiparallax — that is, always resulting from a closer position to the true conjunction—, is always smaller than the needed one to obtain the true lunar longitude whose apparent longitude equals the apparent longitude of the Sun in the true conjunction. In addition, we know that the variation of the parallax in longitude when approaching the true position in the apparent conjunction is also smaller than in the true conjunction, since the altitude of the true longitude of a true conjunction is always greater than that of an apparent conjunction. Hence, the iteration of the procedure an infinite number of times tends to the true lunar longitude whose apparent longitude equals the apparent longitude of the Sun in the true conjunction. In turn, the increment resulting from the interpolation—$\Delta p_{\lambda 3} = \Delta p_{\lambda}{}^{2}_{2} / \Delta p_{\lambda 1}$— is greater than the epiparallax between the situations 2 and 3 ($e_p|^3_2$)—i.e., $e_p|^3_2 = [p_\lambda(☽)_3 - p_\lambda(\odot)_3] - [p_\lambda(☽)_2 - p_\lambda(\odot)_2]$. That is,

$$\Delta p_{\lambda}{}^{2}_{2} / \Delta p_{\lambda 1} > [p_\lambda(☽)_3 - p_\lambda(\odot)_3] - [p_\lambda(☽)_2 - p_\lambda(\odot)_2] \qquad (12).$$

The reason lays on the fact that the obtained slope through previous increments —$m = \Delta p_{\lambda 2} / \Delta p_{\lambda 1}$— is always greater than the real one, since the variation of the parallax in longitude when we tend to the true position in the apparent conjunction is smaller than when we tend to the true conjunction. Hence, the longitude obtained through the interpolation can be smaller, equal, or greater than the one being sought, whereas if we would have used ($e_p|^3_2$), the obtained longitude would be, in the first quadrant, greater, and, in the second quadrant, smaller than the longitude of the true Moon in the apparent conjunction. Whatever the case, the error is very small. In addition, the smaller the increment affected by the slope m —that is, the increment $\Delta p_{\lambda 2}$, since $\Delta p_{\lambda 3} = m\, \Delta p_{\lambda 2}$— is, the error resulting from the difference between m and the real slope is also smaller.

In the second step, Ptolemy should account, as shown by Figure 17, for the additional motion of the Sun during the time in which the Moon traverses Δp_λ. As usual, Ptolemy approximates the motion of the mean Sun, $w\bar{\odot}$, by $1^{o/d}$, and the motion in longitude of the mean Moon, $w\bar{☽}$, by $13^{o/d}$. Thus, the difference of motions between the mean Moon and mean Sun would be of $12^{o/d}$. The steps followed by Ptolemy to determine the position of the apparent conjunction from the true conjunction and the total parallax, Δp_λ, are the following:

- firstly, he divides the total parallax (obtained without taking into account the additional motion of the Sun) by 12 and adds it to itself —that is, $(1+1/12)\,\Delta p_\lambda = 13\,\Delta p_\lambda/12$;[20]
- and he, then, divides the obtained value by the true motion of the Moon in the conjunction, thus obtaining the number of equinoctial hours between the true conjunction and the apparent one.[21]

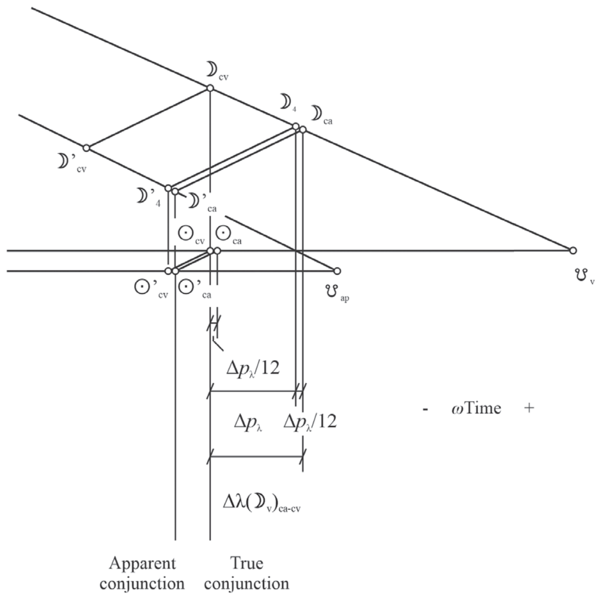


Figure 17. Apparent conjunction in the second quadrant.

Thus, to obtain the increment in longitude of the true Moon in the interval defined by the true conjunction and the apparent one, $\Delta\lambda(☽)_{ca\text{-}cv}$, Ptolemy establishes a lineal equivalence; that is,

20. «To the total parallax in longitude, computed in this way, we add the 1/12th of itself to account for the additional motion of the sun» (Toomer p. 311).

21. «And convert the total to equinoctial hours by dividing it by the moon's true hourly motion at the conjunction» (Toomer p. 311).

$$\Delta\lambda(☽)_{\text{ca-cv}} = {}^{w_☽}/_{(w_☽ - w_\odot)}\ \Delta p_\lambda = {}^{13}/_{12}\ \Delta p_\lambda \qquad (13).$$

However, how does Ptolemy reach this result? To describe this method, we define Δt as the time between situation 4 and the true conjunction —$\Delta t = t_4 - t_{cv}$— and Δt_+ as the time between the apparent conjunction and situation 4 —$\Delta t_+ = t_{ca} - t_4$—. Likewise, as shown in Figure 18, $\Delta\lambda_{\odot☽4}$ is the difference between the longitude of the Sun and the Moon in the fourth situation; $\Delta\lambda_☽$ is the increment in longitude of the apparent Moon between the apparent conjunction and the situation 4; and $\Delta\lambda_\odot$ is the increment in longitude of the apparent Sun between the apparent conjunction and situation 4.

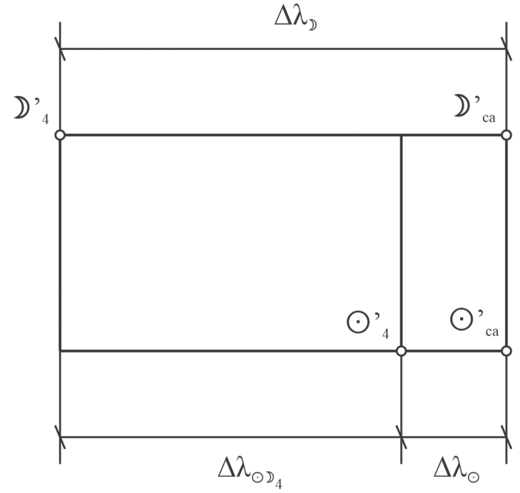


Figure 18. Apparent Sun and Moon between situation 4 and apparent conjunction.

From these variables, we can obtain the time interval, Δt, during which the Moon traverses Δp_λ knowing that

$$\Delta p_\lambda = w_☽\ \Delta t.$$

During this time, the Sun traverses

$$\Delta\lambda_{\odot☽4} = w_\odot\ \Delta t = {}^{w_\odot}/_{w_☽}\ \Delta p_\lambda\ ,$$

so that the increment in longitude of the apparent Moon between the apparent conjunction and the situation 4 —$\Delta\lambda_{☽}$— is

$$\Delta\lambda_{☽} = \Delta\lambda_{☉☽4} + \Delta\lambda_{☉}$$

$$\Delta\lambda_{☽} = w_{☽}\ \Delta t_{+}\ {}^{w_{\bar{☉}}}/_{w_{☽}}\ \Delta p_{\lambda} + w_{\bar{☉}}\ \Delta t_{+}$$

$$\Delta t_{+} = {}^{w_{\bar{☉}}}/_{w_{☽}\ (w_{☽} - w_{\bar{☉}})}\ \Delta p_{\lambda}$$

$$\Delta\lambda_{☽} = w_{☽}\ \Delta t_{+} = {}^{w_{\bar{☉}}}/_{w_{☽}\ (w_{☽} - w_{\bar{☉}})}\ \Delta p_{\lambda} = {}^{1}/_{12}\ \Delta p_{\lambda} \qquad (14).$$

Thus, from $\Delta\lambda_{☽}$, i.e., the increment in longitude of the apparent Moon between the apparent conjunction and situation 4, Ptolemy finds the total increment in longitude of the true Moon between the true conjunction and the apparent one by adding it to the total parallax obtained between the true conjunction and situation 4.

$$\Delta\lambda(☽)_{\text{ca-cv}} = \Delta p_{\lambda} + \Delta\lambda_{☽} = \Delta p_{\lambda} + {}^{w_{\bar{☉}}}/_{w_{☽}\ (w_{☽} - w_{\bar{☉}})}\ \Delta p_{\lambda}$$

$$= {}^{w_{☽}}/_{(w_{☽} - w_{\bar{☉}})}\ \Delta p_{\lambda} = {}^{13}/_{12}\ \Delta p_{\lambda} \qquad (15).$$

Consequently, the lunar longitude in the apparent conjunction is

$$\lambda(☽)_{\text{ca}} = \lambda(☽)_{\text{cv}} \pm {}^{13}/_{12}\ \Delta p_{\lambda} \qquad (16),$$

although Ptolemy obtains this value in a later step. Ptolemy adds $\Delta\lambda_{☽}$, an increment in longitude related to the apparent Moon, to Δp_{λ}, an increment in longitude related to the true Moon. In any case, he should have taken into account the effect of the lunar epiparallax in the interval Δt_{+}. This effect is of opposite sign to that of the epiparallax obtained to compute the total parallax Δp_{λ}. Likewise, he does not take into account the solar epiparallax during the additional motion of the Sun to obtain the apparent longitude of the apparent conjunction. This epiparallax is of opposite sign to that of the lunar epiparallax in this interval. Thus, following Ptolemy's previous methodology, the longitude of the true Moon in the apparent conjunction should have been

$$\lambda(☽)_{\text{ca}} \cong \lambda(☽)_{\text{cv}} \pm \left({}^{13}/_{12}\ \Delta p_{\lambda} - e_{p}\big|_{4}^{\text{ca}}\right) \qquad (17)$$

where $e_{p}\big|_{4}^{\text{ca}}$ is

$$e_p\big|_4^{ca} = e_p(☽)\big|_4^{ca} - e_p(\odot)\big|_4^{ca} \qquad (18),$$

i.e., the difference between the lunar and solar parallaxes in the interval defined by situation 4 and the apparent conjunction.

From the increment of the true Moon in longitude, $\Delta\lambda(☽)_{\text{ca-cv}}$, Ptolemy finds the number of equinoctial hours related to this increment by dividing it by the true motion of the Moon in the conjunction. Thus,

$$\Delta t_{\text{ca-cv}} = \Delta\lambda(☽)_{\text{ca-cv}} / w(☽_{\text{cv}}) \qquad (19).$$

Once he obtains this interval, Ptolemy points out whether it should be added to or subtracted from the time of the true conjunction following the effect of the parallax.

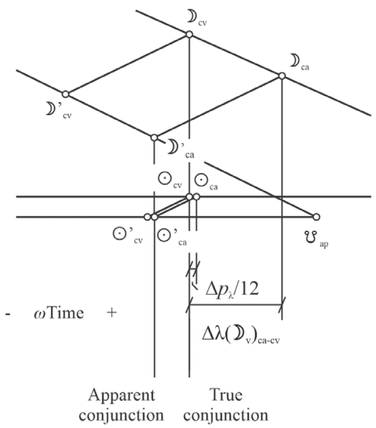


Figure 19. Negative parallax in longitude.

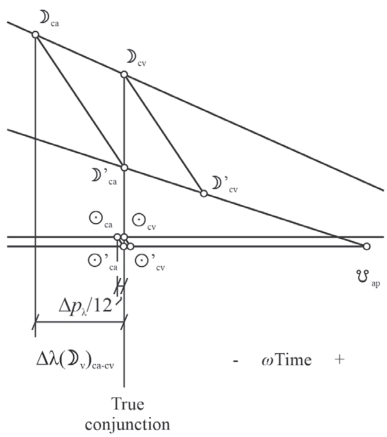


Figure 20. Positive parallax in longitude.

Generally speaking, the distance traversed by the Sun and the Moon during the interval between the true conjunction and the apparent one should cancel the effect of the parallax in the true conjunction. Since the parallax in longitude can be, either, positive or negative, two different cases can take place:

- If the parallax in longitude in the true conjunction is negative —that is, if the apparent longitude is smaller than the true one—, we should consider a positive distance traversed by the Sun and the Moon in longitude that would account for the negative parallax in longitude, so that the time interval corresponding to this distance is positive. As a result, the true conjunction would take place before the time of the apparent conjunction and, consequently, the true longitudes in the true conjunction would be smaller than the true longitudes in the apparent conjunction, as shown in Figure 19.

- If the parallax in longitude in the true conjunction is positive —that is, if the apparent longitude is greater than the true one—, we should consider a negative distance traversed by the Sun and the Moon in longitude that would account for the positive parallax in longitude, so that the time interval corresponding to this distance is negative. As a result, the apparent conjunction would take place before the time of the true conjunction and, consequently, the true longitudes in the apparent conjunction would be smaller than the true longitudes in the true conjunction, as shown in Figure 20.

Ptolemy examines if the parallax in longitude follows the zodiacal signs —that is, if the parallax in longitude is positive— or is opposite to the zodiacal signs —that is, if the parallax in longitude is negative—. If the parallax in longitude is positive, he subtracts the time interval between the apparent conjunction and the true one, $\Delta t_{\text{ca-cv}}$, at the time of the true conjunction, so that the time of the apparent conjunction is

$$t_{\text{ca}} = t_{\text{cv}} - \Delta t_{\text{ca-cv}} \qquad (20).$$

With t_{ca}, he finds the position of the Moon in the apparent conjunction in longitude, latitude and anomaly. Likewise, if the parallax in longitude is negative, the time of the apparent conjunction is

$$t_{\text{ca}} = t_{\text{cv}} + \Delta t_{\text{ca-cv}} \qquad (21).$$

Accordingly, he finds the position of the Moon in the apparent conjunction in longitude, latitude and anomaly.

(iii) Argument in apparent latitude for the apparent conjunction

Once we have obtained the true and apparent longitudes of the Moon in the apparent conjunction, we should address Ptolemy's third step to account for the effect of the parallax in the computation of the magnitude and phases of the solar eclipse. In this third step, Ptolemy should obtain the argument in apparent latitude from the true one. If this argument in latitude is included in the tables,[22] we can infer that a solar eclipse will take place and we can obtain its magnitude. Thus, for any given apparent conjunction close to a node (Figure 21), Ptolemy finds the argument in latitude of the apparent conjunction following these steps:

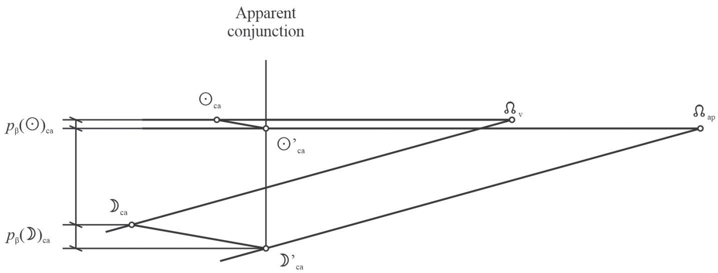


Figure 21. Apparent conjunction.

- firstly, he computes the lunar parallax taking into account the distance in equinoctial hours between the apparent conjunction and the meridian;
- he, then, subtracts the solar parallax from the lunar parallax, $\Delta p_{ca} = p(☽)_{ca} - p(\odot)_{ca}$;
- he finds the value in latitude of the previous parallax, i.e., Δp_β;
- and finally, he obtains the argument in latitude in the lunar inclined orbit from the previous result times 12;

Thus, he, first, finds the lunar parallax in the apparent conjunction, i.e., $p(☽)_{ca}$. Next, he finds the solar parallax, $p(\odot)_{ca}$, and he subtracts it to the lunar one to obtain the difference between the lunar and solar parallax in the apparent conjunction, $\Delta p_{ca} = p(☽)_{ca} - p(\odot)_{ca}$, as shown in Figure 22. From this difference, he finds the component in latitude in the apparent conjunction, Δp_β.

Once he has obtained the component in latitude of the difference between the solar and lunar parallaxes, Δp_β, he multiplies it by 12, so that he finally finds the increment of the argument in latitude resulting from the difference between the solar and lunar parallax, since

$$^{1}/_{\sin i} \cong 11;30 \cong 12 \qquad (22)$$

22. See *Almagest* VI.8 (Toomer pp. 306-308) for these tables.

with $i = 5^{\circ}$, the angle of the lunar inclined orbit relative to the ecliptic. Thus, by dividing the component in latitude of the parallactic difference by sin i, Ptolemy deems that the angle of inclination of the true inclined orbit relative to the ecliptic and the angle of inclination of the apparent course of the Moon are equal, so that both —the ecliptic and the apparent course of the Moon— are parallel, as he takes i to obtain an argument in apparent latitude. One possible explanation for the reason why Ptolemy approximates 1/sin i by 12 and not by 11;30 is that he is introducing a correction after considering i in the apparent course of the Moon. However, this explanation should be dismissed since the angle of inclination of the apparent course of the Moon —let us refer to it as i'— can either be greater or smaller than the one of the true inclined orbit. Hence, the value by which the component in latitude of the parallactic difference, Δp_{β}, should be multiplied to obtain the component in the argument in latitude can be greater or smaller than 11;30. Consequently, Ptolemy's use of 12 instead of 11;30 is the result of an approximation, and not a correction.

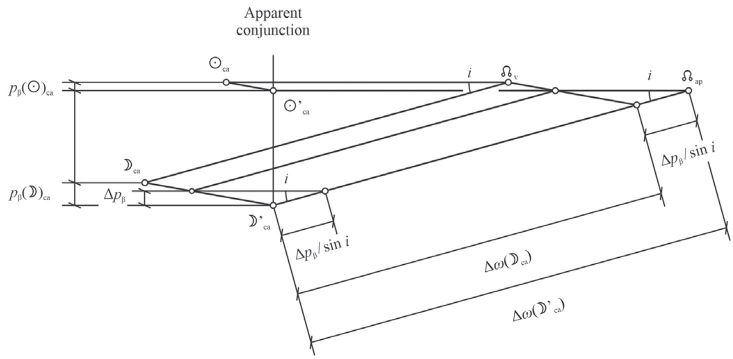


Figure 22. Argument in latitude of the apparent conjunction.

Thus, considering Figure 22, the difference of the argument in latitude of the apparent Moon in the apparent conjunction relative to the apparent node is

$$\Delta\omega(☽'_{ca}) = \Delta\omega(☽_{ca}) \pm {}^{\Delta p_{\beta}}/_{\sin i} \qquad (23),$$

where the signs + or – depend on the position of the observer and on whether the conjunction takes place close to the ascending or descending nodes.

However, Ptolemy is more concerned by the actual argument in apparent latitude from the true argument in latitude, rather than by the difference of the argument in latitude relative to the apparent node, since the former is the value of the argument in latitude needed in the tables of solar eclipses. To obtain the argument in latitude from the previous differences, we should add 90° in case of a descending node, or 270° in case of the ascending one. Thus, the argument in apparent latitude is

$$\omega(☽'_{ca}) = \omega(☽_{ca}) \pm \Delta p_{\beta} / \sin i \qquad (24).$$

As pointed out before, the sign ± in the previous equation depends (i) on the position of the observer and (ii) on whether the conjunction takes place close to the ascending or descending nodes. Let us, first, consider the effect of the position of the observer on whether $\Delta p_{\beta} / \sin i$ should be added or subtracted.[23]

If the observer is located in a geographical latitude above —that is, to the north— of the ecliptic, the zenith is located in a positive latitude. Since the true altitude is always greater than the apparent one, because of the effect of the parallax, and the latitude of the zenith is positive, the apparent latitude is always smaller than the true one.

In turn, if the observer is located in a geographical latitude below —that is, to the south— of the ecliptic, the zenith is located in a negative latitude. Since the true altitude is always greater than the apparent one, because of the effect of the parallax, and the latitude of the zenith is negative, the apparent latitude is always greater than the true one.

Thus, if the observer is located to the north of the ecliptic, the parallax in latitude is negative. Hence, the apparent courses of the Sun and Moon are found, in the figure, below the true ones.

In turn, if the observer is located to the south of the ecliptic, the parallax in latitude is positive. Hence, the apparent courses of the Sun and Moon are found, in the figure, above the true ones.

In addition to the effect of the latitude of the parallax, the sign ± also depends on whether the eclipse takes place close to the ascending or descending nodes, and if the apparent conjunction takes place in positive or negative latitudes. In general, for a same situation, if the eclipse takes place close to an ascending or descending node, the sign of the expression changes. Ptolemy takes into account four situations depending on whether the effect of the parallax in latitude of the Moon in the apparent conjunction takes place to the north or to the south of the

23. See *Almagest* V.19 (Toomer pp. 266-267).

ecliptic, and on whether it is close to the ascending or descending node. The four cases are indicated below:

(1) If the effect of the parallax in latitude takes place to the north of the ecliptic and the Moon is close to the ascending node, the result is added to the argument in latitude of the true Moon in the apparent conjunction.

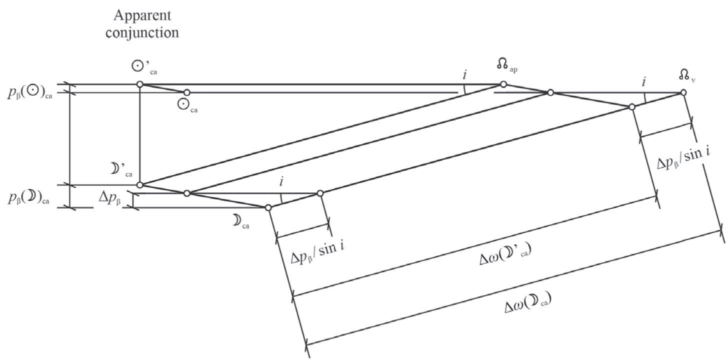


Figure 23. Conjunction before the ascending node
with parallax to the north of the ecliptic.

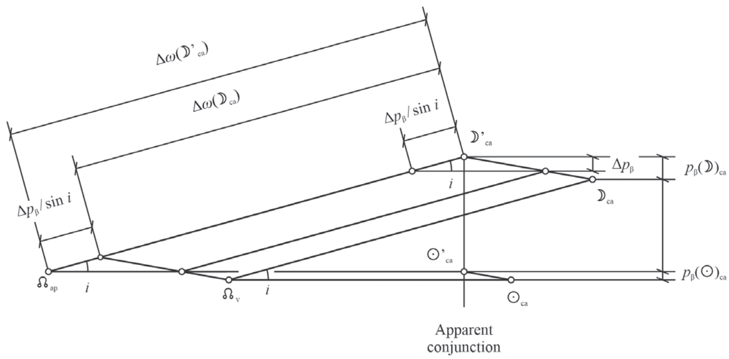


Figure 24. Conjunction after the ascending node
with parallax to the north of the ecliptic.

In this case, the equation is

$$\omega(☽'_{ca}) = \omega(☽_{ca}) + \Delta p_\beta / \sin i \ .$$

If the apparent conjunction takes place before reaching the node, the difference of the argument in apparent latitude of the apparent conjunction relative to the node is smaller than the true one —$\Delta\omega(☽'_{ca}) < \Delta\omega(☽_{ca})$. Hence, the argument in latitude of the apparent Moon in the apparent conjunction is greater than that of the true one —$\omega(☽'_{ca}) > \omega(☽_{ca})$.

In turn, if the apparent conjunction takes place after traversing the node, the difference of the argument in apparent latitude of the apparent conjunction relative to the node is greater than that of the true one —$\Delta\omega(☽'_{ca}) > \Delta\omega(☽_{ca})$. Hence, the argument in latitude of the apparent Moon in the apparent conjunction is also greater than that of the true one —$\omega(☽'_{ca}) > \omega(☽_{ca})$. Consequently, in both cases, $\Delta p_\beta / \sin i$ must be added, as Ptolemy points out.

(2) If the effect of the parallax in latitude takes place to the north of the ecliptic and the Moon is close to the descending node, the result is subtracted from the argument in latitude of the true Moon in the apparent conjunction.

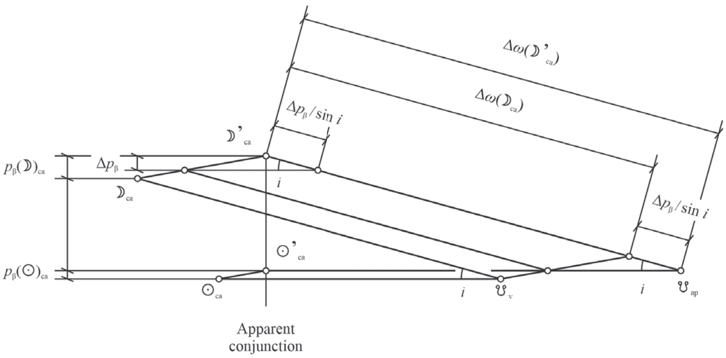


Figure 25. Conjunction before the descending node with parallax to the north of the ecliptic.

In this case, the equation is

$$\omega(☽'_{ca}) = \omega(☽_{ca}) - \Delta p_\beta / \sin i \ .$$

If the apparent conjunction takes place before reaching the node, the difference of the argument in apparent latitude of the apparent conjunction relative to the node is greater than the true one —$\Delta\omega(☽'_{ca}) > \Delta\omega(☽_{ca})$. Hence, the argument

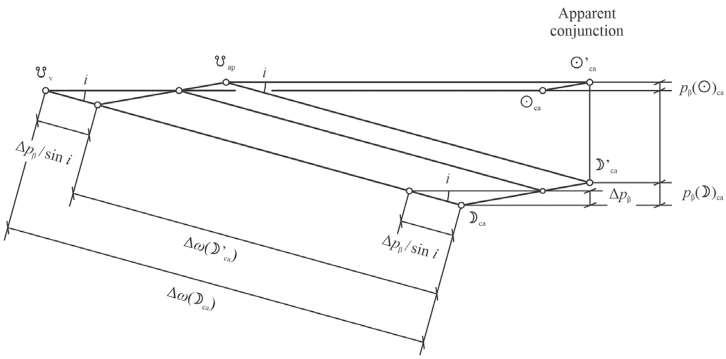


Figure 26. Conjunction after the descending node
with parallax to the north of the ecliptic.

in latitude of the apparent Moon in the apparent conjunction is smaller than that of the true one —$\omega(☽'_{ca}) < \omega(☽_{ca})$.

In turn, if the apparent conjunction takes place after traversing the node, the difference of the argument in apparent latitude of the apparent conjunction relative to the node is smaller than that of the true one —$\Delta\omega(☽'_{ca}) < \Delta\omega(☽_{ca})$—. Hence, the argument in latitude of the apparent Moon in the apparent conjunction is also smaller than that of the true one —$\omega(☽'_{ca}) < \omega(☽_{ca})$—. Consequently, in both cases, Δp_β / sin i must be subtracted, as Ptolemy points out.

(3) If the effect of the parallax in latitude takes place to the south of the ecliptic and the Moon is close to the ascending node, the result is subtracted from the argument in latitude of the true Moon in the apparent conjunction.

In this case, the equation is

$$\omega(☽'_{ca}) = \omega(☽_{ca}) - \Delta p_\beta / \sin i\ .$$

The situation is similar to the apparent conjunction close to the descending node and parallax to the north of the ecliptic. That is, if the apparent conjunction takes place before reaching the node, the difference of the argument in apparent latitude of the apparent conjunction relative to the node is greater than the true one—$\Delta\omega(☽'_{ca}) > \Delta\omega(☽_{ca})$. Hence, the argument in latitude of the apparent Moon in the apparent conjunction is smaller than that of the true—$\omega(☽'_{ca}) < \omega(☽_{ca})$.

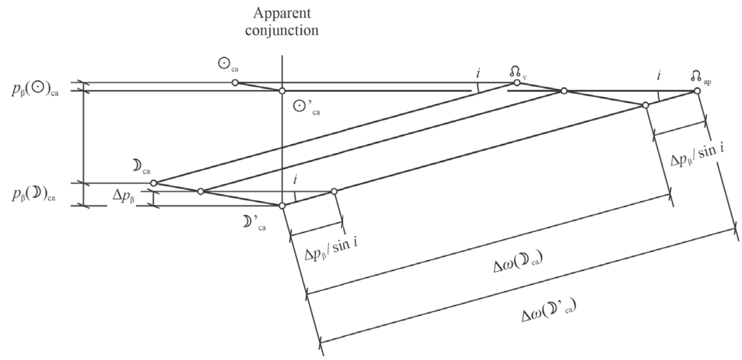


Figure 27. Conjunction before the descending node
with parallax to the north of the ecliptic.

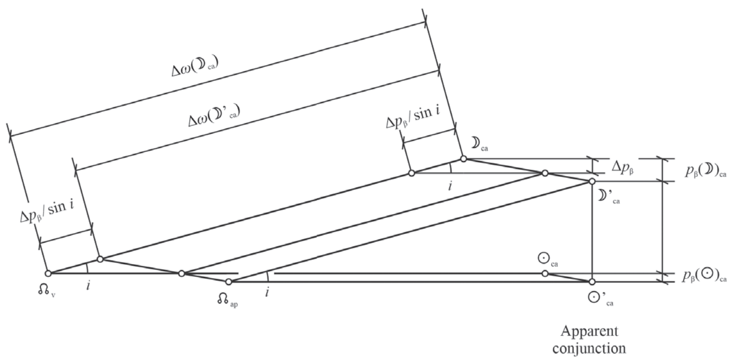


Figure 28. Conjunction after the ascending node
with parallax to the south of the ecliptic.

In turn, if the apparent conjunction takes place after traversing the node, the difference of the argument in apparent latitude of the apparent conjunction relative to the node is smaller than that of the true one —Δω(☽'$_{ca}$) < Δω(☽$_{ca}$)—. Hence, the argument in latitude of the apparent Moon in the apparent conjuction is also smaller than that of the true one —ω(☽'$_{ca}$) < ω(☽$_{ca}$)—. Consequently, in both cases, Δp_β / sin i must be subtracted, as Ptolemy points out.

(4) If the effect of the parallax in latitude takes place to the south of the ecliptic and the Moon is close to the descending node, the result is added to the argument in latitude of the true Moon in the apparent conjunction.

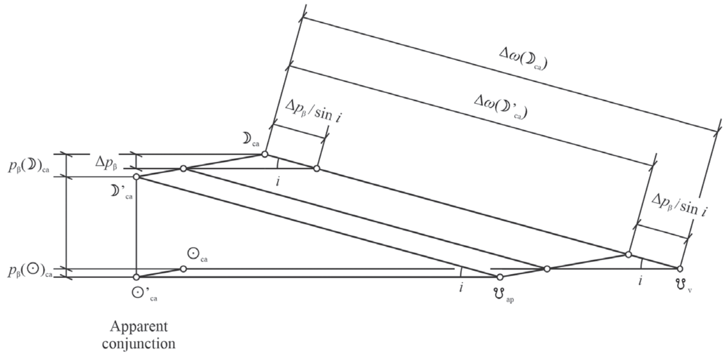


Figure 29. Conjunction before the descending node
with parallax to the south of the ecliptic.

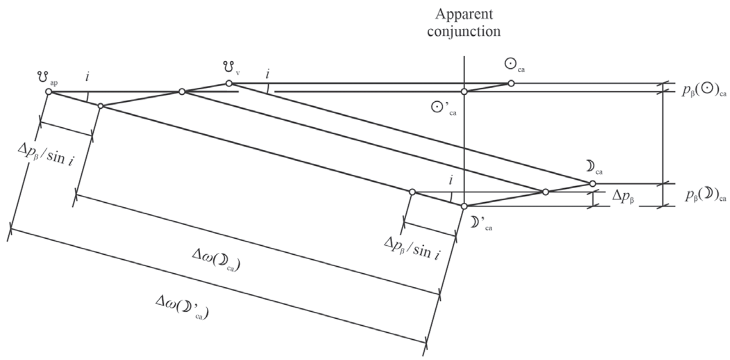


Figure 30. Conjunction after the descending node
with parallax to the south of the ecliptic

In this case, the equation is

$$\omega(\text{☽}'_{ca}) = \omega(\text{☽}_{ca}) + \Delta p_\beta / \sin i \,.$$

This situation is equivalent to the apparent conjunction close to the ascending node and parallax to the north of the ecliptic. That is, if the apparent conjunction takes place before reaching the node, the difference of the argument in apparent latitude of the apparent conjunction relative to the node is smaller than that of the true one —$\Delta\omega(\text{☽}'_{ca}) < \Delta\omega(\text{☽}_{ca})$. Hence, the argument in latitude of the apparent Moon in the apparent conjunction is greater than that of the true —$\omega(\text{☽}'_{ca}) > \omega(\text{☽}_{ca})$.

In turn, if the apparent conjunction takes place after traversing the node, the difference of the argument in apparent latitude of the apparent conjunction relative to the node is greater than that of the true one —$\Delta\omega(☽'_{ca}) > \Delta\omega(☽_{ca})$. Hence, the argument in latitude of the apparent Moon in the apparent conjunction is also greater than that of the true one —$\omega(☽'_{ca}) > \omega(☽_{ca})$—. Consequently, in both cases, Δp_β / sin i must be added, as Ptolemy points out.

Thus, after the above considerations, Ptolemy ends up finding an approximation to the argument in latitude of the apparent Moon in the apparent conjunction.

Finding the magnitude

Once Ptolemy has obtained the argument in latitude of the apparent Moon in the apparent conjunction, that is, once he has accounted for the effect of the parallax, Ptolemy finds the magnitude of the solar eclipse following the same method as with the lunar eclipse.

He uses Tables I and II devoted to solar eclipses, instead.[24] Firstly, he examines if the argument in latitude falls within the limits in which an eclipse can take place.[25] If this is so, a solar eclipse will take place. He, then, introduces the argument in latitude of the apparent Moon in the apparent conjunction and finds in Column III of Table I the magnitude of the eclipse when the Moon is in its apogee. Likewise, with the argument in latitude of the apparent Moon in the apparent conjunction, he finds in Column III of Table II the magnitude of the eclipse when the Moon is in its perigee. Based on both perigee and apogee results, he, then, interpolates them using Table V of interpolation with the lunar anomaly as argument.[26]

2.5. *The* Almagest *on the phases of solar eclipses*

As a first approximation, to obtain the duration of the solar eclipse, Ptolemy's method is the same as with lunar eclipses.[27] That is, with the argument in latitude of the apparent Moon in the apparent conjunction, he finds in column IV of

24. See *Almagest* VI.8 (Toomer p. 306).
25. See *Almagest* VI. 5 (Toomer pp. 282-287).
26. See *Almagest* VI.8 (Toomer p. 308).
27. See *Almagest* VI.10 (Toomer p. 312).

Table I the minutes of immersion of the eclipse when the Moon is located in its apogee. Likewise, with the argument in latitude of the apparent Moon in the apparent conjunction, he finds in column IV of Table II the minutes of immersion of the eclipse when the Moon is in its perigee. Then, the resulting values should be interpolated using Table V with the lunar anomaly as argument. He, then, accounts for the additional motion of the Sun, so that he multiplies by 13/12 the minutes of immersion found with the tables. Finally, he finds the duration of each phase in equinoctial hours by dividing the minutes of immersion obtained before by the true hourly motion of the Moon.

However, the method to find out the phases of the solar eclipse is not exactly the same as with lunar eclipses, since the effect of the parallax should be taken into account. Two are the effects introduced by the parallax:

(i) Firstly, the duration of the phases would be greater than the ones obtained with the tables; and
(ii) secondly, the duration of both phases can be different.

Firstly, let us address the effect of the parallax, i.e., that the duration of the phases would be greater than the ones obtained with the tables. Ptolemy deals with this effect in the following quotation:

> This phenomenon is due to the fact that the effect of the parallax on the moon's apparent motion is always to produce the appearance of motion which would be in advance (if one were to disregard the moon's proper motion towards the rear). For suppose, first, that the moon's apparent position is before [i.e. to the east of] the meridian: then, as it gradually rises higher [above the horizon], its eastward parallax becomes continually smaller than at the moment preceding, and thus its motion towards the rear appears slower. Or suppose, secondly, that its apparent position is after [i.e. to the west of] the meridian: then, again, as it gradually descends [towards the horizon], its westward parallax becomes continually greater than at the moment preceding, and thus, as before, its motion towards the rear appears slower.[28]

Ptolemy's method is not very clear. To obtain the apparent motion, he knows an interval obtained with the use of tables, an increment in the parallax related to

28. See *Almagest* VI.10 (Toomer p. 312).

this interval, and the true motion of the Moon. The above quotation can be interpreted in two different ways:

(i) either, the interval obtained with tables is an apparent interval, so that, through the parallactic correction, he finds the true interval;
(ii) or, the interval obtained with tables is a true interval, so that, through the parallactic correction, he finds the apparent interval.

Since the intervals obtained with the use of the tables related to the minutes of immersion and emersion are equal and the procedure involves apparent radii, the most plausible hypothesis is that Ptolemy understands the intervals obtained with tables as apparent intervals.

If the intervals obtained with tables are apparent, how can he find the apparent motion of the Moon?

Firstly, he should take the minutes of immersion obtained with the tables and correct them with the additional motion of the Sun. He then considers the initial and final parallaxes of the apparent interval relative to the minutes of immersion, and obtains a new interval, that should be deemed as a true one; that is an interval in the lunar inclined orbit. If he applies the true lunar motion to the interval obtained before, he will be able to find the duration of the phase relative to the interval in question. And last, from the duration of the phase and the —apparent— minutes of immersion computed with the tables and corrected with the additional motion of the Sun, he can obtain the apparent motion of the Moon in its apparent inclined orbit.

The method would, then, be the following:

- With the use of tables, he finds a value that he multiplies by 13/12 and obtains $\Delta\omega(☽')$, the interval in the apparent course of the Moon.
- He, then, finds the corresponding true interval, $\Delta\omega(☽)$, as $\Delta\omega(☽) = \Delta\omega(☽') - \Delta p_{\omega}$, where Δp_{ω} is the component in the lunar inclined orbit of the increment of the parallax.
- He, then, finds the duration of the phase by applying the true lunar motion as $\Delta t = \Delta\omega(☽) / w(☽)$.
- And last, he finds the apparent motion, $w(☽')$, as $w(☽') = \Delta\omega_{ap} / \Delta t$.

If, instead of the previous hypothesis, Ptolemy would deem the interval obtained with the use of tables as a true interval and the obtained one after the correction with the parallactic increment as an apparent interval, he would be able to apply the additional information that he has, namely, the true motion of the Moon, either, to the true interval, obtained with tables, or to the apparent interval. In the first case, both phases would be equal, and consequently, it would contradict Ptolemy's premise that the duration of both phases should be different because of the parallax. In the second case, he would apply a true motion to an apparent interval, and this makes no sense.

If the intervals obtained with the tables are apparent, we should compare the results obtained with the aforementioned method with Ptolemy's ones.

Ptolemy considers two situations:

- When the solar eclipse takes place before the meridian, i.e., towards the east, as the altitude of the Moon increases over the horizon, the component in longitude of its parallax towards the east decreases, so that the apparent motion of the Moon appears to advance relative to the true one. Thus, the motion of the Moon in the opposite direction appears to be slower.
- When the solar eclipse takes place after the meridian, i.e., towards the west, as the altitude of the Moon decreases over the horizon, the component in longitude of its parallax towards the west increases, so that, as in the previous case, the apparent motion of the Moon appears to advance relative to the true one. Thus, the motion of the Moon in the opposite direction appears to be slower again.

Consequently, the motion of the Moon in the opposite direction appears to be slower regardless of whether the solar eclipse takes place before or after the meridian. However, what does Ptolemy mean by «its motion towards the rear». At the beginning of the previous quotation, Ptolemy points out:

> This phenomenon is due to the fact that the effect of the parallax on the moon's apparent motion is always to produce the appearance of motion which would be in advance (if one were to disregard the moon's proper motion towards the rear).

The Moon appears to be moving «in advance» since, when advancing towards the meridian, its parallax decreases towards the east; and when descending towards the horizon, the parallax increases towards the west. For this reason, it appears to advance faster in its course over the horizon. Thus, an apparent motion «in advance» is a motion relative to the horizontal motion of the celestial sphere.

Consequently, an apparent motion «towards the rear» should be an apparent motion relative to a motion in longitude, or, in the case of the Moon, a motion in its inclined orbit.[29] Hence, the effect of the parallax results in that the apparent motion of the Moon in longitude —or accordingly, in its inclined orbit— is slower than the true one. That is, $w(☽') < w(☽)$.

We should now consider whether the hypothesis about Ptolemy's method agrees with the previous interpretation, i.e., $w(☽') < w(☽)$.

Let us examine the apparent and true intervals. Since the duration of the phase is the same in the apparent interval and the true one, and the apparent motion is smaller than the true one, the apparent interval should be smaller than the true one, because

$$\Delta\omega(☽') = w(☽') \,/\, \Delta t < w(☽) \,/\, \Delta t = \Delta\omega(☽) \qquad (25).$$

Thus, we can verify if the apparent motion is slower than the true one if the apparent interval is smaller than the true one.

Let us first examine the situation in the first quadrant.

We know that

$$\omega(☽')_1 = \omega(☽)_1 + p_\omega(☽)_1$$
$$\omega(☽')_2 = \omega(☽)_2 + p_\omega(☽)_2.$$

Additionally,

$$p_\omega(☽)_1 > p_\omega(☽)_2\ ,$$

with p_ω the component of the parallax in the inclined orbit. If we take the increments, we find

$$\Delta\omega(☽') = \Delta\omega(☽) + \Delta p_\omega(☽).$$

But since

$$\Delta p_\omega(☽) < 0,$$

29. See Toomer p. 267 and p. 311.

the apparent interval is smaller than the true one

$$\Delta\omega(☽') < \Delta\omega(☽) \qquad (26).$$

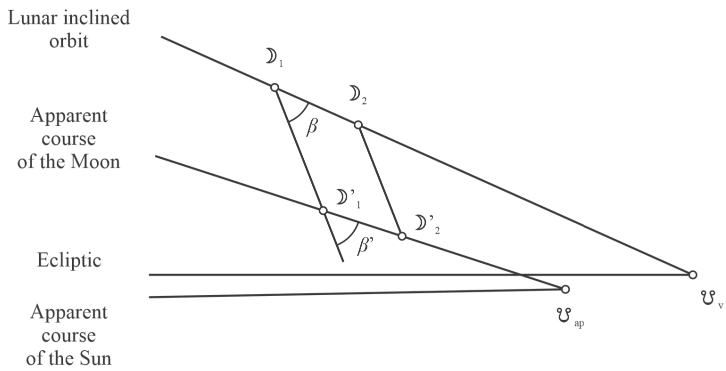


Figure 31. True and apparent intervals in the first quadrant.

Graphically (see Figure 31), in a first approximation, given two parallel parallaxes defining the initial and end times of an interval, if angle β, i.e., the acute angle defined by the lunar inclined orbit and the parallax, is smaller than angle β', i.e., the acute angle defined by the apparent course of the Moon and the parallax, the true intervals should be greater than their apparent intervals. In fact, to be more accurate, if

$$\beta < \beta' < 180^{o} - \beta \qquad (27),$$

the apparent interval is smaller than the true one, so that, accordingly, the apparent motion is also slower that than the true one. In order for the graphical method to be correct, the acute angles defined by the inclined orbit and the parallax, and by the apparent course and the parallax should be taken into account, since they have the two needed characteristics: (i) a negative increment of the parallax as a function of time—$\Delta p_{\omega}(☽) < 0$— and (ii) the addition of the component in the inclined orbit of the parallax to the argument in true latitude to obtain the argument in apparent latitude —$\omega(☽') = \omega(☽) + p_{\omega}(☽)$—.

As to the second quadrant, we know that

$$\omega(☽')_{1} = \omega(☽)_{1} - p_{\omega}(☽)_{1}$$

$$\omega(☽')_2 = \omega(☽)_2 - p_\omega(☽)_2.$$

Additionally,

$$p_\omega(☽)_1 < p_\omega(☽)_2,$$

with p_ω the component of the parallax in the inclined orbit. If we take the increments, we find that

$$\Delta\omega(☽') = \Delta\omega(☽) - \Delta p_\omega(☽).$$

But since

$$\Delta p_\omega(☽) > 0,$$

the apparent interval is smaller than the true one

$$\Delta\omega(☽') < \Delta\omega(☽) \qquad (28),$$

as in the case of the first quadrant.

If we examine the situation in the second quadrant graphically (see Figure 32), given two parallel parallaxes defining the initial and end times of an interval, if angle β, i.e., the acute angle defined by the lunar inclined orbit and the parallax, is smaller than angle β', i.e., the acute angle defined by the apparent course of the Moon and the parallax, the true intervals should be greater than their apparent intervals. In fact, to be more accurate, if

$$\beta < \beta' < 180^\circ - \beta,$$

the apparent interval is smaller than the true one, so that, accordingly, the apparent motion is also slower that than the true one.

For the geometrical procedure to be correct, the following must be taken into account: the acute angles defined by the inclined orbit and the parallax, and by the apparent course and the parallax, since they have the two needed characteristics, namely (i) a positive increment of the parallax as a function of time —$\Delta p_\omega(☽) > 0$— and (ii) the subtraction of the component in the inclined orbit of the parallax from the argument in true latitude to obtain the argument in apparent latitude —$\omega(☽') = \omega(☽) - p_\omega(☽)$—.

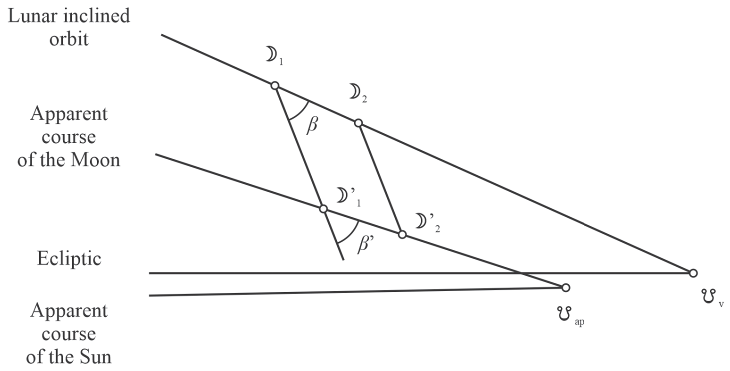


Figure 32. True and apparent intervals in the second quadrant.

To obtain the true interval from the apparent one graphically, let us consider Figure 33. If, as a way of approximation, we regard two different parallaxes as parallel, the true interval is

$$\Delta\omega(☽) = \Delta\omega(☽')\ {}^{\sin\beta'}/_{\sin\beta} \qquad (29).$$

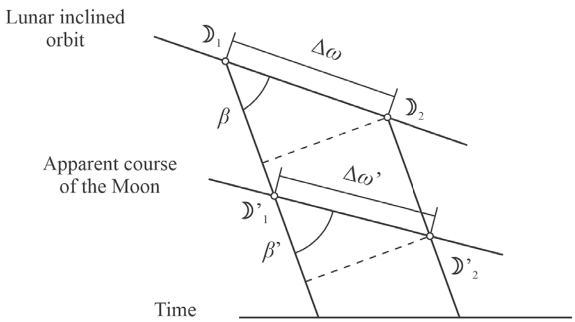


Figure 33. Graphical resolution of the true interval.

Thus, if the previous condition is met, i.e., $\beta < \beta' < 180^{\circ} - \beta$, we obtain that

$$\sin\beta' > \sin\beta \qquad (30)$$

and, consequently, $\Delta\omega(☽) > \Delta\omega(☽')$.

Before examining the true motion relative to the apparent one, it may be interesting to consider under which conditions, in case there are, the true motion would be smaller than the apparent one. In general, we know intuitively that in most cases it is greater. However, there may be some situations in which this is not the case.

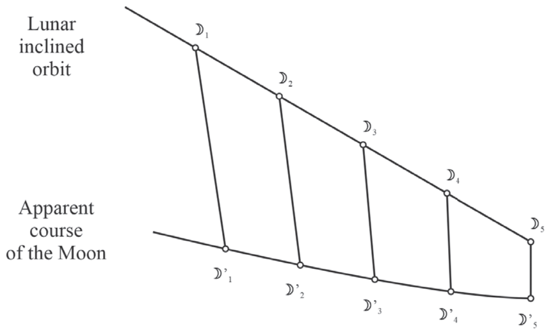


Figure 34. Temporal evolution of the parallax relative to the lunar inclined orbit.

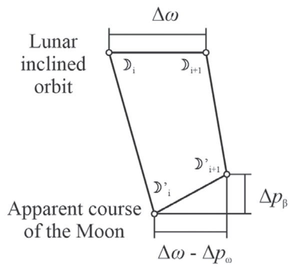


Figure 35. Increment in the evolution of the parallax relative to the lunar inclined orbit.

Figure 34 shows the evolution of the lunar parallax as a function of the lunar inclined orbit. We see that, as the Moon increases its altitude, the parallax decreases and tends to be a perpendicular relative to the inclined orbit. We can examine the conditions in which the apparent motion is greater than the true one if we take one of the figures defined by two successive true and apparent positions of the Moon, as in Figure 35.

We want to know when

$$\Delta\omega(\text{☽}) < \Delta\omega(\text{☽}') \, .$$

From Figure 35, in which all increments are positive, for infinitesimal increments, the previous inequality is

$$w(\text{☽})\,\mathrm{d}t < \left[\left(w(\text{☽})\,\mathrm{d}t - \frac{\mathrm{d}p_\omega}{\mathrm{d}t}\,\mathrm{d}t\right)^2 + \left(\frac{\mathrm{d}p_\beta}{\mathrm{d}t}\,\mathrm{d}t\right)^2\right]^{1/2} \qquad (31),$$

where dp_{ω}/dt corresponds to the parallax in the inclined orbit and dp_{β}/dt corresponds to the component in latitude of the parallax relative to the inclined orbit —and not relative to the ecliptic—. Since all the increments are positive, we can find

$$w(☽) < \frac{1}{2}\frac{\left(\frac{dp_{\omega}}{dt}\right)^2+\left(\frac{dp_{\beta}}{dt}\right)^2}{\frac{dp_{\omega}}{dt}} \qquad (32).$$

If dp_{ω}/dt and dp_{β}/dt tend to zero, the second term of the inequality also tends to zero. When dp_{β}/dt has a significant value and, in turn, dp_{ω}/dt tends to zero, then the second term of the inequality increases exponentially, so that we would obtain the values that make the initial inequality true, and, consequently, the apparent interval would be greater than the true one. Let β be the angle defined by the parallax and the lunar inclined orbit. The variation of the parallax in the inclined orbit is

$$\frac{dp_{\omega}}{dt} = \frac{dp}{dt}\cos\beta - \text{p sen}\,\beta \qquad (33).$$

And the variation of the parallax in latitude relative to the inclined orbit is

$$\frac{dp_{\beta}}{dt} = \frac{dp}{dt}\,\text{sen}\,\beta - \text{p}\cos\beta \qquad (34).$$

When β tends to 0° ($\beta \to 0°$),

$$\frac{dp_{\omega}}{dt} \to \frac{dp}{dt} \qquad \text{and} \qquad \frac{dp_{\beta}}{dt} \to p\ .$$

In addition, the parallax is big, but its variation is minimal. In turn, when β tends to 90° ($\beta \to 90°$),

$$\frac{dp_{\omega}}{dt} \to -p \qquad \text{and} \qquad \frac{dp_{\beta}}{dt} \to \frac{dp}{dt}\ .$$

In this case, the parallax is small and its variation is big. We need a minimal dp_{ω}/dt and a significant dp_{β}/dt. A minimal dp_{ω}/dt is true when β tends to 0°; in turn, a significant dp_{β}/dt is true when β tends to 90°. Thus, according to this first estimate, it would be difficult to find a situation in which the true motion would be smaller than the apparent.

Once we have studied the first effect, we should focus on the second effect pointed out by Ptolemy: that is, that in general the phases of immersion and emersion are different to each other.

Ptolemy points out that the more the Moon approaches the meridian —that is, the more it ascends in altitude—, the differences between successive parallaxes, defined by equal time intervals, increase, as from the table of parallaxes,[30] so that the parallax decreases more rapidly as the Moon approaches the meridian. That is, the absolute value of its variation with the altitude is greater in areas close to the meridian than in areas close to the horizon.

Hence, the duration of the phases to the one obtained through the column of *minutes of immersion* —column IV of the tables I and II— is not only different after the correction because of the additional motion of the Sun, but also because the phases of immersion and emersion of a same eclipse are different between each other, and thus the closer to the meridian is greater than the farther one. Lastly, Ptolemy points out that, if the eclipse mid-time happens at the same time when traversing the meridian, the duration of the phase of immersion should be equal to the phase of emersion.

Finally, Ptolemy illustrates the effect of the parallax in the duration of the different phases to conclude his exposition on solar eclipses.

Let us consider the second thesis by Ptolemy. To do so, we will first examine the first quadrant. We will use subscripts 'i', 'm' and 'f' to refer to the initial, middle and end times of the solar eclipse, and subscripts 'im' and 'em' to refer to the phase of immersion and the of emersion.

We know that

$$\begin{aligned}\omega(☽')_{\mathrm{i}} &= \omega(☽)_{\mathrm{i}} + p_{\omega}(☽)_{\mathrm{i}}\\ \omega(☽')_{\mathrm{m}} &= \omega(☽)_{\mathrm{m}} + p_{\omega}(☽)_{\mathrm{m}}\\ \omega(☽')_{\mathrm{f}} &= \omega(☽)_{\mathrm{f}} + p_{\omega}(☽)_{\mathrm{f}}\,.\end{aligned}$$

Additionally,

$$p_{\omega}(☽)_{\mathrm{i}} > p_{\omega}(☽)_{\mathrm{m}} > p_{\omega}(☽)_{\mathrm{f}}\,,$$

30. See *Almagest* V.18 (Toomer p. 265).

with p_ω the component of the parallax in the inclined orbit. In turn, we know that the variation of the parallax increases when the altitude increases. Hence,

$$\Delta p_\omega(☽)_{em} < \Delta p_\omega(☽)_{im} < 0\ .$$

If we take the increments corresponding to the phases of immersion and emersion, we find

$$\Delta\omega(☽')_{em} = \Delta\omega(☽)_{em} + \Delta p_\omega(☽)_{em}$$
$$\Delta\omega(☽')_{im} = \Delta\omega(☽)_{im} + \Delta p_\omega(☽)_{im}\ .$$

If we take the difference between the intervals of emersion e immersion, we find that

$$\Delta\omega(☽')_{em} - \Delta\omega(☽')_{im} = (\Delta\omega(☽)_{em} - \Delta\omega(☽)_{im}) + (\Delta p_\omega(☽)_{em} - \Delta p_\omega(☽)_{im})\ .$$

But, since

$$\Delta p_\omega(☽)_{em} < \Delta p_\omega(☽)_{im} < 0$$

and, consequently,

$$\Delta p_\omega(☽)_{em} - \Delta p_\omega(☽)_{im} > 0\ ,$$

we find that

$$(\Delta\omega(☽')_{em} - \Delta\omega(☽')_{im}) < (\Delta\omega(☽)_{em} - \Delta\omega(☽)_{im})\ .$$

However, since the apparent intervals are equal, that is

$$\Delta\omega(☽')_{em} = \Delta\omega(☽')_{im}\ ,$$

then

$$0 < (\Delta\omega(☽)_{em} - \Delta\omega(☽)_{im})\ .$$

Hence, the true interval in the phase of immersion is smaller than in the phase of emersion. That is,

$$\Delta\omega(☽)_{em} > \Delta\omega(☽)_{im} \quad (35).$$

Consequently, the duration of the phase of emersion is greater than that of immersion, since

$$\Delta t_{em} = \Delta\omega(☽)_{em} / w(☽) > \Delta\omega(☽)_{im} / w(☽) = \Delta t_{im} \quad (36).$$

Thus, when the solar eclipse takes place in the first quadrant, the duration of the phase of emersion, the closer to the meridian —according to Ptolemy—, is greater than the duration of the phase of immersion.

As for the second quadrant, we know that

$$\omega(☽')_i = \omega(☽)_i - p_\omega(☽)_i$$
$$\omega(☽')_m = \omega(☽)_m - p_\omega(☽)_m$$
$$\omega(☽')_f = \omega(☽)_f - p_\omega(☽)_f \,.$$

Additionally,

$$p_\omega(☽)_i < p_\omega(☽)_m < p_\omega(☽)_f \,,$$

with p_ω the component of the parallax in the inclined orbit. In turn, we know that the variation of the parallax is greater as the altitude increases. Hence,

$$0 > \Delta p_\omega(☽)_{im} > \Delta p_\omega(☽)_{em} \,.$$

If we take increments corresponding to the phase of immersion and emersion, we find

$$\Delta\omega(☽')_{em} = \Delta\omega(☽)_{em} - \Delta p_\omega(☽)_{em}$$
$$\Delta\omega(☽')_{im} = \Delta\omega(☽)_{im} - \Delta p_\omega(☽)_{im} \,.$$

And if we take the difference between the intervals of emersion and immersion, we find

$$\Delta\omega(☽')_{em} - \Delta\omega(☽')_{im} = (\Delta\omega(☽)_{em} - \Delta\omega(☽)_{im}) - (\Delta p_\omega(☽)_{em} - \Delta p_\omega(☽)_{im}) \,.$$

But since

$$0 > \Delta p_{\omega}(☽)_{\text{im}} > \Delta p_{\omega}(☽)_{\text{em}}$$

and consequently

$$\Delta p_{\omega}(☽)_{\text{em}} - \Delta p_{\omega}(☽)_{\text{im}} < 0 \ ,$$

we find that

$$(\Delta\omega(☽')_{\text{em}} - \Delta\omega(☽')_{\text{im}}) > (\Delta\omega(☽)_{\text{em}} - \Delta\omega(☽)_{\text{im}}) \ .$$

However, since the apparent intervals are equal, that is

$$\Delta\omega(☽')_{\text{em}} = \Delta\omega(☽')_{\text{im}} \ ,$$

then

$$0 > (\Delta\omega(☽)_{\text{em}} - \Delta\omega(☽)_{\text{im}}) \ .$$

Or similarly, the true interval in the phase of immersion is greater than in the phase of emersion. That is,

$$\Delta\omega(☽)_{\text{im}} > \Delta\omega(☽)_{\text{em}} \qquad (37).$$

Consequently, the duration of the phase of immersion is greater than that of emersion, since

$$\Delta t_{\text{in}} = \Delta\omega(☽)_{\text{im}} / w(☽) > \Delta\omega(☽)_{\text{em}} / w(☽) = \Delta t_{\text{em}} \qquad (38).$$

Thus, when the solar eclipse takes place in the second quadrant, the duration of the phase of immersion, which, according to Ptolemy, is the closer to the meridian, is greater than the duration of the phase of emersion.

Graphically, we know that, in general, whenever

$$\beta < \beta' < 180^{\circ} - \beta$$

is true, the apparent interval is smaller than the true one, so that the apparent motion is also smaller than the true one.

Given two equal apparent intervals, since

$$\Delta\omega(☽) = \Delta\omega(☽') \; {}^{\sin \beta'}/_{\sin \beta} \; ,$$

the true interval —$\Delta\omega(☽)$— relative to an apparent interval whose angle β' tends to 90º ($\beta' \rightarrow 90º$) is greater than the true interval —$\Delta\omega(☽)$— relative to an apparent interval whose angle β' tends to 0º or 180º ($\beta' \rightarrow 0º$ or $\beta' \rightarrow 180º$). That is,

$$\Delta\omega(☽)|_{\sin \beta' \rightarrow 1} > \Delta\omega(☽)|_{\sin \beta' \rightarrow 0} \, .$$

Hence, considering Figure 36, where we deem that the phase of emersion is closer to the meridian —in fact to the mid-heaven of the ascendant—, we find that the interval whose $\beta' \rightarrow 90º$ corresponds to the phase of emersion, whereas the interval whose $\beta' \rightarrow 0º$ corresponds to the phase of immersion. Thus,

$$\Delta\omega(☽)_{em} = \Delta\omega(☽)|_{\sin \beta' \rightarrow 1} > \Delta\omega(☽)|_{\sin \beta' \rightarrow 0} = \Delta\omega(☽)_{im} \qquad (39).$$

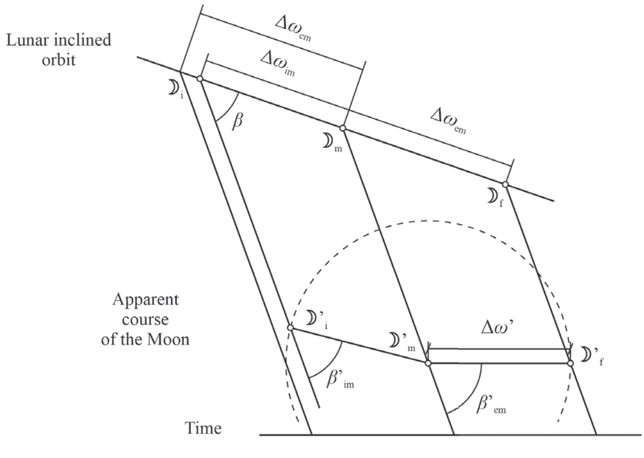


Figure 36. True intervals from apparent equal intervals in the first quadrant.

Consequently, the duration of the phase of emersion is greater than that of immersion; and, since the eclipse takes place in the first quadrant, the duration of the phase closer to the meridian is greater.

In short, all of the above confirms the hypothesis that Ptolemy deems that the intervals obtained with the tables, and corrected with the additional increment of the Sun, should be considered as apparent intervals. Likewise, it also seems to confirm that the motion «*towards the rear*» refers to the lunar motion in longitude or in its inclined orbit.

3. Jābir b. Aflaḥ on solar eclipses

Once we have studied Ptolemy's method to obtain the magnitude and phases of solar eclipses, by expanding Ptolemy's laconic exposition of the topic —particularly, his understanding of the effect of the lunar parallax—, we will address Jābir b. Aflaḥ's method on the topic.

In his Introduction to the *al-Kitāb fī l-Hay'a*, Jābir b. Aflaḥ pointed out a series of reasons that led him to write this work, including his intention to expand on some topics that Ptolemy had overly summarized, and to correct some mistakes that he had found in the *Almagest*. Jābir b. Aflaḥ both expands and corrects Ptolemy's treatment of solar eclipses.

As Ptolemy does, Jābir b. Aflaḥ finds the magnitude and the duration of the phases of solar eclipses by expanding the method dealing with lunar eclipses. He, first, accounts for the effect of the parallax and, then, finds the magnitude and the phases of solar eclipses following the same procedure that he used for the computation of lunar eclipses, avoiding the use of tables.[31]

To understand Jābir b. Aflaḥ's criticisms of Ptolemy on the topic of solar eclipses, we first need to introduce Jābir b. Aflaḥ's own method on the topic.

Jābir b. Aflaḥ divides his exposition in different parts. Firstly, he presents a figure to support his description of the evolution of the solar and lunar parallax over time. Next, he aims to obtain the apparent conjunction from the true one on the basis of a geometric approximation in which he only deals with longitudes. Then, he explains his solution to find the apparent conjunction in order to obtain the apparent magnitude of solar eclipses using times and motions. He, then, examines the different duration of the phases of solar eclipses. And lastly, he presents his criticisms of Ptolemy on the topic.

3.1. Jābir b. Aflaḥ's method for the computation of Solar Eclipses

Preliminary knowledge

Unlike Ptolemy, Jābir b. Aflaḥ provides a figure (see Figure 37) to support his exposition.

31. See Bellver, «Jābir b. Aflaḥ on Lunar Eclipses».

Circle BZGE represents the horizon. Point A is the zenith of this horizon, and line ZAE its meridian. Arcs BDG and TDK represent two possible dispositions of the ecliptic above the horizon at two specific times. Arc BDG represents a disposition of the ecliptic in which the mid-heaven of the ascendant —in this case point W— is to the east of the meridian —line ZAE—, whereas arc TDK represents a disposition of the ecliptic in which the mid-heaven of the ascendant —in this case point H— is to the west of the meridian —line ZAE—. Points G or K are the ascendant in the true conjunction for both dispositions. Arcs AW and AH of great circle pass through the poles of arcs BDG and TDK —the visible arcs of the ecliptic— and the zenith. Thus, points H and W divide the visible part of the ecliptic into two halves or quadrants. The remaining points on the figure are needed to present Jābir b. Aflaḥ's procedure for the computation of solar eclipses.

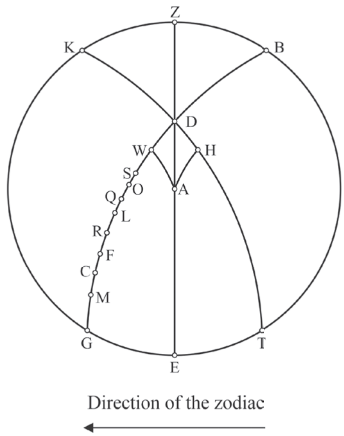


Figure 37. MS Ea 66r.

Jābir b. Aflaḥ then provides a short introduction in which he accounts for the effect of the parallax depending on the location of the true conjunction.

He divides the visible section of the ecliptic into two quadrants: the first one defined by the ascendant and the mid-heaven of the ascendant —either arcs GW or HK—; and the second one defined by the mid-heaven of the ascendant and the descendant —either arcs WB or HT—.

If the conjunction takes place in the first quadrant, the solar and lunar parallaxes in longitude take place along the direction of the zodiac; in turn, if the conjunction takes place in the second quadrant, the solar and lunar parallaxes in longitude take place in the opposite direction of the zodiac, as shown by Figure 38.

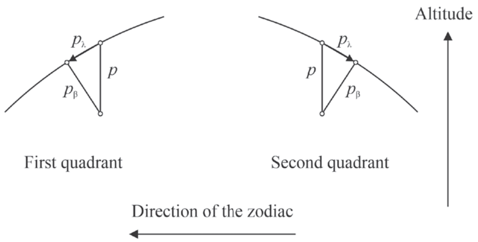


Figure 38. Parallax in longitude according to the direction of the zodiac.

Next, he points out that the parallaxes decrease with altitude. For instance, when the Moon reaches the mid-heaven of the ascendant, points W or H, the parallax in longitude is zero and its true position agrees with the apparent one. When the Moon traverses the mid-heaven of the ascendant and its altitude begins to decrease, its parallax increases, and its parallax in longitude lays along the direction of the zodiac.

He, then, points out that, if in the true conjunction the parallax in longitude lays along the direction of the zodiac, the apparent conjunction takes place before the time of the true conjunction; whereas, if the parallax in longitude lays in the opposite direction of the zodiac, the apparent conjunction takes place after the time of the true conjunction. This is so because of the time correction needed to account for the effect of the parallax, as shown in Figure 39.

If the apparent conjunction, Jābir b. Aflaḥ continues, takes place before the time of the true conjunction, as in the first quadrant, the parallax in longitude in the apparent conjunction is greater than in the true conjunction. Nevertheless, he does not point out, as this may seem obvious, that the motion of the celestial sphere from east to west in the horizon is greater than the solar and lunar motions in longitude. Likewise, if the apparent conjunction takes place after the time of the true conjunction, as in the second quadrant, the parallax in longitude in the apparent conjunction is greater than in the true one, because of the motion of the celestial sphere from east to west, as in the previous case.

He, then, concludes that the parallax in longitude in the apparent conjunction is always greater than in the true conjunction. That is,

$$p_\lambda\big|_{\mathrm{ca}} > p_\lambda\big|_{\mathrm{cv}} \qquad (40).$$

Next, Jābir b. Aflaḥ aims to obtain the longitude of the apparent conjunction from the true one.

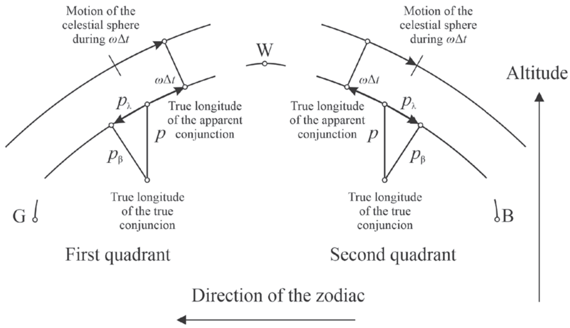


Figure 39. True and apparent conjunctions in both quadrants.

Method to find the apparent conjunction from the true one

Jābir b. Aflaḥ's approach to the topic differs from Ptolemy's on some points, although, broadly speaking, he follows the same steps. The main difference is that Jābir b. Aflaḥ provides a figure that clarifies Ptolemy's method, since Jābir b. Aflaḥ adds a number of points aiding in the understanding of Ptolemy's method. In addition, the above introduction by Jābir b. Aflaḥ helps the reader to understand the evolution of solar eclipses in the sky, and provides him with a basis to present his criticisms of Ptolemy.

After his short introduction, Jābir b. Aflaḥ focuses on finding the apparent conjunction from the true one, as a preliminary step to find the magnitude of solar eclipses. However, in his first approach to the topic, Jābir b. Aflaḥ presents his method without making any reference to the evolution of the eclipse over time.

The method he presents is independent from the quadrant in which the true conjunction takes place. He only needs to know that the parallax in longitude in the apparent conjunction is greater than in the true one. However, in the provided figure (see Figure 37), the conjunction takes place in the first quadrant. Thus, we will show Jābir b. Aflaḥ's method if the conjunction takes place in the first quadrant.

Before mentioning what Jābir b. Aflaḥ says, it is worth pointing out what he does not. For Ptolemy, the first step to account for the effect of the parallax is to obtain the position of the true conjunction in the horizon of the observer. To do so, he sought the difference in equinoctial hours between the true conjunction and the meridian of Alexandria to find the difference in longitude between the meridian of Alexandria and that of the observer in equinoctial hours. He, then, found the difference in equinoctial hours between the true conjunction and the meridian of the observer.

Jābir b. Aflaḥ does not mention this point, although this is a necessary step to compute a solar eclipse. In addition, he only uses parallaxes in longitude without pointing out how to find them. As in previous occasions, Jābir b. Aflaḥ's interest is purely theoretical; and does not take into account any practical application.

To find the longitude of the apparent conjunction, Jābir b. Aflaḥ needs the longitude of the true Moon whose apparent longitude is equal to the apparent longitude of the Sun in the true conjunction. Firstly, he mentions a number of points needed to find the time of the true conjunction, as shown in Figure 40.

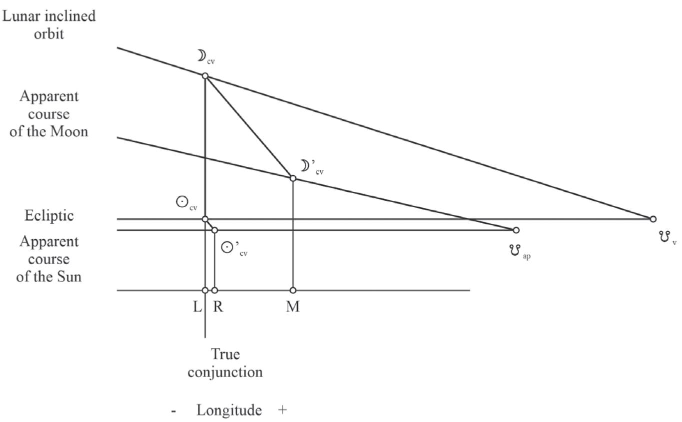


Figure 40. True conjunction.

In Figure 40, point L is the longitude of the true conjunction, M the longitude of the apparent Moon in the true conjunction, and R the longitude of the apparent Sun in the true conjunction. That is,

$$\lambda(☽)_{cv} = \lambda(\odot)_{cv} = L$$
$$\lambda(☽')_{cv} = M$$
$$\lambda(\odot')_{cv} = R\ .$$

Consequently, the solar and lunar parallaxes in the true conjunction are

$$p_{\lambda}(☽)_{cv} = LM$$
$$p_{\lambda}(\odot)_{cv} = LR\ .$$

Jābir b. Aflaḥ seeks to find the point of the ecliptic that corresponds to the true position of the Moon when its apparent position is point R.

Firstly, he finds the difference between the lunar and solar parallaxes in the true conjunction, $\Delta p_{\lambda 1}$. That is,

$$\Delta p_{\lambda 1} = p_{\lambda}(☽)_{cv} - p_{\lambda}(\odot)_{cv} \qquad (41).$$

And according to the points in Jābir b. Aflaḥ's figure,

$$\Delta p_{\lambda 1} = \mathrm{LM} - \mathrm{LR} = \mathrm{RM} .$$

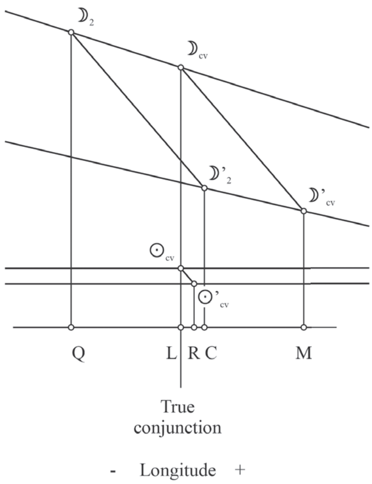


Figure 41. Finding the apparent conjunction.

Once Jābir b. Aflaḥ knows $\Delta p_{\lambda 1}$ = RM, he seeks to compensate the effect of the parallax in longitude in the true conjunction by finding a point in the ecliptic in the opposite direction to the parallax in longitude at a longitude $\Delta p_{\lambda 1}$ = RM from that of the true conjunction. He, thus, seeks point Q (see Figure 41), so that

$$\mathrm{RM} = \mathrm{QL} = \Delta p_{\lambda 1} .$$

And, consequently,

$$\mathrm{QR} = \mathrm{LM} = p_{\lambda}(☽)_{cv} \text{ , since } \mathrm{QR} = \mathrm{QL} + \mathrm{LR} = \mathrm{RM} + \mathrm{LR} = \mathrm{LM} = p_{\lambda}(☽)_{cv} .$$

In our notation, point Q refers to $\lambda(☽_2)$. Once Jābir b. Aflaḥ has obtained this point, he deems a second situation, in which the Moon's longitude is point Q, that is $\lambda(☽_2)$, to examine the parallax effect. If the lunar parallax in longitude on point Q —he points out— is equal to its parallax on point L, the apparent longitude of the Moon is point R, and thus point Q is the one being sought. That is

$$\because\ p_\lambda(☽)_2 = p_\lambda(\mathrm{Q}) = \mathrm{LM} = p_\lambda(☽)_{\mathrm{cv}}$$

$$\therefore\ \lambda(☽'_2) = \mathrm{R}\ .$$

But, because of the epiparallactic effect of point Q relative to point L, $p_\lambda(☽)_2 = p_\lambda(\mathrm{Q}) \neq \mathrm{LM} = p_\lambda(☽)_{\mathrm{cv}}$. Consequently, $\lambda(☽'_2) \neq \mathrm{R}$.

Hence, the parallax on point Q —he continues— is greater than the parallax on point L. Jābir b. Aflaḥ, thus, deems point C as the longitude of the apparent Moon in this second situation. That is

$$\lambda(☽'_2) = \mathrm{C}$$

$$p_\lambda(☽)_2 = p_\lambda(\mathrm{Q}) = \mathrm{QC}\ .$$

He then finds a second increment aiming to account for the effect of the epiparallax between point Q and point L. The increment to account for the epiparallax is RC.

We can obtain RC from QC and QR as

$$\mathrm{RC} = \mathrm{QC} - \mathrm{QR}\ .$$

But we know that QC is the lunar parallax in longitude in this second situation and that QR is the lunar parallax in longitude in the true conjunction. Thus,

$$\mathrm{RC} = \mathrm{QC} - \mathrm{QR} = p_\lambda(☽)_2 - p_\lambda(☽)_{\mathrm{cv}}\ , \qquad \text{since QR} = \mathrm{LM} = p_\lambda(☽)_{\mathrm{cv}}\ .$$

We have referred to RC as $\Delta p_{\lambda 2}$. However, we have seen that Ptolemy deems $\Delta p_{\lambda 2}$ as

$$\Delta p_{\lambda 2} = [p_\lambda(☽)_2 - p_\lambda(\odot)_2] - [p_\lambda(☽)_{\mathrm{cv}} - p_\lambda(\odot)_{\mathrm{cv}}] = e_p(☽)|^2_{\mathrm{cv}} - e_p(\odot)|^2_{\mathrm{cv}} \qquad (42),$$

with

$$e_p(☽)|^2_{\mathrm{cv}} = p_\lambda(☽)_2 - p_\lambda(☽)_{\mathrm{cv}} \qquad \text{and} \qquad e_p(\odot)|^2_{\mathrm{cv}} = p_\lambda(\odot)_2 - p_\lambda(\odot)_{\mathrm{cv}}\ .$$

Thus, Jābir b. Aflaḥ does not take into account the effect of the solar epiparallax during the interval defined by this second situation and the true conjunction, $e_p(\odot)|_{cv}^{2}$.

Once Jābir b. Aflaḥ has obtained $\Delta p_{\lambda 2}$, he needs to account for the effect of the epiparallax, as Ptolemy already did. He, thus, considers a third situation in which the Moon is at a longitude $\Delta p_{\lambda 2} = RC$ from $\lambda(☽_2) = Q$ in the opposite direction to that of the parallax. To do so, he seeks point O, see Figure 42, so that

$$OQ = RC = \Delta p_{\lambda 2} .$$

And consequently

$$OR = QC = p_\lambda(☽)_2 \qquad \text{since } OR = OQ + QL + LR = QL + LR + RC$$
$$= QC = p_\lambda(☽)_2 .$$

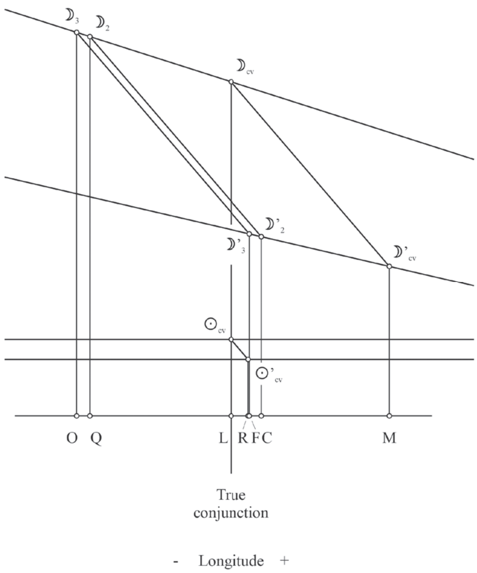


Figure 42. Finding the apparent conjunction.

Next, he finds the parallax in this new situation. If the parallax in longitude of the Moon on point O —he points out— is equal to its parallax on point Q, the apparent longitude of the Moon is point R, and thus point O would be the one being sought. That is

$$\because p_\lambda(☽)_3 = p_\lambda(\mathrm{O}) = \mathrm{QC} = p_\lambda(\mathrm{Q}) = p_\lambda(☽)_2$$
$$\therefore \lambda(☽'_3) = \mathrm{R}\ .$$

But since the effect of the epiparallax of point O relative to point Q is $p_\lambda(☽)_3 = p_\lambda(\mathrm{O}) \neq \mathrm{QC} = p_\lambda(☽)_2$, consequently, $\lambda(☽'_3) \neq \mathrm{R}$.

The parallax on point Q —he adds— is greater than the parallax on point L. Thus, the point obtained is not R, but F, namely the longitude of the apparent Moon in this third situation. That is

$$\lambda(☽'_3) = \mathrm{F}$$
$$p_\lambda(☽)_3 = p_\lambda(\mathrm{O}) = \mathrm{OF}\ .$$

Since the apparent longitude of the Moon when its true longitude is O is not R, but F, Jābir b. Aflaḥ ops for a new approximation. Jābir's procedure is close to the one followed by Ptolemy. According to Ptolemy,

> we take the longitudinal parallax of this by itself, plus an additional amount (if it is significant) which is the same fraction of the latter as the latter is of the original [longitudinal] parallax.[32]

That is, the total epiparallax, e_p, is

$$e_p = \Delta p_{\lambda 2} + l \quad (43)$$

with l

$$l = \Delta p_{\lambda 3} = \Delta p_{\lambda 2}{}^2 / \Delta p_{\lambda 1} \quad (44).$$

In turn, Jābir b. Aflaḥ points out that (see Figure 45):

فليكن اختلاف منظره في نقطة عين هو قوس عين فاء فإن أضفنا إلى قوس ر ف[33] الجزء منها إن كان محسوسا مثل جزءها من قوس ر ص[34] وحملنا ذلك على نقطة عين كأنّه قوس سين عين كانت على التقريب نقطة سين هي النقطة المطلوبة.

32. *Almagest* VI.10 (see Toomer p. 311).
33. MSs Ea and Eb give فاء صاد.
34. MSs Ea and Eb give فاء ميم.

And in translation:

> Be its parallax on point O arc OF. If we add to arc RF [or FC][35] a section of itself (*al-juz' minhā*), if it is significant, in the same proportion of itself to arc RC [or FM],[36] and we add this value to point O, as if [resulting in] arc SO, point S is approximately the point we are looking for.

MSs Ea and Eb differ from MS B on the notation of relevant points. The transmission of the two Escorial manuscripts—MSs Ea and Eb— gives arcs FC and FM, whereas MS B gives, instead, arcs RF and RC. We will, firstly, study the transmission of the Escorial manuscripts, and then the transmission by the Berlin manuscript. We will, then, check which is the correct one.

We will refer to the longitude we are seeking with letter *l*. Jābir b. Aflaḥ, as in the transmission of the Escorial manuscripts, points out that this longitude is «a section of itself [that is, of FC], if it is significant, in the same proportion of itself [that is of FC] to arc FM». That is,

$$l / \mathrm{FC} = \mathrm{FC} / \mathrm{FM}$$

so that *l* is

$$l = \mathrm{FC}^2 / \mathrm{FM} .$$

In Figure 43, we show the graphical resolution of *l*. It is surprising that the transmission of the Escorial manuscripts takes into account ratio FC / FM to find *l*, since FM includes the solar parallax whereas FC does not include the effect of the solar epiparallax.

But longitude *l* should be added to FC. The obtained arc, after adding *l* to FC, should, then, be added, Jābir b. Aflaḥ keeps on, to point O—logically, in the opposite direction to that of the parallax, although Jābir does not point this out—, resulting in arc SO. That is,

$$\mathrm{SO} = \mathrm{FC} + \mathrm{FC}^2/\mathrm{FM} .$$

35. MS B gives RF; MSs Ea, Eb give arc FC.

36. MS B gives RC; MSs Ea, Eb give arc FM.

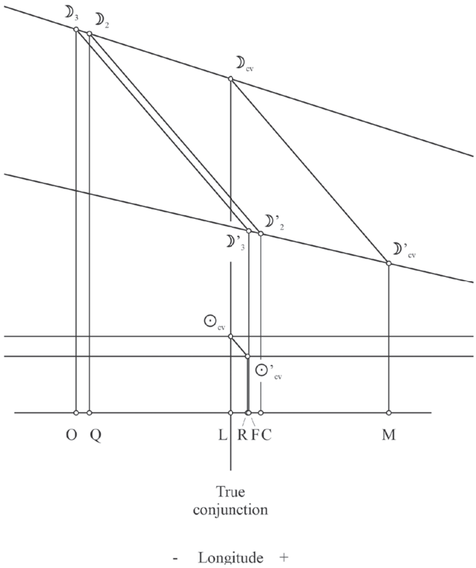


Figure 43. Graphical resolution of *l*.[37]

He, thus, finds point S in the fourth situation. In this new situation, we should consider which would be the apparent longitude of the Moon if it would be placed on longitude S. Thus, SO corresponds to the third increment in longitude, which we will refer to as

$$\mathrm{SO} = \Delta p_{\lambda_3}{}^{(\text{J-Es})} \ ,$$

in which the superscript $^{(\text{J-Es})}$ refers to the increment obtained by Jābir according to the transmission of the Escorial manuscripts.

Lastly, Jābir remarks that point S is the one he was seeking; that is, the true longitude of the Moon whose apparent longitude agrees with the apparent longitude of the Sun in the true conjunction —point R—, that is

$$\lambda(☽) = \mathrm{S} \qquad \text{so that} \qquad \lambda(☽') \cong \lambda(\odot')_{cv} = \mathrm{R} \ .$$

37. To facilitate the graphical representation, the scale of the abscissae does not agree with that of the ordinates.

However, longitudes FC and FM approximately correspond to the following increments:

$$FC \cong RC = \Delta p_{\lambda 2}$$
$$FM \cong RM = \Delta p_{\lambda 1}$$

so that the obtained longitude SO corresponds approximately to

$$SO = \Delta p_{\lambda_3}{}^{(J\text{-}Es)} \cong \Delta p_{\lambda}+_2 \Delta p_{\lambda 2}{}^2 / \Delta p_{\lambda 1} \qquad (45).$$

In turn, the third increment obtained according to Ptolemy's method—which we refer to as $\Delta p_{\lambda_3}{}^{(P)}$— does not correspond to the one obtained by Jābir b. Aflaḥ, since

$$SO = \Delta p_{\lambda_3}{}^{(J\text{-}Es)} \cong \Delta p_{\lambda}+_2 \Delta p_{\lambda 2}{}^2 / \Delta p_{\lambda 1} = \Delta p_{\lambda 2} + \Delta p_{\lambda_3}{}^{(P)} \qquad (46).$$

Consequently, both increments differ in $\Delta p_{\lambda 2}$. To find out if the obtained increment makes sense, it should be approximately equal to RF. However, since

$$SO = \Delta p_{\lambda_3}{}^{(J\text{-}Es)} = FC + FC^2/FM \cong RC + RC^2/RM >> RF ,$$

the suggested increment $SO = \Delta p_{\lambda_3}{}^{(J\text{-}Es)}$ is far greater than the one we are looking for, that is RF. Thus, the increment $\Delta p_{\lambda_3}{}^{(J\text{-}Es)}$ in the Escorial transmission does not seem to make sense, as we shall see below.

In turn, in the transmission of the Berlin manuscript, this new increment — which we refer to as $\Delta p_{\lambda+}{}^{(J\text{-}B)}$, that is the «additional increment in longitude obtained by Jābir in the transmission of the Berlin manuscript»— is

$$SO = \Delta p_{\lambda+}{}^{(J\text{-}B)} = RF + RF^2/RC .$$

However, what does RF refer to? In Figure 42, we see that RF corresponds to the epiparallactic increment between the third situation and the second one; that is, between the lunar parallaxes of longitudes O and Q, since

$$RF = OF - OR .$$

We know that

$$OF = p_\lambda(☽)_3$$
$$OR = QC = p_\lambda(☽)_2$$

since $OR = OQ + QL + LR = QL + LR + RC = QC = p_\lambda(☽)_2$. Hence, RF is

$$RF = p_\lambda(☽)_3 - p_\lambda(☽)_2 \ .$$

As with $\Delta p_{\lambda 2}$, here the procedure in the transmission of the Berlin manuscript does not take into account the solar epiparallax either. Since Jābir b. Aflaḥ adds RF to point O —and, although he does not point this out, as before, the sign of the addition should be in the opposite direction to that of the parallax—, he is, in fact, taking a fourth situation into account, which entails a new epiparallactic increment relative to point O. Thus, RF stands as increment Δp_{λ_3} in Ptolemy's procedure. We will refer to RF as $\Delta p_{\lambda_3}{}^{(J\text{-}B)}$, that is «the third increment obtained by Jābir in the transmission of the Berlin manuscript». The difference between Jābir and Ptolemy's methods regarding this increment is that Ptolemy finds Δp_{λ_3} through an interpolation, whereas the Berlin manuscript applies the procedure used to obtain $\Delta p_{\lambda 2}$, now, to obtain Δp_{λ_3}.

Consequently, let be S' a point in longitude —Jābir b. Aflaḥ does not make reference to this point— at distance $\Delta p_{\lambda_3}{}^{(J\text{-}B)} = RF$ from O, and in the direction opposite to the lunar parallax. This point yields a new situation, the fourth one. Once the Moon is placed on this point, Jābir b. Aflaḥ deems whether the apparent longitude of the Moon is close to R —which corresponds to the apparent longitude of the Sun in the true conjunction—. Once he realizes that it does not match R, he considers a new increment, which we will refer to as $\Delta p_{\lambda_4}{}^{(J\text{-}B)}$. This time, Jābir b. Aflaḥ, as in the version in the Berlin manuscript, finds a new increment through an interpolation. Jābir b. Aflaḥ suggests this new increment to be

$$\Delta p_{\lambda_4}{}^{(J\text{-}B)} = RF^2/RC \qquad (47).$$

We know that

$$RF = \Delta p_{\lambda_3}{}^{(J\text{-}B)}$$
$$RC = \Delta p_{\lambda 2} \ .$$

Thus, the increment $\Delta p_{\lambda 4}{}^{(J\text{-}B)} = RF^2/RC$ corresponds to

$$\Delta p_{\lambda 4}{}^{(J\text{-}B)} = \Delta p_{\lambda 3}{}^{(J\text{-}B)^2} / \Delta p_{\lambda 2} \qquad (48),$$

which clearly follows Ptolemy's interpolation obtained for $\Delta p_{\lambda 3}{}^{(P)} = \Delta p_{\lambda 2}{}^2 / \Delta p_{\lambda 1}$. Figure 44 provides a graphical explanation of this interpolation.

Nevertheless, is this new approximation really needed —that is, $\Delta p_{\lambda 4}{}^{(J\text{-}B)}$— if compared with the one, $\Delta p_{\lambda 3}{}^{(P)}$, given by Ptolemy, considering the accepted errors in the astronomy of the time? The answer should be negative. The reason behind this new increment introduced by Jābir b. Aflaḥ seems to owe to a feeling of mathematical scrupulousness and elegance. Jābir b. Aflaḥ should have been aware that, by dealing with longitudes and not time intervals, the geometrical approximation entailed to disregard the solar epiparallax, and this from the $\Delta p_{\lambda 2}$ situation. Thus, making use of Ptolemy's interpolation in $\Delta p_{\lambda 3}$ entails to link an exclusively lunar epiparallactic increment —as in the case of $\Delta p_{\lambda 2}$— with a lunisolar parallactic increment —as in the case of $\Delta p_{\lambda 1}$—. This time, with $\Delta p_{\lambda 4}{}^{(J\text{-}B)}$, Jābir links two epiparallactic increments, and both exclusively lunar —as in the case of $\Delta p_{\lambda 2}$ and $\Delta p_{\lambda 3}{}^{(J\text{-}B)}$—. Such an interpolation seems to meet Jābir b. Aflaḥ's standards of scrupulousness.

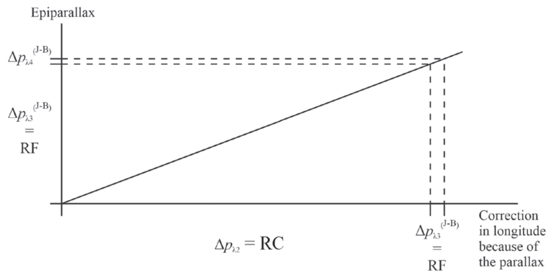


Figure 44. Graphical resolution of $\Delta p_{\lambda 4}{}^{(J\text{-}B)}$.[38]

Thus, after he obtained $\Delta p_{\lambda +}{}^{(J\text{-}B)}$ as

$$\Delta p_{\lambda +}{}^{(J\text{-}B)} = \Delta p_{\lambda 3}{}^{(J\text{-}B)} + \Delta p_{\lambda 4}{}^{(J\text{-}B)} = \Delta p_{\lambda 3}{}^{(J\text{-}B)} + \Delta p_{\lambda 3}{}^{(J\text{-}B)^2} / \Delta p_{\lambda 2} =$$

38. To facilitate the graphical representation, the scale of the abscissae does not agree with that of the ordinates.

$$= RF + RF^2/RC = SO \qquad (49),$$

Jābir b. Aflaḥ can obtain a new situation —the fifth one— which corresponds to point S. Jābir b. Aflaḥ regards this point as the sought one, since it corresponds to the true longitude of the Moon whose apparent longitude, approximately, matches the apparent longitude of the Sun in the true conjunction —point R—, that is

$$\lambda(☽_5) = S \qquad \text{so that} \qquad \lambda(☽'_5) \cong \lambda(\odot')_{cv} = R \qquad (50).$$

Thus, the total parallax according to the transmission of the Berlin manuscript is

$$\Delta p_\lambda = \sum_i \Delta p_{\lambda i} \text{ with } i = 1..4 \qquad (51).$$

Figure 45 shows the result of Jābir. Aflaḥ's method according to the transmission of the Berlin manuscript.

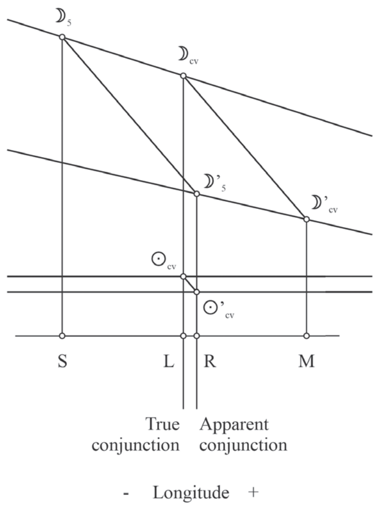


Figure 45. Jābir b. Aflaḥ's method to obtain the apparent conjunction.

We have pointed out above that Ptolemy bases his approximation to obtain the apparent longitude of the Moon in the apparent conjunction by adding increments in longitude corresponding, firstly, to the difference of parallaxes —$\Delta p_{\lambda 1}$—, secondly, to the epiparallactic difference —$\Delta p_{\lambda 2}$— between the Moon and the Sun, and, lastly, to an interpolation of the previous increments—$\Delta p_{\lambda 3}$—. The true longitude of the Moon, whose apparent longitude matches the apparent longitude of the Sun in the apparent conjunction, can be obtained through the indefinite iteration of the procedure used above to obtain the first two non-interpolated increments. Since Jābir b. Aflaḥ, as in the transmission of the Berlin manuscript, iterates the correction one more time than Ptolemy, his approximation is more accurate than the latter's. Likewise, we have already pointed out that the slope obtained through interpolation —in this case, $m = \Delta p_{\lambda 3}^{(J\text{-}B)} / \Delta p_{\lambda 2}$— is always greater than the one obtained by the iterative addition of successive epiparallactic increments. However, since the interpolation is applied to the increment $\Delta p_{\lambda 3}^{(J\text{-}B)}$ —since $\Delta p_{\lambda 4}^{(J\text{-}B)} = m \cdot \Delta p_{\lambda 3}^{(J\text{-}B)}$ with $m = \Delta p_{\lambda 3}^{(J\text{-}B)} / \Delta p_{\lambda 2}$—, the error introduced by the difference between m and the real slope is smaller than in Ptolemy's approximation. Thus, point S obtained as in the transmission of the Berlin manuscript is far more accurate than the one obtained by Ptolemy, and, no doubt, far more correct than Jābir b. Aflaḥ's method as in the transmission of the Escorial manuscripts.

An analysis based on Figure 42 helps us show the degrees of accuracy of the different methods to find point R. These methods include the one by Ptolemy and the ones in the two transmissions of the *al-Kitāb fī l-Hay'a*. Figure 46 illustrates the accuracy of the different methods.

In this Figure, be point J_{Es} the value obtained by Jābir b. Aflaḥ according to the transmission of the Escorial manuscripts. To facilitate the graphical representation, the scale in Figure 42 has been augmented ten times. Ptolemy's approximation with increment $\Delta p_{\lambda 2}$ alone is far more accurate than the one in the transmission of the Escorial manuscripts. In turn, be point Pt the result obtained by Ptolemy, and point J_B the result obtained by Jābir b. Aflaḥ in the transmission of the Berlin manuscript. To compare both accuracies in this second case, the scale in Figure 42 has been augmented five hundred times. As presumed above, the more accurate procedure is the one in the transmission of the Berlin manuscript, so that it can be presumed that Jābir b. Aflaḥ's actual method is the one in the transmission of the Berlin manuscript. It is more accurate and it agrees with Jābir b. Aflaḥ's scrupulousness.

However, Jābir b. Aflaḥ claims that the result obtained, point S, corresponds to the true longitude of the Moon in the apparent conjunction, since its apparent longitude agrees with point R, the apparent longitude of the Sun in the true conjunction. However, Ptolemy adds a further step by which he multiplies the total obtained parallax times 13/12 to find the longitude of the true Moon in the apparent

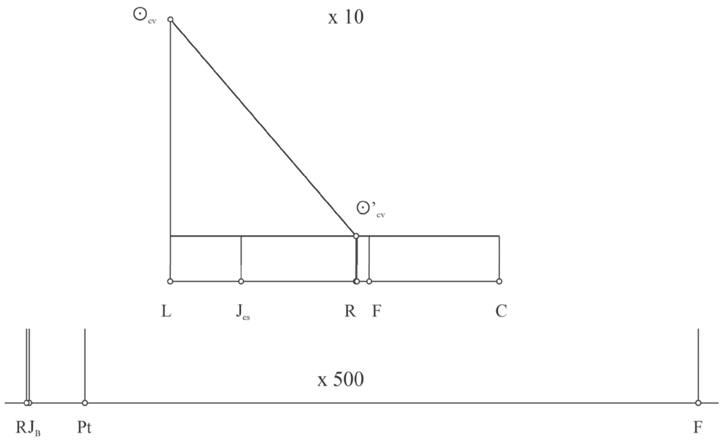


Figure 46. Graphical accuracy
of the different methods.

conjunction, since the solar motion during the time the Moon traverses a longitude equivalent to the total parallax should be taken into account.

Nevertheless, as pointed out above, in this preliminary stage, Jābir b. Aflaḥ presents his method without making any reference to time intervals, which he will introduce next.

Magnitudes of the solar eclipse

Jābir b. Aflaḥ follows the same steps as in Ptolemy's method. Firstly, he aims to compute the longitude, latitude and lunar anomaly in the apparent conjunction. Then, he computes the apparent latitude. And, lastly, he finds the magnitude of the solar eclipse following the same steps of the lunar eclipse.

Longitude, latitude and lunar anomaly in the apparent conjunction

In this second theoretical approximation to the problem, in which he does take into account time increments, Jābir b. Aflaḥ's method is the same as the one presented above, although the main difference, this time, is that he turns parallaxes and epiparallaxes into time increments. To do so, he divides these longitudes by the true motion of the Moon in the true conjunction. Depending on angle γ, the time increments obtained are added to or subtracted from the time of the true conjunction to find the time of the apparent conjunction. He then finds the longitude, latitude and lunar anomaly using the above time as an argument in tables. Let us see this method when time intervals are considered instead (see Figure 47).

a. First time increment (Δt_{I})

The steps to find the first-time increment, Δt_{I}, which corresponds to $\Delta p_{\lambda\mathrm{I}}$, are the following ones:

- Firstly, he finds the total lunar parallax in the true conjunction —$p(☽)_{cv}$— and subtracts the total solar parallax —$p(\odot)_{cv}$—. That is, $p(☽)_{cv} - p(\odot)_{cv}$.
- He, then, finds the lunar parallax in longitude —$p_{\lambda}(☽)_{cv}$—, that is, arc RM. Jābir b. Aflaḥ —or maybe a scribe— makes a mistake, since in the previous step he found the difference between the parallaxes of the Moon and the Sun in the true conjunction. That is, $\Delta p_{\mathrm{I}} = p(☽)_{cv} - p(\odot)_{cv}$. It is the component in longitude of this parallatic difference which, in fact, corresponds to arc RM. That is,

$$\mathrm{RM} = \Delta p_{\lambda\mathrm{I}} = p_{\lambda}(☽)_{cv} - p_{\lambda}(\odot)_{cv}\,.$$

- Next, he finds the first time increment —Δt_{I}— by dividing RM by the motion of the true Moon in the true conjunction —$w(☽)_{cv}$ —. That is,

$$\Delta t_{\mathrm{I}} = \Delta p_{\lambda\mathrm{I}} \,/\, w(☽)_{cv} = \mathrm{RM} \,/\, w(☽)_{cv}\,.$$

- Once he has obtained Δt_{I}, he adds or subtracts it to or from the time of the true conjunction according to angle γ' so that,
  - if the parallax in longitude takes place in the direction of the zodiacal signs across the ecliptic, he subtracts Δt_{I} from the time of the true conjunction; that is,

    $$t_2 = t_{cv} - \Delta t_{\mathrm{I}}\,;$$

  - and, if the parallax takes place in the opposite direction of the zodiacal signs across the ecliptic, he adds Δt_{I} to the time of the true conjunction; that is,

$$t_2 = t_{cv} + \Delta t_1 \, .$$

The difference with the procedure presented before is mainly that, instead of finding the time increment after obtaining the total parallax corrected with the solar motion in the interval and adding or subtracting it to or from the time of the apparent conjunction according to the orientation of the parallax, he does this step for each of the increments.

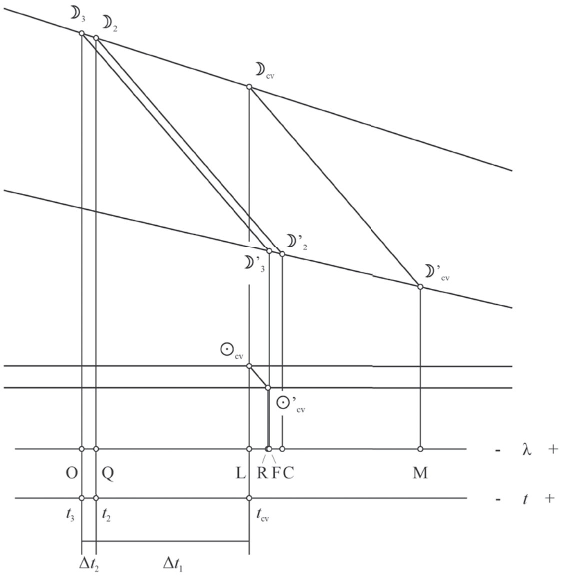


Figure 47. Correspondences between increments in longitude and time.

b. Second time increment (Δt_2)

The steps to find the second time increment Δt_2, which corresponds to $\Delta p_{\lambda 2}$, are the following ones:

- He, first, finds the lunar parallax in longitude at time t_2, which corresponds to arc QC. That is, $p_\lambda(☽)_2$ = QC.
- He, then, finds the parallactic difference in longitude between time t_2 and the time of the true conjunction. This difference corresponds to arc CR. That is,

$$\Delta p_{\lambda 2} = p_{\lambda}(☽)_2 - p_{\lambda}(☽)_1 = \mathrm{CR}.$$

- Next, he finds the time increment related to $\Delta p_{\lambda 2}$ —i.e., Δt_2— by dividing it by the true motion of the Moon. That is,

$$\Delta t_2 = \Delta p_{\lambda 2} / w(☽)_{cv} = \mathrm{CR} / w(☽)_{cv}.$$

Jābir b. Aflaḥ understands that the true motion of the Moon, although he does not point it out, is the true motion of the Moon in the true conjunction, since this is the one used to find the two subsequent increments.

- Once he has obtained Δt_2, he adds or subtracts it to or from time t_2 according to angle γ, although he presents this in a slightly different way. In this case, he makes reference to the distance of the position of the true conjunction relative to the degree of the ascendant at this specific time, so that,
  - if the parallax in longitude takes place in the direction of the zodiacal signs across the ecliptic —or if it takes place in the second quadrant, that is if the distance of the true conjunction relative to the degree of the ascendant is greater than 90°—, he subtracts Δt_2 from time t_2; that is,

    $$t_3 = t_2 - \Delta t_2 ;$$

  - and, if the parallax takes place in the opposite direction of the zodiacal signs across the ecliptic —or if it takes place in the first quadrant, that is if the distance of the true conjunction relative to the degree of the ascendant is smaller than 90°—, he adds Δt_2 to time t_2; that is,

    $$t_3 = t_2 + \Delta t_2 .$$

Even though, at a first glance, in this case Jābir's method does not seem to make sense, since he takes angle γ as the reference value to obtain the distance of the true conjunction relative to the degree of the ascendant at time, t_2, and not the longitude of the Moon at this time, it is always true that the Moon finds itself in the same quadrant during the interval defined by the true conjunction and the apparent one. Thus, all the time increments have the same sign. Hence, he can establish the sign of the time increments by using the angle γ of any longitude of the Moon between the true conjunction and the apparent conjunction at any time between the true conjunction and the apparent one.

c. Additional time increment (Δt_+)

Next, Jābir finds the additional time increment, Δt_+, which corresponds to the increment in longitude $\Delta p_{\lambda+}$, in our notation. This additional increment includes the increments $\Delta p_{\lambda 3}$ and $\Delta p_{\lambda 4}$, mentioned above. The procedure to find this additional increment is the following:

- He, first, finds the lunar parallax in longitude at time t_3, which corresponds to arc OF. That is, $p_\lambda(☽)_3 = \text{OF}$.
- He, then, finds the parallatic difference in longitude between time t_3 —with $p_\lambda(☽)_3 = \text{OF}$— and time t_2 —with $p_\lambda(☽)_2 = \text{OR}$—. This difference corresponds to arc RF. That is,

$$\Delta p_{\lambda 3} = p_\lambda(☽)_3 - p_\lambda(☽)_2 = \text{RF} .$$

- Next, he adds a longitude which, relative to RF, has the same ratio as RF relative to RC. In our notation, $\Delta p_{\lambda 4}$. That is,

$$\Delta p_{\lambda 4} = \Delta p_{\lambda 3}{}^2 / \Delta p_{\lambda 2} = \text{RF}^2 / \text{RC} .$$

And, thus, the additional longitude is

$$\Delta p_{\lambda+} = \Delta p_{\lambda 3} + \Delta p_{\lambda 4} = \Delta p_{\lambda 3} + \Delta p_{\lambda 3}{}^2 / \Delta p_{\lambda 2} = \text{RF} + \text{RF}^2 / \text{RC} \qquad (52).$$

In this case, all the manuscript versions agree and follow the method in the Berlin manuscript regarding the increments in longitude. This adds to the hypothesis that this is Jābir's original method, and not the one presented in the Escorial manuscripts.

- He, then, adds the increment in longitude obtained, $\Delta p_{\lambda+}$, to arc OR and finds arc SR.
- Point S is approximately the point of the position of the true Moon when its apparent position is point R. MS Ea gives point Z, which refers to the south in the Figure, instead of point R, making clear that the scribe did not understand the procedure.

d. Finding the time difference between the true and apparent conjunctions and the longitude, latitude and lunar anomaly in the apparent conjunction.

- Once he has obtained point S, he divides arc SL —the total parallax— by the true motion of the Moon in the true conjunction —$w(☽)_{cv}$— and finds the time difference between the true conjunction and the apparent one. That is,

$$\Delta t_{\text{ca-cv}} = \Delta\lambda(☽)_{\text{ca-cv}} / w(☽)_{\text{cv}} \qquad (53).$$

- He, then, adds or subtracts $\Delta t_{\text{ca-cv}}$ to or from the time of the true conjunction according to angle γ , so that,
  - if the parallax in longitude takes place in the direction of the zodiacal signs across the ecliptic, he subtracts $\Delta t_{\text{ca-cv}}$ from the time of the true conjunction; that is,

$$t_{\text{ca}} = t_{\text{cv}} - \Delta t_{\text{ca-cv}} \ ;$$

  - and, if the parallax takes place in the opposite direction of the zodiacal signs across the ecliptic, he adds $\Delta t_{\text{ca-cv}}$ to the time of the true conjunction; that is,

$$t_{\text{ca}} = t_{\text{cv}} + \Delta t_{\text{ca-cv}} \ .$$

- Thus, we find —according to Jābir b. Aflaḥ— the positions of the Moon in longitude, latitude and anomaly at this time, that is, at the time of the apparent conjunction.

At this point, it becomes completely clear that Jābir b. Aflaḥ deems point S as the true longitude of the Moon in the apparent conjunction. Hence, he does not take into account the additional motion of the Sun during the added time interval, as he does not multiply the time interval added because of the parallactic correction 13/12 times. This important mistake does not seem to have been introduced by a scribe, since all three Arabic manuscripts in Arabic script agree on this point, and both Jābir b. Aflaḥ's introductory section to his method and his actual method clearly state that point S is the true longitude of the Moon in the apparent conjunction. Additionally, Jābir b. Aflaḥ does not seem to regard Ptolemy's correction because of the additional motion of the Sun as a mistake. He neither mentions this correction in the criticisms he levels at Ptolemy on the latter's treatment of solar eclipses, nor when Jābir presents his own method. It is also unlikely that Jābir b. Aflaḥ would have missed this correction because of a hasty reading of the *Almagest*. The text is difficult and can only be understood after studying it very carefully. In addition, Jābir b. Aflaḥ's list of criticisms of Ptolemy's treatment of solar eclipses, that will be addressed below, shows his painstaking punctiliousness. The only plausible explanation seems to be that his copy of the *Almagest* would have a textual lacuna together with the fact that Jābir b. Aflaḥ should have missed this error in the manuscript he owned. It does call the attention that this mistake remained in the different versions of Jābir's *al-Kitāb fī l-Hay'a* across his different editions over the years.

Apparent latitude

Once Jābir b. Aflaḥ knows the degree of the Moon in longitude, latitude and anomaly in the apparent conjunction, he obtains the apparent latitude to find the magnitude of the solar eclipse following his method for the lunar eclipse, which, as in the case of the lunar eclipse, completely differs from Ptolemy's.

To illustrate Jābir's method, let us consider Figure 48, where Jābir's incorrect apparent conjunction is referred to with subscript ca*. Jābir's method is as follows:

- Firstly, he states that the true latitude of the Moon —$\beta(☽)_{ca}$— and its total parallax —$p(☽)_{ca}$— in the apparent conjunction is known.
- He, then, points out that the parallax in latitude of the Sun should be subtracted to find the parallax in latitude. The result seems to refer to the parallactic difference in latitude between the Moon and the Sun in the apparent conjunction —Δp_{β}—. Thus, the solar parallax in latitude should not be subtracted from the total lunar parallax in the apparent conjunction —the immediate antecedent in the text—, but from the lunar parallax in latitude in the apparent conjunction. That is,

$$\Delta p_{\beta} = p_{\beta}(☽)_{ca} - p_{\beta}(\odot)_{ca} \ .$$

- Hence, he finds the apparent latitude of the Moon in the apparent conjunction. That is,

$$\beta'(☽')_{ca} = \beta(☽)_{ca} - \Delta p_{\beta} \ .$$

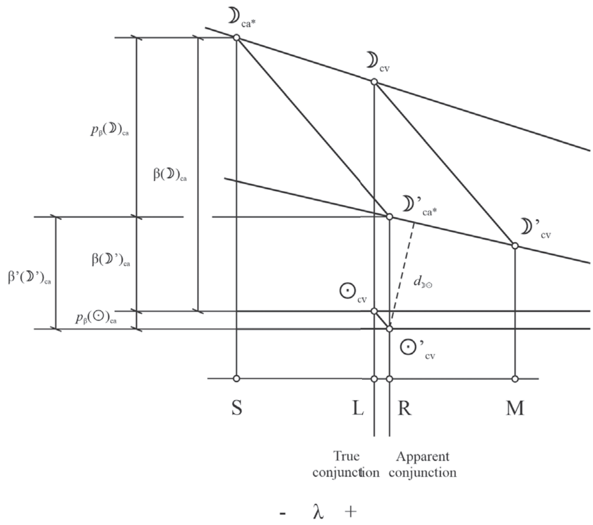


Figure 48. Apparent latitude of the apparent Moon
in the apparent conjunction.

Magnitude of the solar eclipse

Once he has accounted for the effect of the solar and lunar parallax, and, thus, he has found the apparent latitude in the apparent conjunction, Jābir b. Aflaḥ seeks to find the magnitude of solar eclipses. Since the method is equivalent to that for lunar eclipses, he summarizes the needed steps as follows:

- Firstly, he finds the distance between the apparent centers of the Sun and the Moon ($d_{☽\odot}$) at the eclipse mid-time (Figure 48).
  Next, he obtains the lunar radius ($r_{☽}$) from the lunar anomaly in its epicycle in the apparent conjunction.
- He, then, adds the lunar and solar radii. That is, $r_{☽} + r_{\odot}$.
- And, lastly, he obtains the difference between the previous sum of the lunar and solar radii —that is, $r_{☽} + r_{\odot}$— and the distance between their centers at the eclipse mid-time —$d_{☽\odot}$— and finds the immersion at the eclipse mid-time (μ), i.e.,

$$\mu = r_{☽} + r_{\odot} - d_{☽\odot} \qquad (54).$$

In this case, the immersion formula at the eclipse mid-time is correct, contrary to this formula in Jābir b. Aflaḥ's treatment of lunar eclipses, where he pointed out that the distance between the centers should be subtracted from the sum of the diameters of the Moon and, in the case of lunar eclipses, the Earth shadow cone.

Lastly, the magnitude of the eclipse, m, can be obtained from the immersion, μ, as

$$m = 12\, \mu / d_{☽} \quad (55),$$

where $d_{☽}$ is the lunar diameter.

Phases of the solar eclipse

Once Jābir b. Aflaḥ has obtained the value of the magnitude of the solar eclipse, he aims to study the duration of the phases of the solar eclipse. Contrary to the lunar eclipse, the phases of the solar eclipse are only two: immersion and emersion. Jābir b. Aflaḥ divides the topic in two different steps. Firstly, he finds the minutes of immersion of both phases without taking into account the effect of the parallax in their duration. Then, he studies the effect of the parallax in the duration of both phases.

The first step is similar to the solution of the minutes of immersion of the phases of the lunar eclipse, although in the latter case he considered four phases. The two phases of the solar eclipse correspond to the first and last phases of the lunar eclipse. Jābir b. Aflaḥ finds the duration of the phases of the solar eclipse as follows:

Firstly, he assumes that the Sun is motionless during the eclipse. Since the distance between the center of the Sun and the Moon at the eclipse mid-time and the sum of the radii of the Sun and the Moon are known, the arcs between the initial time of the eclipse and its middle time and between its middle time to its end time are known.

Given Figure 49,[39] BD is the value of the minutes of immersion, i.e.,

$$BD = [AB^2 - AD^2]^{1/2} = [(r_{☽} + r_{\odot})^2 - (r_{☽} + r_{\odot} - \mu)^2]^{1/2} =$$
$$= [\, \mu^2 - 2\mu\, (r_{☽} + r_{\odot})]^{1/2}\ .$$

39. See HAMA p. 136.

Once Jābir b. Aflaḥ has obtained the minutes of immersion, he, then, finds the course of the Moon with its apparent motion by adding a twelfth part of arc BD to it —that is, he multiplies arc BD 13/12 times—. He, thus, finds the arc relative to the course of the Moon with its apparent motion between the eclipse initial and middle times and between its middle and end times.

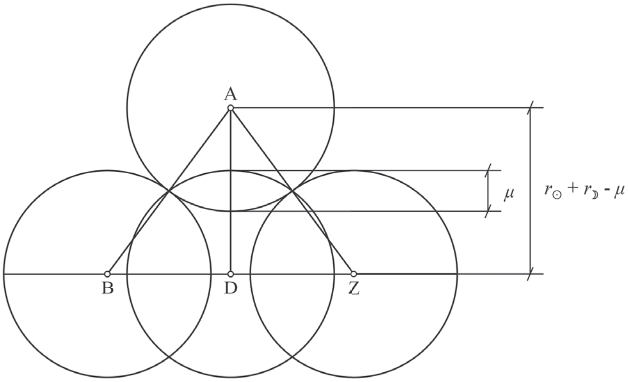


Figure 49. Minutes of immersion
in the solar eclipse.

However, Jābir b. Aflaḥ points out, following Ptolemy, that the parallax affects the duration of each of the eclipse phases differently. To account for this effect, Jābir b. Aflaḥ describes, first, how the parallax affects the apparent motion of the Moon and the duration of the phases in a broad way. Then, he describes this effect in detail. Thus, we will first study his broad approach. In the following steps, we will indicate the initial, middle and end times of the eclipse with subscripts i, m and f.

Firstly, he shows that the apparent motion in the phase of immersion is different from that of the phase of emersion:

> ∵ Parallax in longitude is different in the initial, middle and end times of the eclipse. That is,
>
> $$p_{\lambda}(☽)_{\mathrm{i}} \neq p_{\lambda}(☽)_{\mathrm{m}} \neq p_{\lambda}(☽)_{\mathrm{f}} \neq p_{\lambda}(☽)_{\mathrm{i}} .$$
>
> ∴ Apparent motion relative to the phase of immersion is different from the apparent motion relative to the phase of emersion. That is,
>
> $$w(☽')_{\mathrm{em}} \neq w(☽')_{\mathrm{im}} \qquad (56).$$

And, then, he deduces that the duration of the phase of immersion must be different from that of the emersion:

∵ The arcs that correspond to equal phases of immersion and emersion are equal. That is,

$$\Delta\omega(☽')_{em} = \Delta\omega(☽')_{im}$$

∵ Apparent motions relative to the phase of immersion and emersion are different.
∴ Durations of the phases of immersion and emersion are different.

$$\Delta t_{em} \neq \Delta t_{im} \qquad (57).$$

Having stated that both the apparent motion and the duration of each phase must be different from the true one, Jābir b. Aflaḥ aims to study how these differences occur, relying on a detailed proof in Figure 50, which we reproduce in what follows. In this Figure, points Z and H appear repeated twice. This is because Jābir b. Aflaḥ brings together in a single Figure the two situations that can occur in a solar eclipse: (i) when the parallax in longitude takes place along the direction of the zodiac across the ecliptic, that is, if the eclipse takes place in the first quadrant of the ecliptic; and (ii) when the parallax takes place in the opposite direction of the zodiac across the ecliptic, that is, if the eclipse takes place in the second quadrant of the ecliptic.

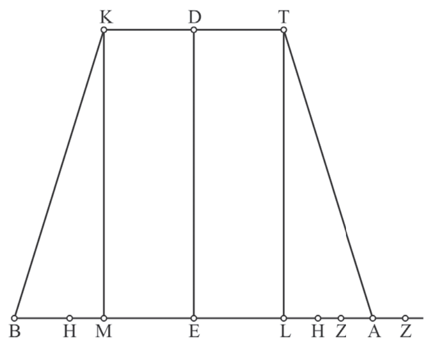


Figure 50. MS Ea 66v.

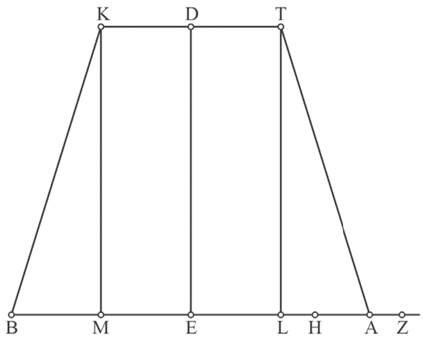


Figure 51. Solar and lunar positions for an eclipse in the first quadrant.

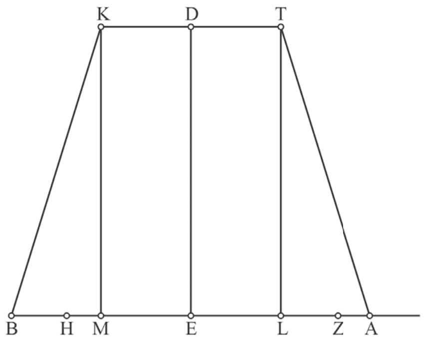


Figure 52. Solar and lunar positions for an eclipse in the second quadrant.

Firstly, he defines the apparent positions of the Sun and the Moon across the solar eclipse. Arc AB represents the lunar inclined orbit. In turn, he does not define arc TK, although it represents the apparent course of the Sun, since he defines points T, D and K as the argument in latitude —and we add apparent— of the center of the solar disk in the initial, middle and end times of the eclipse respectively —that is, $\omega(\odot')_i = T$, $\omega(\odot')_m = D$ and $\omega(\odot')_f = K$—. Points A, E and B, in turn, refer to the apparent positions of the center of the lunar disk at the initial, middle and end times of the eclipse, that is, $\omega(☽')_i = A$, $\omega(☽')_m = E$ and $\omega(☽')_f = B$.

For this disposition, arcs AT and BK refer to the sum of the solar and lunar radii. That is, $AT = BK = r_☽ + r_\odot$.

Consequently,

$\because AT = BK$
$\therefore AE = EB$.

However, although it is not indicated, for the above deduction to be fulfilled it is necessary that TD = DK; that is, that the arcs that the Sun traverses during the phases of immersion and emersion are equal.

Next, once the apparent dispositions of the Sun and the Moon across the solar eclipse are defined, he indicates the true positions of the Moon and the parallaxes at the three significant times. For a given apparent disposition, he considers two possible true positions that depend on the direction of the parallax, that is on the quadrant of the ecliptic in which the eclipse takes place. This gives two true positions of the Moon for each of the initial and middle times of the eclipse.

Points Z and H are the true positions of the Moon at the initial and middle times respectively, that is, $\omega(☽)_i = Z$ and $\omega(☽)_m = H$. Then, Jābir b. Aflaḥ points out that arc AZ is the lunar parallax in longitude at the initial time of the eclipse and arc EH is its parallax at the middle time. Lastly, he points out that both parallaxes are different —that is, $p_\lambda(☽)_i = AZ \neq p_\lambda(☽)_m = EH$—. Jābir b. Aflaḥ is approximating longitudes by arguments in latitude by representing parallaxes in longitude in the lunar inclined orbit.

Once the apparent and true positions of the Sun and the Moon are defined, Jābir b. Aflaḥ proceeds to illustrate the demonstration by dealing, firstly, with the apparent motion of the Moon in both phases.

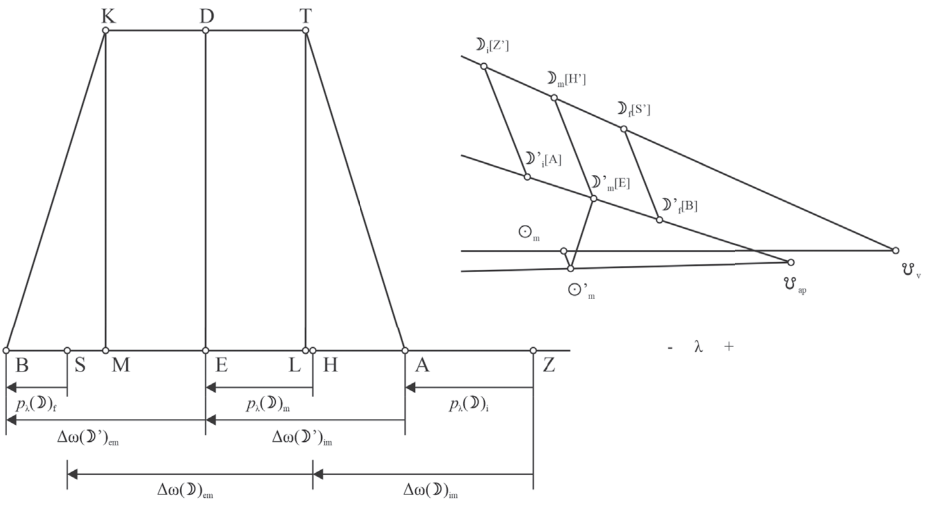
- Firstly, he points out that, during the time in which the Moon apparently traverses arc AE, its true course is arc ZH. That is,

$$\Delta\omega(☽')_{im} = AE \text{ and } \Delta\omega(☽)_{im} = ZH .$$

- Then, he establishes the difference between arcs AE and ZH depending on the initial and middle parallaxes in longitude as:

$$AE - ZH = \Delta\omega(☽')_{im} - \Delta\omega(☽)_{im} = p_\lambda(☽)_i - p_\lambda(☽)_m = AZ - EH .$$

At this point, Jābir b. Aflaḥ considers the demonstration in terms of the two situations that can occur. These two situations are: (i) if the parallax in longitude takes place in the direction of the zodiac across the ecliptic —that is, if its component in longitude is positive or, alternatively, if the eclipse takes place in the first quadrant of the ecliptic; and (ii) if the parallax takes place in the opposite direction of the zodiac signs across the ecliptic —that is, if its component in longitude is negative, or, alternatively, if the eclipse takes place in the second quadrant of the ecliptic. Let us remember that Ptolemy takes the meridian as a reference. This will be one of the criticisms that Jābir b. Aflaḥ will make of him.



- Fig. a - - Fig. b -

Figure 53. Duration of the phases of the solar eclipse with parallax in the direction of the zodiac.

Firstly, he considers the case in which the parallax takes place in the direction of the zodiacal signs across the ecliptic to obtain the ratio between the apparent and true motions of the Moon in the first quadrant. Figure 53 shows Jābir b. Aflaḥ's method. Figure -a- is similar to the one presented in the manuscript, although we have added point S corresponding to the true position of the Moon at the final time of the eclipse, that is, $\omega(☽)_f = S$.

Figure -b-, in turn, shows the same previous arrangement based on the series of Figures that have been used throughout this study. In it, the true positions are indicated as Z', H' and S' to distinguish them from points Z, H and S, since Jābir b. Aflaḥ projects them on the same line in which the apparent positions of the Moon are. Thus, to obtain the ratio between the true and apparent motions of the Moon in the first quadrant, Jābir b. Aflaḥ points out that:

- if the parallax takes place in the direction of the zodiacal signs across the ecliptic, the parallax at the initial time of the eclipse should be greater than the parallax at the middle time, so that the arc ZH is greater than the arc AE.

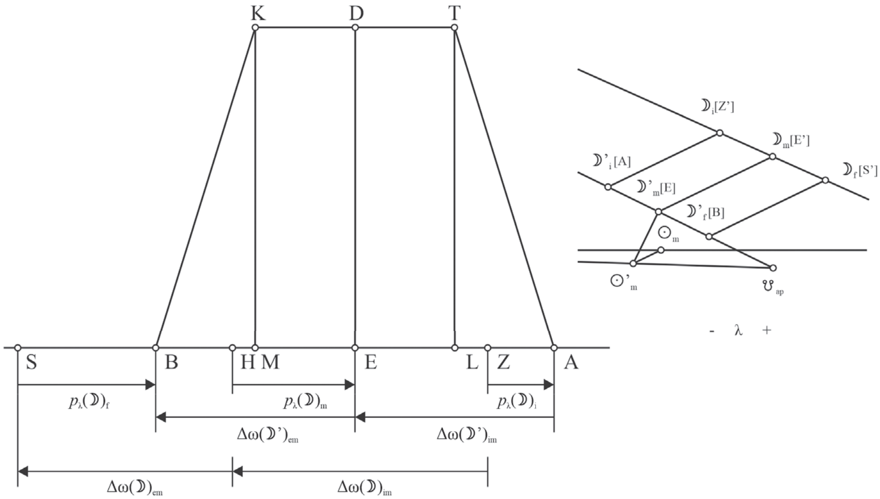
$\because\ p_{\lambda}(\rightmoon) > 0$ (in the direction of the zodiacal signs)
$\therefore\ p_{\lambda}(\rightmoon)_{i} > p_{\lambda}(\rightmoon)_{m}$
$\therefore$ ZH > AE .

And the conclusion that he draws from this is that, in this first quadrant,

- the apparent motion is slower than the true one:

$$w(\rightmoon')_{im} < w(\rightmoon)_{im} .$$

Next, he considers the case in which the parallax takes place in the opposite direction of the zodiacal signs across the ecliptic to obtain the ratio between the apparent and true motions of the Moon in the second quadrant of the ecliptic. Figure 54 shows Jābir b. Aflaḥ's method. Figure -a-, as in the previous case, is similar to the one presented in the manuscript, although we have added a new point S again. Figure -b- is similar to the one that appears in the previous case. Thus, to obtain the ratio between the apparent and true motions of the Moon in the first quadrant of the ecliptic, Jābir b. Aflaḥ points out that:



- Fig. a - - Fig. b -

Figure 54. Duration of the phases of the solar eclipse with parallax in the opposite direction of the zodiacal signs across the ecliptic.

- if the parallax takes place in the opposite direction of the zodiacal signs across the ecliptic, the parallax at the initial time of the eclipse is smaller than the one at the middle time, so that, as in the first quadrant of the ecliptic, arc ZH is also greater than arc AE.

$$\begin{aligned}&\because\ p_\lambda(☽) < 0 \text{ (opposite direction of the zodiacal sings)}\\&\therefore\ p_\lambda(☽)_\mathrm{i} < p_\lambda(☽)_\mathrm{m}\\&\therefore\ \mathrm{ZH} > \mathrm{AE}\ .\end{aligned}$$

And the conclusion that he draws from this is that,

- in this second quadrant of the ecliptic, the apparent motion is slower than the true one; so it is true that in both quadrants the apparent motion is always slower than the true one. That is,

$$w(☽')_{\mathrm{im}} < w(☽)_{\mathrm{im}}\ .$$

The next step consists of extending to the phase of emersion the results that have been obtained for the phase of immersion. Jābir b. Aflaḥ points out, shortly, that

- the same goes exactly for arc EB.

That is, in the phases of emersion is true that, in both quadrants,

$$w(☽')_{\mathrm{em}} < w(☽)_{\mathrm{em}}\ .$$

And, therefore, throughout the solar eclipse, it is true that

$$w(☽') < w(☽) \qquad (58).$$

Once he has obtained the ratio between the apparent and true motions, he aims to obtain the duration of the phases of immersion and emersion. To do this, he computes the true course of the Moon, from the apparent one and the parallaxes at the initial and end times of the phase. Once the true course is obtained —the increment in its argument in latitude—during the phase, he divides it by the true motion of the Moon and finds the duration of the phase. These steps are indicated as follows:

- Firstly, he finds arc ZH from the difference between arcs AZ and EH and adds it to arc AE. Arc ZH corresponds to the true course of the Moon during the phase of immersion. That is,

$$\mathrm{ZH} = \mathrm{AE} + (\mathrm{AZ} - \mathrm{EH}) \, .$$

- Then, he divides ZH —the true course of the Moon during the phase of immersion— by the true motion of the Moon —w(☽)—. The result is the time during which the Moon traverses with its apparent motion arc AE.

$$\mathrm{ZH} \,/\, w(☽) = \Delta t_{\mathrm{im}} \text{ and thus } \Delta\omega(☽')_{\mathrm{im}} = \mathrm{AE} \, .$$

- And, lastly, he extends the previous result to the phase of emersion pointing out that the same thing happens exactly in case of arc EB when adding the parallax at points E and B to the arc EB. To explain this last step, it should be recalled that, in Figure 53 and Figure 54, point S has been added, and thus $\omega(☽)_{\mathrm{f}} = \mathrm{S}$, $p_{\lambda}(☽)_{\mathrm{f}} = \mathrm{BS}$ and $\Delta\omega(☽)_{\mathrm{em}} = \mathrm{HS}$. Thus, the duration of the phase of emersion can be found, since

$$\mathrm{HS} = \mathrm{EB} + (\mathrm{EH} - \mathrm{BS})$$
$$\mathrm{HS} \,/\, w(☽) = \Delta t_{\mathrm{em}} \text{ and thus } \Delta\omega(☽')_{\mathrm{em}} = \mathrm{EB} \, .$$

Finally, once he has obtained the duration of each phase separately, he only has to compare the duration of both phases to determine whether they are equal or not, and, in this second case, which of them is greater than the other.

As a premise, he points out that parallaxes in longitude vary the most in areas close to the mid-heaven of the ascendant and the least in areas close to the degrees of the ascendant or the descendant.

He, then, takes into account if the distance of the Moon to the ascendant, throughout the eclipse, is smaller or greater than 90º, or if the distance of the Moon to the ascendant at the eclipse mid-time is equal to 90º (the mid-heaven of the ascendant).

And he draws the conclusion that if the distance of the Moon to the ascendant is smaller than 90º, the duration of the immersion is smaller than that of the emersion; if it is greater than 90°, the duration of the immersion is greater than that of the emersion; and if, at the eclipse mid-time, is equal to 90°, the duration of the immersion is equal to that of the emersion.

Firstly, to explain his premise, i.e., that parallaxes in longitude vary the most in areas close to the mid-heaven of the ascendant, Jābir b. Aflaḥ points out that this fact can be considered analogous to the variation (*tafāḍul*) of the angles of the anomaly (*ikhtilāf*) of the hypothesis in eccentricity (*al-falak al-khārij al-markaz*).

That is, he establishes a very interesting comparison between the parallax and the hypothesis in eccentricity. The phenomenon of the parallax is due to a difference between the observed position and the true one, since the observer is on the Earth's surface. In turn, the hypothesis in eccentricity addresses a difference between the observed and true motions, since the center of the motion of the heavenly body is eccentric relative to the center of the universe. Thus, a series of correspondences occur between the parallax and a hypothesis in eccentricity, which are listed in the table below.

Graphically, given Figure 55, where O is the point of the observer —in the eccentricity, the center of the universe, and, in the parallax, the Earth's surface— and C is the reference center—in the eccentricity, the center of the eccentric circle, and, in the parallax, the center of the Earth—, then h is the geocentric altitude and h_{ob} is the observed altitude —so that, $h_{ob} = h - p$—, the eccentricity —e— is equivalent to the Earth radius —r_T—, the angle of the equation of the center —q— is equivalent to the angle of the parallax —p— and the radius of the eccentricity —R_{ex}— is equivalent to the distance of the Moon to the center of the Earth — $d_{T☽}$—.

Correspondences between the hypothesis in eccentricity and parallax		
	Hypothesis in eccentricity	Parallax
Anomalous variable	Apparent Motion of the heavenly body	Apparent position
Point of reference	Center of the motion	Center of the universe
Point of observation	Center of the universe	Surface of the Earth
Difference between the point of reference and the point of observation	Eccentricity	Earth radius

Angle of the difference	Equation of the center	Parallax
Minimum variation of the anomalous variable	In the mesogee	In the horizon
Maximum variation of the anomalous variable	In the perigee	In the zenith

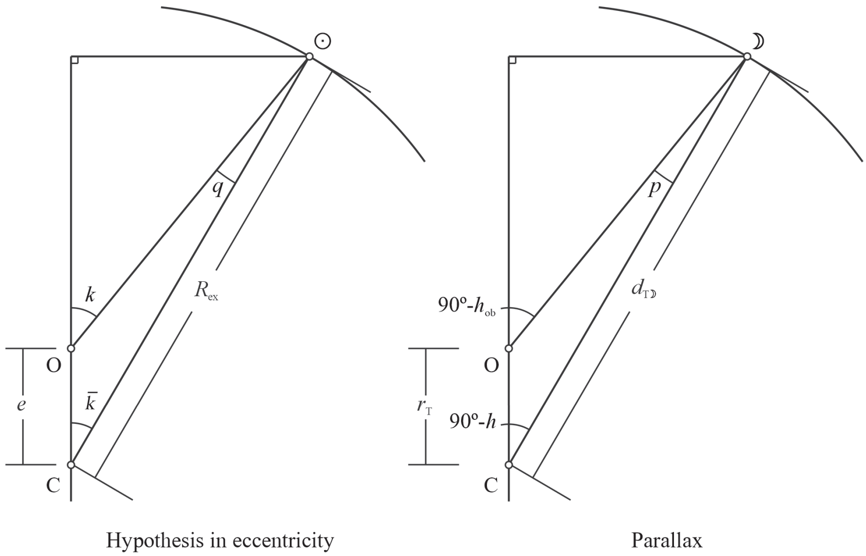


Figure 55. Geometric correspondences between the eccentricity and the parallax.

To prove that parallaxes in longitude vary the most in areas close to the mid-heaven of the ascendant, Jābir b. Aflaḥ refers to a section in the *al-Kitāb fī l-Hay'a* in which he addresses the variation in the eccentricity. His text is as follows:

> And this is explained by what we have mentioned about the variation (*tafāḍul*) of the angles of the anomaly (*ikhtilāf*) relative to the eccentricity (*al-falak al-khārij al-markaz*).

Jābir b. Aflaḥ is referring to the section in which he studies the variation of the angles of the equation, whether it is the hypothesis in epicycle or eccentricity. The text is as follows:

ويستبين بذلك أن التفاضل في زوايا الاختلاف على كلّ واحد من الأصلين أعظم ما يكون الكوكب في البعدين الأبعد والأقرب له ولا يزال التفاضل يقلّ حتّي ينتهي الكوكب إلى مجازه الأوسط وهي النقطة التي بعدها من نقطة البعد الأبعد بالرؤية ربع دائرة.[40]

The above text can be translated as:

> And it is made clear by this that the variation of the angles of the equation according to both hypotheses [i.e., in epicycle or in eccentricity] is maximum when the heavenly body is in the apogee or the perigee and the [said variation] gradually decreases until the heavenly body is situated in its mesogee (*al-majāz al-awsaṭ*), that is the point whose distance from the apogee is in appearance a quarter of a circle. [i.e., 90°].

Jābir b. Aflaḥ develops *Almagest* III.3, where Ptolemy studies the correspondence between the hypotheses in epicycle and in eccentricity. Specifically, Jābir b. Aflaḥ addresses the argument in which Ptolemy shows that, given a hypothesis in eccentricity, the maximum differences between the mean and true motion occur in the mesogees, at ±90° from the apogee in the apparent circle.[41]

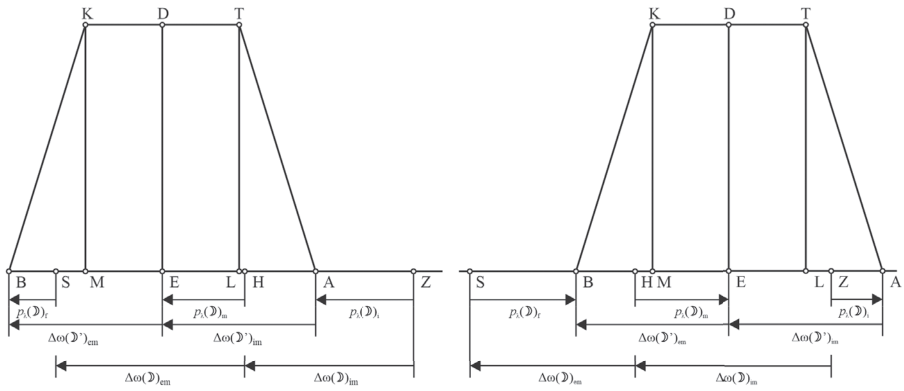
Jābir b. Aflaḥ points out that the maximum variation of the lunar parallax in latitude occurs when the Moon is in the mid-heaven of the ascendant. Indeed, as the mid-heaven of the ascendant is determined by the intersection with the ecliptic of the great circle that passes through the zenith and would be orthogonal to it, the mid-heaven of the ascendant is the point of the ecliptic with maximum altitude. Hence, it is the point of the ecliptic with minimum parallax and in which its variation would be maximum.

Having studied the premise, i.e., that the maximum variation of the lunar parallax in latitude occurs when the Moon is in the mid-heaven of the ascendant, and its geometrical equivalence with the hypothesis in eccentricity, we shall finally consider how this premise affects the duration of the phases according to the criterion that Jābir b. Aflaḥ has defined. Jābir gives the conclusion without demonstrating it, perhaps aware that it follows from what he has indicated previously, but in contrast to the certain care that he has shown until now. We shall fill this gap.

We know that the higher the altitude, the lower the parallax, and the lower it is, the greater its variation.

40. See MS Ea 34r-34v for this quotation and MS Ea 34r-35r for the complete section.
41. See *Almagest* III.3 (Toomer pp. 145-147).

Firstly, we will study what happens when the solar eclipse takes place in the first quadrant, that is when the distance to the ascendant is less than 90°. To do this, we will consider Figure 59a. In it, the parallax in longitude is greater at the eclipse initial time than at the end time and, therefore, the variation of the parallax is greater in the phase of emersion than in the phase of immersion.



- Fig. a - - Fig. b -

Figure 56. Duration of the phases depending on the quadrant.

Hence,

$$\Delta p_{\lambda}(☽)_{\text{im}} < \Delta p_{\lambda}(☽)_{\text{em}} \qquad (59).$$

And consequently

$$\Delta\omega(☽)_{\text{im}} < \Delta\omega(☽)_{\text{em}} \quad \text{or, alternatively,} \quad HZ < SH \qquad (60).$$

Hence, since the duration of the phase of immersion —Δt_{im}— is $\Delta\omega(☽)_{\text{im}}$ / $w(☽)$ and the duration of the phase of emersion —Δt_{em}— is $\Delta\omega(☽)_{\text{em}} / w(☽)$, the duration of the phase of immersion should be smaller than that of the phase of emersion. That is,

$$\Delta t_{\text{im}} < \Delta t_{\text{em}} \qquad (61).$$

Secondly, we will study what happens when the solar eclipse takes place in the second quadrant, that is when its distance to the ascendant is greater than 90°. To do this, we will consider Figure 56b. In it, the parallax in longitude is smaller at the eclipse initial time than at the final time and, therefore, the variation of the parallax is grater in the phase of immersion than in the phase of emersion.

Hence,

$$\Delta p_\lambda(☽)_{\text{im}} > \Delta p_\lambda(☽)_{\text{em}} \qquad (62).$$

And, consequently,

$$\Delta\omega(☽)_{\text{im}} > \Delta\omega(☽)_{\text{em}} \quad \text{or, alternatively,} \quad HZ > SH \qquad (63).$$

Hence, since the duration of the phase of immersion —Δt_{im}— is $\Delta\omega(☽)_{\text{im}}$ / $w(☽)$ and the duration of the phase of emersion —Δt_{em}— is $\Delta\omega(☽)_{\text{em}}$ / $w(☽)$, the duration of the phase of immersion should be greater than that of the phase of emersion. That is,

$$\Delta t_{\text{im}} > \Delta t_{\text{em}} \qquad (64).$$

When the middle time of the eclipse takes place in the mid-heaven of the ascendant, contrary to Ptolemy's statement, the variation of the parallax in both phases of the eclipse is the same, and therefore the duration of both phases as well.

In summary, Jābir b. Aflaḥ's method is much clearer than that of Ptolemy. Jābir b. Aflaḥ's method broadly agrees with the one presented by Ptolemy, except for the correction of the meridian by the mid-heaven of the ascendant, which we will see below. Since Jābir b. Aflaḥ only criticizes this aspect and his interpretation agrees with that which we have made of Ptolemy's procedure, Jābir b. Aflaḥ considered that Ptolemy interpreted the minutes of immersion obtained with the tables and corrected with the additional motion of the Sun as apparent intervals.

3.2. Jābir b. Aflaḥ's criticisms of Ptolemy

We have just studied Jābir b. Aflaḥ's method to find the magnitude and phases of solar eclipses. After presenting his method, Jābir b. Aflaḥ then introduces some criticisms of Ptolemy's method, as he already advanced in his introduction. Jābir b.

Aflaḥ raises three criticisms, although in the introduction he groups them into two: the first and second criticism on the one hand, and the third on the other. He groups the first and second criticism since they are due to the same cause. We will study these first two criticisms grouped together; and we will finally focus on the third.

In his Introduction to the *al-Kitāb fī l-Hay'a*, Jābir b. Aflaḥ points out:

> There is another mistake in the computation of solar eclipses and in the values of their phases. All this is mentioned in Book V of this work.

The text of his first criticism in Book V is as follows:

> This matter is not as Ptolemy thinks, for he said that if the middle time of the eclipse takes place at noon, both times are equal. But this is a mistake, for between the degree [in longitude] of the mid-heaven and the degree [in longitude] of the mid-heaven of the ascendant in the northern countries there may be an arc with a value [which cannot be neglected] and which in the seventh climate reaches up to 37°. Thus, if the Moon during the eclipse is on this arc, its distance from the ascendant after noon would be less than 90°, or its distance from the ascendant before noon would be greater than 90°. Hence, the matter about the duration of the phases (*azmina*) [of solar eclipses] differs from what [Ptolemy] mentioned.

And the second criticism follows right after:

> Likewise, Ptolemy makes a mistake when he adds that the times that correspond to the arcs of the parallaxes in longitude always depend on the distance of the true conjunction to the meridian, be it before it or after it. This is never the case except in an eclipse whose ascendant would be the head of Aries or Libra. [Only] in this case, the degree [in longitude] of the mid-heaven is [the same as that of] the mid-heaven of the ascendant. In turn, when the ascendant is not one of these two points, these two degrees [in longitude] are different. If the position of the true conjunction is between these two degrees [in longitude], as [if the true conjunction] takes place before noon and its distance to the degree [in longitude] of the ascendant is greater than 90°, or [as if it] takes place after noon and its distance to the ascendant is less than 90°, then the time interval which corresponds to the parallax in longitude should be subtracted from the time interval which corresponds to the distance [in longitude] between the true conjunction and the meridian, although [Ptolemy] adds it. Therefore, there is a mistake in the apparent conjunction with a [non negligible] error, since the parallax in longitude in the northern countries has a significant value. Thus, the error (*khilāf*) [introduced by

Ptolemy] in the apparent conjunction is a time [difference] that corresponds to the double of the parallax in longitude.

For Jābir b. Aflaḥ, the reason behind Ptolemy's mistake is that he takes the meridian as a reference instead of the mid-heaven of the ascendant.

This mistake, according to Jābir b. Aflaḥ, affects two places and hence the two criticisms. Firstly, by taking the mid-heaven as a reference, Ptolemy makes a mistake in pointing out that the phases of immersion and emersion of an eclipse that takes place in the mid-heaven are equal. In any case, he should have pointed out, according to Jābir b. Aflaḥ, that only the phases of immersion and emersion of an eclipse that takes place in the mid-heaven of the ascendant are equal.

Secondly, this mistake also affects the resolution of the apparent conjunction from the true one, since Ptolemy —according to Jābir b. Aflaḥ—takes the mid-heaven, and not the mid-heaven of the ascendant, as the reference to decide whether to add or subtract the correction in time to obtain the apparent conjunction.

We will study, below, the effect of choosing the mid-heaven, instead of the mid-heaven of the ascendant, as reference to compute the magnitude and phases of solar eclipses. The meridian is the great circle that passes through the pole of the equator, P_e, and the zenith, Z. The mid-heaven is the point of the ecliptic that cuts the meridian. Likewise, the mid-heaven of the ascendant is the great circle that passes through the pole of the ecliptic, P_ε, and the zenith, Z. Since the pole of the ecliptic is inclined relative to the pole of the equator at an angle equal to the obliquity of the ecliptic and revolves around it on an approximately daily basis, the mid-heaven and the mid-heaven of the ascendant only coincide twice during the day when the poles of the ecliptic, the equator and the zenith are aligned. In those times, the ascendant coincides with ♈$_{0^\circ}$ or ♎$_{0^\circ}$ and vice versa in case of the descendant. On all other occasions, the mid-heaven of the ascendant, when we face the south, falls to the east or west of the meridian. The angle determined by the mid-heaven and the mid-heaven of the ascendant is maximum when the angle $P_\varepsilon P_e Z$ is approximately 90°. The transit of the Sun through the meridian — which drags with it the ecliptic and consequently also the Moon, as its longitude is close to that of the Sun in solar eclipses — defines the time in which the Sun reaches its maximum altitude. Likewise, approximately at this time, the Moon also reaches its maximum altitude. Consequently, the absolute value of the parallax of both heavenly bodies at this time is minimal. In turn, the transit of the Sun through the mid-heaven of the ascendant defines the time in which the parallax in longitude of the Sun—and therefore also of the Moon in solar eclipses—is minimal, although the total parallax —that is the resultant of the components of the parallax in longitude and latitude— is not.

Thus, we will consider the first case in which, when facing towards the south, the mid-heaven of the ascendant falls to the east of the meridian (Figure 57). We can define three intervals in the courses of the Sun and the Moon. Firstly, we will define the interval determined by the ascendant and the solar transit through the mid-heaven of the ascendant. In this interval, the total parallax is decreasing, while the parallax in longitude is positive and decreasing. In turn, in the second interval determined by the transit of the Sun through the mid-heaven of the ascendant and by its transit through the meridian, the total parallax is decreasing, but in turn, the parallax in longitude is negative and its absolute value is increasing. Finally, in the third interval, determined by the solar transit through the meridian and the descendant, the total parallax is increasing, the parallax in longitude is negative and its absolute value is increasing.

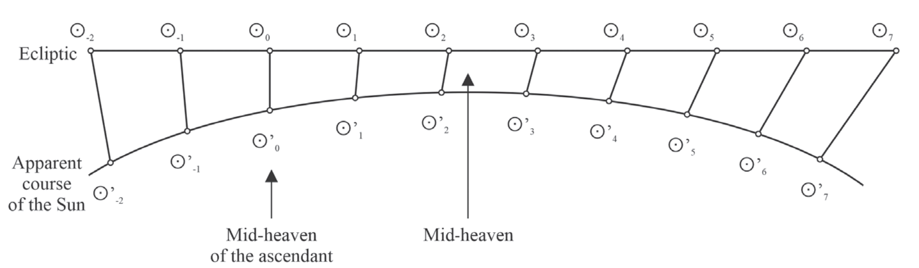


Figure 57. Mid-heaven of the ascendant east of the meridian.

If, in turn, we consider a second case in which, facing south, the mid-heaven of the ascendant falls to the west of the meridian (Figure 58), we can define three intervals in the paths of the Sun and the Moon. First, we define the interval determined by the ascendant and its transit through the meridian. In this interval, the total parallax is decreasing, while the parallax in longitude is positive and decreasing. In turn, in the second interval determined by the Sun's transit through the meridian and by its transit through the mid-heaven of the ascendant, the total parallax is increasing, but in turn, the parallax in longitude is positive and decreasing. Finally, in the third interval, determined by the solar transit through the mid-heaven of the ascendant and by its transit through the descendant, the total parallax is increasing and the parallax in longitude is negative but its absolute value is increasing.

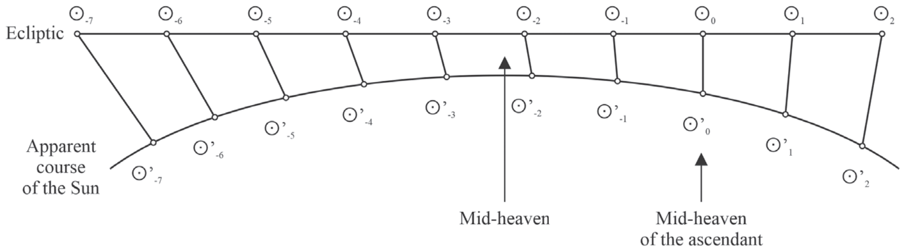


Figure 58. Mid-heaven of the ascendant west of the meridian.

Since time corrections depend on longitudes, any time increment that Ptolemy has referenced relative to the meridian, should in fact be referenced a priori relative to the mid-heaven of the ascendant, since it is with respect to this point that the parallax in longitude changes from positive to negative or vice versa. Ptolemy uses time increments determined by increments in longitude twice: (i) when he finds the apparent conjunction from the true one, which is the topic of the second criticism by Jābir b. Aflaḥ; and (ii) when he studies the different durations of the phases of immersion and emersion in the solar eclipse, the topic of the first criticism. We will study now the first of these two criticisms.

Ptolemy finds the longitude of the apparent conjunction from the longitude of the true conjunction. To do this, he uses a correction in longitude from the difference between the parallaxes of the Moon and the Sun in the true conjunction to which he adds successive epiparallactic corrections. To this increment in longitude, he adds the additional motion of the Sun and, from the total correction in longitude, he obtains a time increment dividing the correction in longitude by the true motion of the Moon in the true conjunction. Jābir b. Aflaḥ's criticism focuses on the criterion applied by Ptolemy to add or subtract this correction in time to or from the time of the true conjunction to obtain the apparent one. Jābir b. Aflaḥ points out that Ptolemy is wrong because he should have taken as reference the mid-heaven of the ascendant and not the meridian. Let us compare the text by Ptolemy to see if Jābir b. Aflaḥ's criticism of him makes sense. Ptolemy points out:

> If the longitudinal parallax we found is towards the rear [i.e. in the order] of the signs (we explained previously how to determine this), we subtract the amount in degrees which we had converted into equinoctial hours from the moon's position, as previously determined, at the moment of the true conjunction, in longitude, latitude and anomaly (each separately): this gives us the [corresponding] true positions of the

> moon at the moment of apparent conjunction, while the number of hours itself [resulting from the above computation] tells us by how much the apparent conjunction precedes the true one. But if the longitudinal parallax we found is in advance [i.e., in the reverse order] of the sings, contrariwise, we add the amount in degrees to the position, as previously determined, at the moment of true conjunction, in longitude, latitude and anomaly (each separately); and the number of hours will give us the amount by which the apparent conjunction is later than the true one.[42]

Thus, from this quotation it does not follow that Ptolemy uses the meridian as a criterion to add or subtract the correction in time to or from the time of the true conjunction to obtain the apparent one. The criterion that Ptolemy uses is if the parallax in longitude is positive, for which he uses the expression «towards the rear of the signs» or if it is negative, for which he uses the expression «in advance of the signs». The point that delimits both situations is the intersection of the ecliptic with the mid-heaven of the ascendant and not with the meridian. Hence, Ptolemy is using the mid-heaven of the ascendant and not the meridian as a criterion. Consequently, the criticism of Jābir b. Aflaḥ is surprisingly baseless. So, does Jābir b. Aflaḥ refer to this fragment? And if so, do the Arabic translations of the *Almagest* refer to the meridian?

The answer to the first question is unequivocally affirmative. The elements in this section of the *al-Kitāb fī l-Hay'a* point to this. For instance, Jābir b. Aflaḥ speaks about adding or subtracting time intervals determined by the parallax in longitude to or from the time of the true conjunction, as well as about the apparent conjunction. These elements do not appear in any other paragraph of Ptolemy's study of solar eclipses in the *Almagest*. It could be argued that Jābir b. Aflaḥ refers to the section in which Ptolemy transfers the position of the true syzygy according to the meridian of Alexandria to the local meridian,[43] but in this section no parallaxes in longitude appear yet. Consequently, it should be ruled out that Jābir b. Aflaḥ refers to any other section of the *Almagest* study on solar eclipses.

Thus, we must ask ourselves if Jābir b. Aflaḥ's criticism is due to the fact that he was using a poor Arabic translation of the *Almagest*?

The text in the Isḥāq/Thābit version is the following one:

42. See Toomer p. 311.
43. See Toomer p. 310.

فإن كان اختلاف المنظر في الطول قد وقع على توالي البروج فإنّا قد بيّنّا فيما تقدّم كيف يمكننا تمييز ذلك: نقصنا الأجزاء التي رددناها إلى الساعات الاستوايية من أجزاء القمر التي تقدّمنا فقوّمنا للزمان الحقيقي الذي للاجتماع كلّ صنف منها على حيل له أعني من الأجزاء التي في الطول ومن الأجزاء التي للعرض ومن الأجزاء التي للاختلاف فيحصل لنا مسيرات القمر الحقيقيّة في زمان الاجتماع الذي يرى وتكون تلك الساعات التي وجدناها الساعات التي تقدّم الاجتماع الذي يرى الاجتماع الحقيقي وإن كان اختلاف المنظر في الطول يوجد قدما للبروج فعلنا عكس ذلك فزدنا تلك الأجزاء على المسيرات التي تقدّم تقويمها للزمان الحقيقي للاجتماع لكلّ واحد من الطول أيضا والعرض والاختلاف وكانت لنا تلك الساعات هي التي بها يتأخّر الاجتماع الذي يرى على الاجتماع الحقيقي.[44]

Whereas the text in the version by al-Ḥajjāj is the following one:

فإن كان اختلاف المنظر الذي في الطول على توالي البروج فإنّا قد بيّنّا فيما تقدّم لتمييز ذلك أمّا الأجزاء التي قسمناها بالساعات المعتدلة فتنقصها من أجزاء القمر المحصلة التي في زمان الاجتماع الحقيقي ونصير كلّ واحد من الطول والعرض ومسير الاختلاف على حدّته ويكون ما بقي هو مجازات القمر الحقّيّة التي تكون في زمان اجتماع القمر الذي يرى ويكون قد وجدنا الساعات التي بها يتقدّم الاجتماع الذي يرى قبل الحقّي وإن كان اختلاف المنظر الموجود في الطول على خلاف توالي البروج أمّا الأجزاء فنزيدها على نكس المجازات التي تقدّم تحصل كلّ واحد منها في زمان الاجتماع الحقّي من الطول أيضا والعرض ومسير الاختلاف ويكون قد وجدنا الساعات اللواتي بعدهنّ يكون الاجتماع الذي يرى بعد الاجتماع الحقّي.[45]

Reading both translations shows that the Arabic *Almagest* makes no reference at any point to the meridian, so the criticism by Jābir b. Aflaḥ seems to make no sense. If we continue reading the Arabic *Almagest* at the point where we left off —that is, when Ptolemy begins the resolution of the argument on the apparent latitude of the apparent conjunction—, we find the following in the version by Isḥāq/Thābit:

فننظر من أمر البعد بين الاجتماع الذي يرى وبين انتصاف النهار من الساعات المستوية بتلك الأبواب بأعيانها كم، أوّلا، اختلاف منظر القمر في الدائرة العظمى التي ترسم مارّة بالنقطة التي على سمت الرأس فننقص ممّا نجده من ذلك اختلاف منظر الشمس الذي بإزاء ذلك العدد.[46]

44. MS Paris, BnF, Ar. 2482, f. 126r.

45. MS London, British Museum, Add. 7474, f. 178v, and MS Leiden, Universiteitsbibliotheek, Or. 680, ff. 101v-102r.

46. MS Paris, BnF, Ar. 2482, f. 126r.

The translation by al-Ḥajjāj, is as follows:

وأيضا نبحث بهذه الأبواب كم بعد ما بين الاجتماع الذي يرى وبين فلك نصف النهار من الساعات المعتدلة، أوّلا، كم يختلف منظر القمر في الفلك العظيم المخطوط عليه وعلى نقطة سمت الرؤوس ونلقى مما نجد اختلاف منظر الشمس الذي يقابل ذلك العدد.[47]

That is, the paragraph in the *Almagest* that follows up the text which is the object of Jābir b. Aflaḥ's criticism points out that, in order to obtain the difference between the parallax in latitude of the Moon and the Sun, the distance between the apparent conjunction and the meridian should be taken into account. Thus, we can venture two hypotheses: either Jābir b. Aflaḥ was working with a abridgment of the *Almagest* that was not sufficiently accurate at this point; or the criterion that Ptolemy used to discriminate the addition or subtraction of the correction in time was missing from the manuscript of the *Almagest* with which Jābir was working, and, after the gap, the text linked with the resolution of the argument in apparent latitude of the Moon at the point at which Ptolemy speaks about the meridian. It is also possible that he did not understand the text well and interpreted it in light of the first criticism, where he considers that Ptolemy should have used the mid-heaven of the ascendant as a reference.

Jābir b. Aflaḥ's criticism contains other surprises. Although, as regards the addition or subtraction of the correction in time, the criterion used by Jābir b. Aflaḥ is exactly the same as that used by Ptolemy, the author of the *al-Kitāb fī l-Hay'a* differs in the correction he proposes in his criticism of Ptolemy from the method he has just previously presented in his study.

Jābir b. Aflaḥ proposes in his study of solar eclipses that, to obtain the time of the apparent conjunction, the time increment must be subtracted from the time of the true conjunction when the true conjunction takes place in the first quadrant, before the mid-heaven of the ascendant, and must be added when it takes place in the second quadrant, after the mid-heaven of the ascendant.

However, in his criticism he points out that, when a true conjunction takes place in the interval defined by the meridian and the mid-heaven of the ascendant regardless of whether the conjunction takes place after reaching 90º or before reaching 90º of the ascendant —so that the mid-heaven of the ascendant can be

47. MS Leiden, Universiteitsbibliotheek, Or. 680, f. 102r.

found east or west of the meridian—, the time increment corresponding to the parallax in longitude is subtracted. However, according to Ptolemy's method and Jābir b. Aflaḥ's, as in his study, the time increment can only be subtracted when the true conjunction takes place less than 90º from the ascendant; that is, when it occurs in an interval defined by the meridian, as the closest limit to the east, and the mid-heaven of the ascendant, as the closest limit to the west (Figure 58).

However, in the other situation—when the true conjunction takes place in an interval defined by the mid-heaven of the ascendant, as the closest limit to the east, and the meridian, as the closest limit to the west (Figure 57)—, the time increment must be added, contrary to what Jābir b. Aflaḥ says in his criticism. However, this correction to his criticism is in agreement with what Jābir b. Aflaḥ himself says in his study of solar eclipses.

Finally, it only remains for us to clarify some consequences that follow from this criticism. At the end of his criticism, Jābir b. Aflaḥ points out that the error committed amounts to twice the final correction in time. This is so because the correction in time was added instead of being subtracted.

Once we have seen Jābir b. Aflaḥ's second criticism focused on the resolution of the apparent conjunction from the true one, we will study the first criticism in which Jābir b. Aflaḥ discusses the disposition on the horizon that makes the duration of the phases of immersion and emersion equal. Jābir's criticism is as follows:

> This matter is not as Ptolemy thinks, for he said that if the middle time of the eclipse takes place at noon, both times are equal. But this is a mistake, for between the degree [in longitude] of the mid-heaven and the degree [in longitude] of the mid-heaven of the ascendant in the northern countries there may be an arc with a value [which cannot be neglected] and which in the seventh climate reaches up to 37º. Thus, if the Moon during the eclipse is on this arc, after noon its distance from the ascendant would be less than 90º, or before noon its distance from the ascendant would be greater than 90º. Hence, the matter about the duration of the phases (*azmina*) [of solar eclipses] differs from what [Ptolemy] mentioned.

When the middle time of the eclipse takes place in the meridian, Ptolemy points out that the duration of the phases of immersion and emersion is the same. Ptolemy says:

> For this reason, the only situation in which the time of immersion is approximately equal to the time of emersion is when mid-eclipse occurs precisely at noon, for then

the appearance of motion in advance resulting from the parallax is about equal on both sides [of mid-eclipse].[48]

Jābir b. Aflaḥ points out that the middle time of the eclipse should take place in the mid-heaven of the ascendant for the duration of both phases to be equal. Along these lines, in MS Paris, BnF, Ar. 2482, containing Isḥāq/Thābit's translation of the *Almagest*, we find the following marginal gloss:

جعل هنا الدائرة التي تمرّ بسمت الرأس وبموضع القمر في وسط الكسوف القائمة على فلك البروج على زوايا قائمة دائرة نصف النهار ليست على الأكثر دائرة نصف النهار وإنما يكون دائرة نصف النهار إذا كان موضع القمر في وسط الكسوف أحد المنقلبين.[49]

Additionally, when Ptolemy presents an example in which he shows the effect of the parallax in the duration of the phases,[50] we find the following marginal gloss in MS Paris, BnF, Ar. 2482:

نقص هاهنا أجزاء الساعة من أجزاء بعد القمر من سمت الرأس وزادها عليها وإنما كان ينبغي أن ينقصها من الساعات التي بين وسط زمان الكسوف ونصف النهار أو يزيدها عليها ثمّ ننظر كم بعد القمر من سمت الرأس في ذلك الوقت وليس يصحّ علمه إلاّ أن يكون القمر في رأس الحمل أو الميزان ويكون سمت الرأس تحت دائرة معدل النهار وإلاّ فليس يصحّ.[51]

Both marginal glosses criticize Ptolemy for taking the meridian as a reference. Let us consider whose statement is the correct one, either Ptolemy's or Jābir b. Aflaḥ's.

For the duration of the phase of immersion to be equal to the phase of emersion, it should be true that:

- the variation of the lunar altitude as a function of time should be zero —$\mathrm{d}h_{☽}/\mathrm{d}t = 0$—, and that

48. See Toomer p. 312.
49. MS Paris, BnF, Ar. 2482, f. 126v.
50. See Toomer p. 313.
51. MS Paris, BnF, Ar. 2482, f. 127r.

- the variation of the lunar motion in altitude as a function of time should be equal on both sides of the eclipse.

We will only consider the first condition, since it is the only one necessary to discriminate between Ptolemy and Jābir b. Aflaḥ's theses. Thus, we look for a point on the horizon at which the tangent of the lunar course would be parallel to the horizon. Consequently, we must analyze the different components of the lunar motion.

We will momentarily consider that the Moon moves across the ecliptic and not in its inclined orbit. The lunar motion is the composition of the motion of the ecliptic over the horizon and the motion of the Moon across the ecliptic.

Given Figure 59, where line ZS is the meridian with Z the zenith and S the geographical South, line ESO is the horizon with points E and O the geographical East and West points, points E_v and E_i the solar rising in the summer and winter solstices, and points O_v and O_i the solar setting in both solstices, then arc $E_iM_aO_v$ represents the apparent ecliptic at the time in which the ascendant is E_i and the descendant is O_v, and point M_a is the mid-heaven of the ascendant. Let the Moon be on point L. We will consider two components of the lunar motion on point L: one parallel to the equator; and a second one following along the ecliptic in the opposite direction to the motion in the horizon of the ecliptic. The absolute value of the motion of the Moon in the ecliptic is always smaller than the motion of the ecliptic in the horizon, and, thus, the resultant is always a motion in the opposite direction of the zodiacal signs over the horizon.

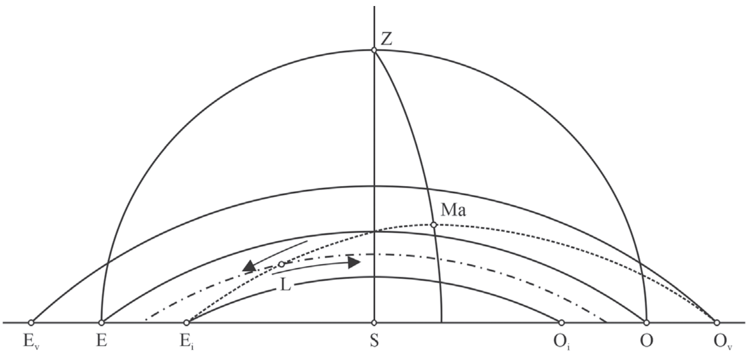


Figure 59. Mid-heaven of the ascendant over the horizon.

Be ☽$_m$ the Moon at the eclipse mid-time and be l the vector of the position of the Moon over the horizon. Thus, $l = (h(☽)\ , a(☽))$ where h indicates the component in altitude and a the component in azimuth. Let us consider as an initial condition that the vector position of the Moon at the eclipse mid-time is $l(☽_m) = (h(☽_m), a(☽_m))$. For a given time increment, Δt, the Moon is on point $l(☽_m) + \Delta l = (h(☽_m) + \Delta h(☽)\ , a(☽_m) + \Delta a(☽))$. We know that the lunar motion over the horizon is the composition of the lunar motion over the ecliptic, i.e., $\Delta\lambda$, and the motion of the ecliptic around the equator, i.e., $\Delta\tau$, with τ the hourly angle. Thus, for a given time increment, Δt, $\Delta l \cong (\Delta\lambda_h + \Delta\tau_h\ , \Delta\lambda_a + \Delta\tau_a)$ with $\Delta\lambda_h$ and $\Delta\lambda_a$ the components in altitude and azimuth of the increment in longitude, and $\Delta\tau_h$ and $\Delta\tau_a$ the components in altitude and azimuth of the increment in the hourly angle. If we only take an infinitesimal time increment of the component in altitude of the lunar motion, we will obtain $\mathrm{d}h = \mathrm{d}\lambda_h + \mathrm{d}\tau_h$, where $\mathrm{d}\lambda_h$ and $\mathrm{d}\tau_h$ are the components in altitude of the lunar motion in longitude and in the hourly angle. Since $\mathrm{d}h$, $\mathrm{d}\lambda_h$ and $\mathrm{d}\tau_h$ are motions linked to a same $\mathrm{d}t$, we will obtain the variation in altitude of the Moon as a function of time as

$$\mathrm{d}h_{☽}/\mathrm{d}t = \mathrm{d}\lambda_h(☽)/\mathrm{d}t + \mathrm{d}\tau_h(☽)/\mathrm{d}t \qquad (65).$$

Thus, the first condition —i.e., that the variation of the altitude as a function of time would be zero— is

$$\mathrm{d}h_{☽}/\mathrm{d}t = \mathrm{d}\lambda_h(☽_m)/\mathrm{d}t + \mathrm{d}\tau_h(☽_m)/\mathrm{d}t = 0 \qquad (66).$$

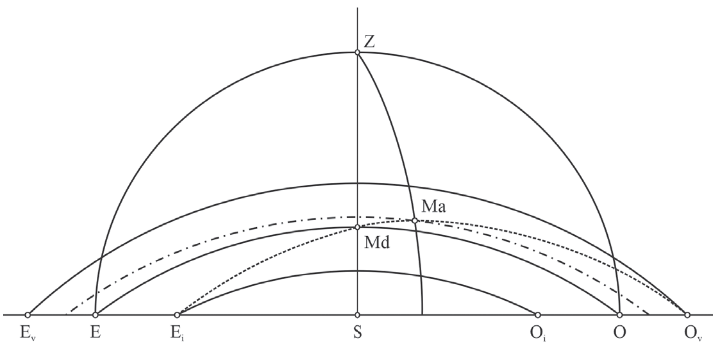


Figure 60. Eclipse mid-time in the meridian or in the mid-heaven of the ascendant.

Let us examine the thesis by Ptolemy. Given Figure 60, let us consider that the middle time of the eclipse takes place on point M_d in the local meridian. In this case, the variation in altitude of the hourly angle of the Moon as a function of time is zero, but not the variation in altitude of its longitude as a function of time. That is,

$$\frac{d\tau_h(☽_m)}{dt} = 0; \qquad \frac{d\lambda_h(☽_m)}{dt} \neq 0\ .$$

Consequently, the thesis by Ptolemy —the phases of immersion and emersion are equal when the middle time of the eclipse takes place on the local meridian— does not meet the condition that the variation of the lunar altitude as a function of time would be zero.

Now, let us examine the thesis by Jābir b. Aflaḥ. Given Figure 60, let us consider that the middle time of the eclipse takes place on the mid-heaven of the ascendant, point M_a. In this case, the variation in altitude of the longitude of the Moon as a function of time is zero, but not the variation in altitude of its hourly angle as a function of time. That is,

$$\frac{d\tau_h(☽_m)}{dt} \neq 0; \qquad \frac{d\lambda_h(☽_m)}{dt} = 0\ .$$

Consequently, the thesis by Jābir b. Aflaḥ — i.e., the phases of immersion and emersion are equal when the middle time of the eclipse takes place on the mid-heaven of the ascendant— does not meet the condition that the variation of the lunar altitude as a function of time would be zero again.

Thus, neither Ptolemy's nor Jābir b. Aflaḥ's theses meet the first condition needed for the phases of immersion and emersion to be equal. Interestingly, one of the solutions that makes zero the variation in altitude of the Moon as a function of time at the eclipse mid-time is when it occurs in a lunar transit through the meridian and the mid-heaven of the ascendant at the same time. That is, the duration of the immersion phase is equal to that of emersion —at least as far as the first criterion is concerned— when the middle time of the eclipse occurs at noon of the summer solstice or at noon of the winter solstice (Figure 61). Both situations are probably the only ones in which the second criterion is met —that the variation of the lunar motion in altitude as a function of time would be equal on both sides of the eclipse.

If, instead of the ecliptic, we take into consideration the lunar inclined orbit, the first criterion would be

$$\frac{dh_{☽}}{dt} = \frac{d\omega_h(☽_m)}{dt} + \frac{d\tau_h(☽_m)}{dt} = 0 \qquad (67).$$

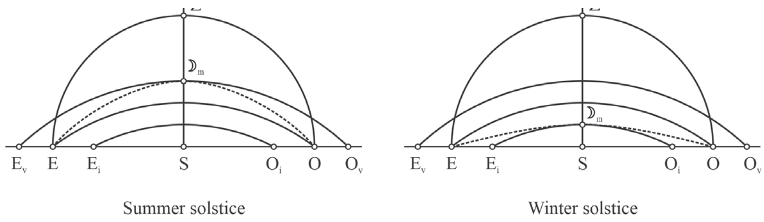


Figure 61. Eclipse mid-time with equal phases of immersion and emersion.

That is, we must take into account the argument in latitude of the ecliptic and not the longitude. In addition, since this is a solar eclipse, it is true that

$$\mathrm{d}\omega_h(☽_m)/\mathrm{d}t \neq \mathrm{d}\lambda_h(☽_m)/\mathrm{d}t \qquad (68),$$

except when the middle time of the eclipse coincides with the node in the local zenith. Except in this case, the middle time of the eclipse that makes the phases of immersion and emersion equal does not coincide with the mid-heaven of the ascendant.

In short, assuming the approximation of the ecliptic by the lunar inclined orbit at the eclipse mid-time, the solution provided by Jābir b. Aflaḥ is only correct if taken as a complement to the solution given by Ptolemy.

We have just seen the first two critiques of Jābir b. Aflaḥ on Ptolemy grouped together. We will, now, study the third criticism of the author of the *al-Kitāb fī l-Hay'a*. In his index of criticisms in the introduction, Jābir b. Aflaḥ points out:

> There is another mistake in the computation of the solar eclipse regarding the delimitation of the lunar parallax in latitude, where he adds it to the ecliptic, whereas he should have added it to the Moon itself. However, we will not mention this in our book, since it is only necessary for the composition of tables used in the computation of the solar eclipse, and this belongs to the realm of practical questions (*umūr ʿamaliyya*).

At the end of his study of solar eclipses, we find the text of the criticism that Jābir mentioned in his Introduction to his *al-Kitāb fī l-Hay'a*:

> Likewise, the same thing happened to him in the delimitation of the side of the parallax in latitude to obtain from it the apparent latitude of the Moon. Ptolemy pointed out:

> If the parallax in latitude is northwards with respect to the ecliptic, we consider the matter. If [the position of] the Moon moves towards the node of the head [of the dragon], we add [this value], and if it moves towards the node of the tail [of the dragon], we subtract [it]. If the parallax in latitude is southwards with respect to the ecliptic, we will act in the opposite way.[52]

> Thus, he added (*aḍāfa*) the parallax in latitude in this position to the ecliptic. However, it must be added (*yajib an yuḍīfahu*) to the Moon itself, not to the ecliptic. Therefore, he introduces an error (*khilāf*) in the distance to the node, and he enters the table with [an argument] smaller or greater than the one that should be entered with in reality. Hence, for its apparent latitude —i.e., the [value] opposite the [argument] with which one enters the table— there will necessarily be a large error (*khilāf kathīr*). The same goes for the degrees (*ajzā'*) of the phase of immersion and the phase of emersion [obtained with the table]. And it is for this reason that we have drawn attention to it here.

Jābir b. Aflaḥ refers to Ptolemy's resolution of the argument in apparent latitude to be able to enter the tables to obtain the magnitude and the duration of the phases of the solar eclipse, once the apparent longitude of the apparent conjunction is found. Briefly, the argument in apparent latitude in the apparent conjunction —$\Delta\omega(☽'_{ca})$— is obtained from the argument in true latitude in the apparent conjunction —$\Delta\omega(☽_{ca})$— after the addition or subtraction of the argument in latitude that corresponds to the difference of the parallax in latitude between the Moon and the Sun in the apparent conjunction —$\Delta p_\beta / \sin i$—; that is,

$$\Delta\omega(☽'_{ca}) = \Delta\omega(☽_{ca}) \pm {}^{\Delta p_\beta}/_{\sin i} \qquad (69).$$

If the direction of the parallax in latitude is towards the north of the ecliptic, the equivalent argument in latitude is added when the Moon is close to the ascending node, and is subtracted if it is close to the descending node; and if the

52. «If the effect of the latitudinal parallax is northwards with respect to the ecliptic, we add the result to the previously determined true position in [argument of] latitude at the moment of apparent conjunction when the moon is near the ascending node, but subtract it when the moon is near the descending node. Contrariwise, if the effect of the latitudinal parallax is southwards with respect to the ecliptic, we subtract the distance derived from the parallax from the apparent conjunction, when the node is near the ascending node, but add it when the moon is near the descending node». See Toomer p. 311.

direction of the parallax in latitude is towards the south of the ecliptic, it should be done in the opposite way.

Jābir b. Aflaḥ does not need the argument in apparent latitude of the apparent conjunction. However, since he believes that Ptolemy has made a mistake, he addresses the topic in this criticism. However, the text of his criticism is not completely clear. Jābir b. Aflaḥ's quotation of the Arabic *Almagest* is as follows:

إن كان اختلاف المنظر في العرض ممّا يلي الشمال عن فلك البروج نظرنا فإن كان القمر نحو عقدة الرأس زدنا وإن كان نحو عقدة الذنب نقصنا وإن كان اختلاف المنظر في العرض ممّا يلي الجنوب عن فلك البروج فعلنا ضدّ ذلك.[53]

Jābir b. Aflaḥ quotes Isḥāq/Thābit's version although with some lacunae. Isḥāq/Thābit's version is as follows:

إن كان اختلاف المنظر في العرض ممّا يلي الشمال عن فلك البروج نظرنا فإن كان القمر نحو عقدة الرأس زدناه على المسير في العرض [...] منا فقوّمنا لزمان الاجتماع الذي يرى وإن كان نحو عقدة الذنب نقصناه منه [...] فإن كان اختلاف المنظر في العرض ممّا يلي الجنوب عن فلك البروج فعلنا ضدّ ذلك.[54]

The main difference between the manuscript used by Jābir b. Aflaḥ and Isḥāq/Thābit's version of the *Almagest* is that the result should be added —according to the *Almagest*— to the *al-masīr fī l-'arḍ*, that is to the argument in latitude, determined for the time of the apparent conjunction, whereas in the quotation by Jābir b. Aflaḥ, the text does not indicate the complement of the verb *zidnā*, i.e., that to which the result should be added.

The lacuna in the base manuscript used by Jābir b. Aflaḥ prevented him from understanding that Ptolemy added, or subtracted, the argument in latitude determined by the parallax in latitude to, or from, the argument in latitude of the apparent conjunction to obtain the argument in apparent latitude of the apparent conjunction.

An additional source for the study of this criticism is a gloss in the margin of MS Paris, BnF, Ar. 2482, containing Isḥāq/Thābit's version. The marginal gloss is as follows:

53. Ibid.

54. MS Paris, BnF, Ar. 2482, f. 126r.

ما في الكتاب خطاء والصحيح أن يضاف اختلاف المنظر للقمر نفسه لا لفلك البروج أعني أن يقال شمالا
عن القمر وعن فلك المائل أو جنوبا عنهما.[55]

The wording of this gloss is similar to the criticism by Jābir b. Aflaḥ. The same verb used by Jābir in this criticism appears in the marginal gloss of the Paris manuscript containing Isḥāq/Thābit's version.

We should now consider if this criticism makes sense. It should be weighed whether the method by Ptolemy is incorrect or whether the criticism is due to mistakes in the transmission.

The method by Ptolemy does not seem to contain any mistake. It could, perhaps, be argued that he uses plane trigonometry instead of spherical trigonometry, or that he considers the apparent course of the Moon to be parallel to its inclined orbit, but neither Jābir b. Aflaḥ nor the criticism in the margin of the Paris manuscript allude to these approximations. In addition, the criticisms leveled at Ptolemy on this topic do not seem to square with the extant text of the *Almagest*. Thus, the criticisms leveled by Jābir b. Aflaḥ and the Paris manuscript should have their origin in a mistake of interpretation. Jābir b. Aflaḥ's criticism is probably due to a lacuna in his manuscript of the *Almagest*. To understand the rationale behind Jābir b. Aflaḥ's criticism, we can propose three possible hypotheses of interpretation.

The first hypothesis, and the most plausible, is that Jābir b. Aflaḥ considers that Ptolemy's method operates with latitudes and not with arguments in latitude. According to this hypothesis, Jābir b. Aflaḥ interprets Ptolemy as saying that the difference between the parallax in latitude of the Moon and the Sun must be added—or subtracted depending on the situation— to the true latitude of the Moon, to obtain the apparent latitude of the Moon. That is,

$$\beta(☽'_{ca}) = \beta\,(☽_{ca}) \pm \Delta p_{\beta} \qquad (70).$$

The next step would be to obtain the argument in apparent latitude as

$$\Delta\omega(☽'_{ca}) = \beta(☽'_{ca}) \,/\, \sin i \qquad (71).$$

Thus, in this context, «adding to the ecliptic» should be understood as adding the component in latitude of the difference between the lunar parallax and the

55. Ibid.

solar parallax to the null latitude of the ecliptic. In turn, «adding to the Moon» should be understood as adding the difference of the parallax in latitude to the latitude of the true Moon. Thus, if the difference of the parallax in latitude is added to the ecliptic, and not to the true Moon, there would be an error of $\pm 2\Delta p_{\beta}$.

Let us list the different reasons supporting this hypothesis:

- The expression «delimitation of the side of the parallax» makes sense because the fact of adding —or alternatively subtracting— a parallax to the ecliptic, and not to the true Moon, is equivalent to adding —or alternatively subtracting— a parallax of opposite sign to the true Moon, which would result in a change of sign and therefore a change of side between the north and south hemisphere as defined by the plane of the ecliptic.
- Jābir b. Aflaḥ points out that he aims to obtain the apparent latitude (*ʿarḍ al-marʾī*) of the Moon. Sometimes, the term *ʿarḍ* refers to the argument in latitude. However, shortly after Jābir b. Aflaḥ seems to allude to the argument in latitude as «distance to the node» (*al-buʿd min al-ʿuqda*).
- Jābir b. Aflaḥ considers that Ptolemy added the parallax in latitude to the ecliptic and not the argument in latitude that corresponds to the parallax in latitude, since after quoting the *Almagest,* he says: «Thus, he added (*aḍāfa*) the parallax in latitude in this position to the ecliptic».
- Jābir b. Aflaḥ could not have known that the distance in question —which he interpreted as a parallax in latitude— should be added to, or subtracted from, an argument in latitude, since at this point there was a gap in his manuscript.
- The criticism that appears in the margin of the Paris manuscript is congruent with this hypothesis.
- And, in his method for the computation of the magnitude of the eclipse, Jābir b. Aflaḥ uses the apparent latitude of the apparent Moon, which is the value that he considers Ptolemy should use.

In turn, the following arguments weaken this hypothesis:

- Just before the quotation from the *Almagest* that appears in the *al-Kitāb fī l-Hayʾa*, Ptolemy makes it clear that the variable to be added was an argument in latitude. Either the manuscript in use by Jābir b. Aflaḥ had also a lacuna on this point or this fact went unnoticed by him. In turn, the Paris manuscript contains these references alongside the criticism.

- Jābir b. Aflaḥ must have expected when reading the *Almagest* that Ptolemy, after operating with latitudes according to his interpretation, would convert the apparent latitude of the Moon into an argument in latitude, but this was not the case because Ptolemy operates with arguments in latitude and not with latitudes. Jābir b. Aflaḥ must have been aware that this step was not in the text. However, he did not draw attention to it, perhaps because he did not want to extend himself when dealing with a practical point, or perhaps because the operation was obvious.
- The gloss in the Paris manuscript is written in the margin of a copy of the *Almagest* without gaps on this point. The scribe must have copied the criticism in the margin from some previous manuscript without understanding it.

Despite these negative factors, this interpretation seems to us to be the most correct, since it does not force the texts. These negative factors are generally due to the fact that the text of the criticism is not congruent with the text of the *Almagest*, but this may be due to problems in the transmission of the *textus receptus* that Jābir b. Aflaḥ was using.

The second hypothesis of interpretation is closer to the text of the *Almagest* and, consequently, does not accurately represent the texts of the criticisms. This hypothesis basically understands the reference to the ecliptic that appears in the texts as an allusion to a longitude relative to the node, and the reference to the Moon as an allusion to the argument in latitude. In this sense, the criticism could be understood as Jābir b. Aflaḥ calling attention to the fact that Ptolemy is adding —or alternatively subtracting— an argument in latitude —i.e., the correction Δp_β / $\sin i$— to a longitude relative to the node, and thus the reference to the ecliptic. The criticism would, thus, point out that the correction Δp_β / $\sin i$ should be added to the argument in latitude of the true Moon —and thus the reference to the Moon— to obtain the argument in apparent latitude of the apparent Moon.

In general, the arguments that support this hypothesis are the same as those that discredit the previous one, and the arguments that discredit it are the same as those that support the previous one.

The third hypothesis is suggested by the marginal gloss in the Paris manuscript. A possible interpretation of this gloss —*aʿanī an yuqāla*—would suggest that we should not take the ecliptic as a reference to add or subtract the argument in latitude corresponding to the parallax in latitude, but rather we should take the lunar inclined orbit as a reference.

Let us see if this possible interpretation makes sense. In most cases, since the angle of inclination of the lunar inclined orbit relative to the ecliptic is small, when the effect of the parallax would be towards the north of the ecliptic, it will also be towards the north of the inclined orbit. Likewise, in general, when the effect of a parallax would be towards the south of the ecliptic, it will also be towards the south of the inclined orbit. But how would the method of Ptolemy respond when the parallax takes place, for instance, to the north of the ecliptic and to the south of the lunar inclined orbit, or to the north of the lunar inclined orbit and to the south of the ecliptic. This possible interpretation of the criticism in the margin of the Paris manuscript would refer to these cases.

Let us consider Figure 62. In this figure, we can see a case close to the ascending node with parallax in latitude to the north of the ecliptic. Likewise, the parallax in latitude takes place to the north of the inclined orbit. To obtain the argument in apparent latitude we must subtract Δp_{β} / sin i from the argument in latitude of the true Moon. The criticism in the marginal gloss of the Paris manuscript leads us to study a situation in which, as in the previous case, the parallax in latitude takes place to the north of the ecliptic and to the south of the inclined orbit. We show this case in Figure 63.

As in the previous case, Δp_{β} / sin i must be subtracted to obtain the argument in apparent latitude from the argument in latitude of the true Moon in the apparent conjunction, i.e., $\Delta\omega(☽_{ca})$. Consequently, it is not significant to take into consideration the lunar inclined orbit as a reference to determine if Δp_{β} / sin i must be added or subtracted instead of taking the ecliptic as reference. This result is logical, since the parallax in latitude is orthogonal to the ecliptic. Let us suppose that the parallax in latitude is null. In this case, the argument in latitude of the true Moon in the apparent conjunction —$\Delta\omega(☽_{ca})$— and the argument in apparent latitude of the apparent Moon are equal —$\Delta\omega(☽'_{ca})$—. Thus, a null component in latitude of the parallax indicates a point of inflection in the sign of the correction Δp_{β} / sin i, so that it is the orthogonality relative to the ecliptic that determines the sign, that is the addition or subtraction of Δp_{β} / sin i. Hence, the plane of reference is the ecliptic and not the inclined orbit.

In short, this third interpretation of the criticism leads us to a correction that makes no sense. In addition, some of the textual references in the text by Jābir b. Aflaḥ do not agree with this interpretation, and this makes us to reject it.

Considering these three interpretations, we opt for the first one, as it is the one that best fits the text of the criticism. However, this interpretation assumes that the text of the *Almagest* used by Jābir b. Aflaḥ had lacunae. An additional element that should be highlighted is the close relationship between the criticisms in the margin of the Paris manuscript and those by Jābir b. Aflaḥ. The text of the Paris manuscript does not present any lacunae, as does that of Jābir b. Aflaḥ. Furthermore, the marginal gloss is written by the same hand as in the main text of the manuscript, so that the scribe probably copied it from the base manuscript.

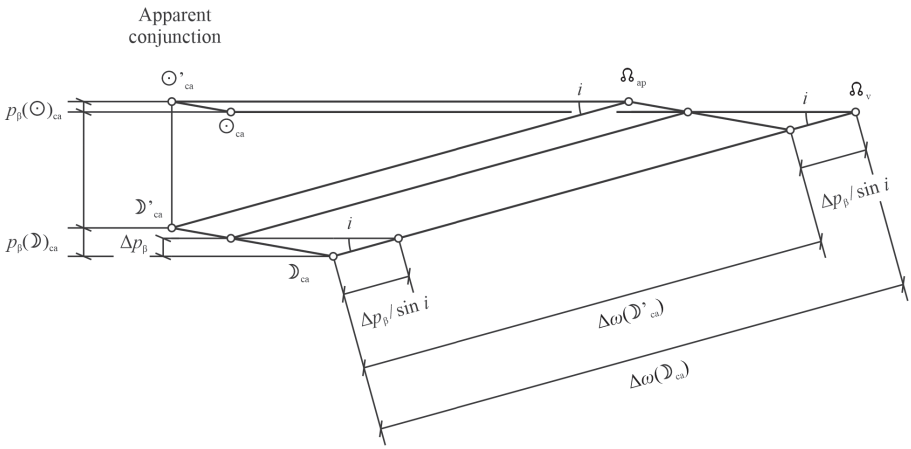


Figure 62. Argument in apparent latitude with parallax in latitude to the north of the ecliptic and the inclined orbit and close to the ascending node.

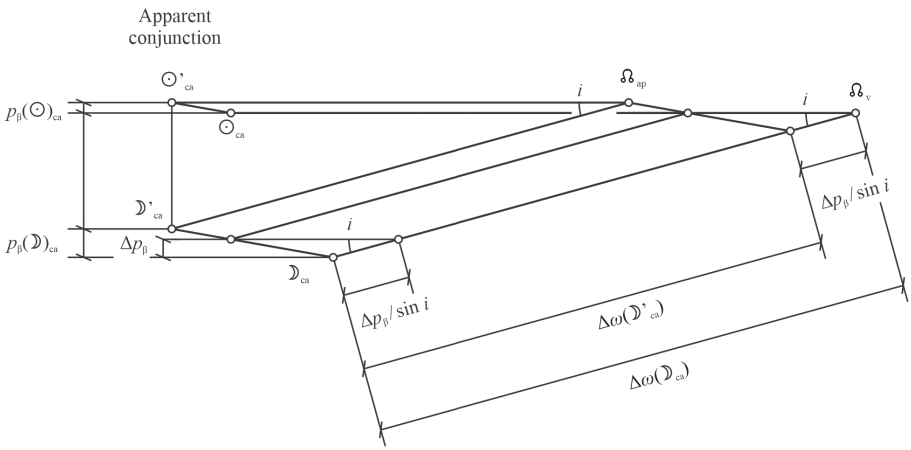


Figure 63. Argument in apparent latitude with parallax in latitude to the north of the ecliptic and to the south of the inclined orbit and close to the ascending node.

We have already pointed out the agreement between the contents and terminology of the gloss and Jābir b. Aflaḥ's criticism in the *al-Kitāb fī l-Hay'a*. All these elements lead us to think that ultimately Jābir b. Aflaḥ was the source of this criticism, although a common source cannot be ruled out.

In short, only the first criticism, discussed in the second place, levelled by Jābir b. Aflaḥ is pertinent. However, the solution he proposes is not adequate either. Two seem to be the main reasons why these three criticisms are not justified: (i) a deficient text of the *Almagest* used by Jābir, and (ii) a geometric oversimplification that allowed him to overstress the motion of the ecliptic over the horizon to find the point in which the phases or immersion and emersion are equal.

4. Conclusions

Even though two of the three criticisms of Ptolemy's method to find out the magnitude and phases of solar eclipses in the *Almagest* leveled by Jābir b. Aflaḥ do not seem justified, they provide us with valuable information about him.

In order to understand and discard these criticisms, we have devoted a long study to Ptolemy's method, since Neugebauer did not address this topic at length in his *History of Ancient Mathematical Astronomy*.

Jābir b. Aflaḥ works with a manuscript of Isḥāq/Thābit's version of the *Almagest* very defective. For instance, the second criticism—which we have dealt with firstly—as well as the third seem to be due to lacunae in his base manuscript of the *Almagest*. Likewise, the fact that he did not take into account the additional motion of the Sun in the resolution of the longitude of the apparent conjunction seems the result of a lacuna in his manuscript of the *Almagest*. It is also possible that Jābir b. Aflaḥ would be working with a particularly poor abridgement of the *Almagest*.

Jābir b. Aflaḥ advances in his work on the *Almagest* by understanding and checking what Ptolemy says. He is in no way a mere scribe who makes an uncritical reading of the text. For instance, his improvements in the successive time increments due to the various parallactic and epiparallactic effects in the Berlin manuscript show that he understood perfectly the procedure followed by Ptolemy.

Jābir b. Aflaḥ tends to reduce the complexity of the celestial motions to geometric models rather than relying on them to understand that complexity. Thus, Jābir b. Aflaḥ can easily lose sight of the celestial motions, as when he does not take into account the solar epiparallax in the resolution of the apparent conjunction.

Jābir b. Aflaḥ is a good mathematician, but a novice in the art of astronomy, as evidenced by the fact that he missed the additional motion of the Sun in the resolution of the apparent conjunction. This shows that Jābir b. Aflaḥ has never computed a solar eclipse.

Jābir b. Aflaḥ seems to work alone, since the *al-Kitāb fī l-Hay'a* does not seem to have been corrected by any professional astronomer who could draw his attention to the fact that he did not take into account the additional motion of the Sun or that his criticisms due to textual problems could ultimately be unjustified. In short, Jābir b. Aflaḥ seems to be an excellent mathematician, but a novice though creative astronomer, capable of the best and the worst. He, nevertheless, is one of the very few medieval astronomers able to gain a thorough understanding of the *Almagest*.

5. Translation

/Ea 64v, Eb 78r, and B 66v/

On the solar eclipse

As for solar eclipses, the magnitudes (*maqādīr al-munkasif*) and durations of the phases of the eclipse (*maqādīr azmān al-kusūf*) are obtained from the arc that passes through the centers of the apparent Sun and Moon, which is the apparent conjunction (*ijtimā'*). To do so, it is necessary to establish the time of the true conjunction and its position, the time of the apparent conjunction and its position for the desired place, the true positions of the Moon in longitude, latitude and anomaly at the time of this apparent conjunction. The computation of all this needs an introduction, which I will describe [in what follows].

[See Figure 64.] Be /Ea 65r/ circle BZGE the circle of the horizon, point A the zenith of this horizon and line ZAE its meridian. Be arcs BDG and TDK two semicircles of the ecliptic. Be the ascendant in the true conjunction one of the two points G or K. Let two arcs of great circle —that is, arcs AW and AH— pass

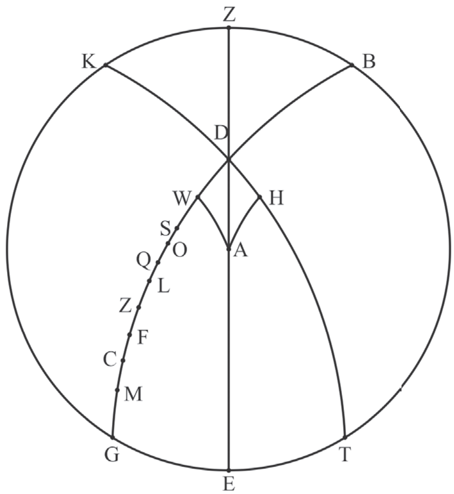


Figure 64. Ea 66r

through the poles of arcs BDG and TDK, and the zenith. Points H and W divide the semicircles in two halves. If the position of the true conjunction takes place in one of arcs GW or HK, that is if its distance to the ascendant is less than 90°, the parallax in longitude takes place in the direction of the zodiacal signs across the ecliptic; whereas if it takes place in one of arcs BW or HT, that is if its distance to the ascendant is greater than 90°, /B 67r/ the parallax in longitude takes place in the opposite direction of the zodiacal signs across the ecliptic. When [the Moon] is above the eastern horizon, the parallax in longitude is maximum and decreases gradually as the Moon rises with the universal motion until the Moon reaches the mid-heaven of the ascendant, that is one of points W or H. At that point, the parallax in longitude becomes null and its apparent position becomes exactly its true position. When the Moon moves with the universal motion and its distance to the degree of the ascendant is greater than 90°, the parallax begins to increase with the universal motion and continues in this way until it reaches the western horizon. When, in the conjunction, the parallax in longitude takes place in the direction of the zodiacal signs across the ecliptic, the apparent conjunction occurs before the true conjunction, and, thus, the parallax in longitude in the apparent conjunction is greater than in the true conjunction. If the parallax takes place in the opposite direction of the zodiac, the apparent conjunction occurs after the true conjunction, and, thus, the parallax /Eb 78v/ in longitude in the apparent conjunction is greater than in the true conjunction. For this reason, the parallax in longitude in the apparent conjunction is always greater than in the true conjunction.

[See Figure 64.] We will consider as a given condition that the position of the true conjunction takes place in one of these two situations: either, that its distance to the ascendant is less than 90°; or that its distance to the ascendant is greater than 90°, as [in the figure] point L on arc WG. Be arc LM the parallax in longitude, point M its apparent position, point R the position of the apparent Sun and arc LR its parallax [i.e., of the Sun] in longitude. We want to know the point of the ecliptic which corresponds to the true position of the Moon when its apparent position is point R. If we take the lunar parallax in longitude of point L and we subtract from it /Ea 65v/ the total solar parallax, we will determine from the difference the arc of the total lunar parallax in longitude, that is, arc RM. If we take an arc from point L, of value equal to arc RM, but in the opposite direction, which results in arc LQ, arc QR is similar to arc LM, which is the parallax of point L. Let us imagine that the Moon is on point Q: if its parallax in longitude on point Q is equal to the parallax on point L, which corresponds to arc LM, its apparent position would be on point R, which is the result we are looking for. However, the parallax on point Q is greater than the parallax on point L, which [in the figure] corresponds to arc QC, and exceeds the result we are looking for by arc RC. If we take from point Q [an arc] as arc RC, which will result in arc QO, arc RO will be like arc CQ. Let us imagine [now] that the Moon is on point O: if the parallax on point O is equal to its parallax on point Q, which corresponds to [arc] QC, its apparent position would be point R, which is the result we are looking for. However, its parallax /B 67v/ on point O is greater than on point Q. Be its parallax on point O arc OF. If we add to arc RF a section of itself (*al-juzʾ minhā*), if it is significant, in the same proportion of itself to arc RC, and we add this value to point O, as if [resulting in] arc SO, point S is approximately the point we are looking for; that is, [the position] of the true Moon when its apparent position is point R, which is, in turn, the apparent position of the Sun. Thus, point R will be the position of the apparent conjunction. *Quod erat demonstrandum*.

/Eb 79r/ Let us present the method regarding the solar eclipse to confirm what we have mentioned and [to explain it] in an easier way. Let us, also, clarify the mistakes that Ptolemy committed in his method regarding this eclipse and in the delimitation of its phases.

We say: firstly, we will obtain the total lunar parallax in the true conjunction and we will subtract the total solar parallax. From the difference, we will know the lunar parallax in longitude, which corresponds to arc RM in the figure. We divide it by the true motion of the Moon in the true conjunction and we save the resulting time in hours. If the parallax in longitude takes place in the direction of the zodiacal signs across the ecliptic, and we have already explained this, we subtract this time from the time of this true conjunction. If the parallax takes place in the opposite direction of the zodiacal signs across the ecliptic, we add [this time] to the time (*zamān*) of this true conjunction. From the result after the addition or the subtraction in hours, we obtain the lunar parallax in longitude for the second time, which is arc QC. We take the difference between the parallaxes —that is, arc CR—, so that we know the time in which the Moon traverses arc CR with its true motion. We add this time to the time (*zamān*) in which the Moon is on /Ea 66r/ point Q, or we subtract it depending on the difference in longitude between the true conjunction and the degree of the ascendant at this time. From the resulting time, we obtain the parallax in longitude for the third time, which is arc OF. We take the difference between this and the parallax of point Q, which is equal to arc OR, [and this difference] is arc RF. We add to it the section of it [i.e., arc RF], if it is significant, in the same proportion of itself to arc RC. We add it to arc OR and we obtain arc SR. Point S is approximately the point of the true position of the Moon when its apparent position is point R. Once we know this point —i.e, arc SL—, we divide it by the true motion /B 68r/ of the Moon in the true conjunction and we consider the resulting time. If the parallax in longitude takes place in the direction of the zodiacal signs across the ecliptic, we subtract this time from the time (*zamān*) of the true conjunction; and if it takes place in the opposite direction of the zodiacal signs across the ecliptic, we add it. The result /Eb 79v/ of the sum or the subtraction is the time of the apparent conjunction.

Thus, we will know the positions of the Moon in longitude, latitude and anomaly for this time. Therefore, we will know its true latitude and its total parallax. We subtract from it the solar parallax, and we know, from the difference, its parallax in latitude. Thus, we will know its apparent latitude. Then, we will know, thanks to the apparent latitude [of the Moon], according to what we have explained in the previous step, the distance (*miqdār*) between the centers [of the heavenly bodies] at the eclipse mid-time. Next, we will know, from the position of the Moon in its epicycle, the value of the lunar radius that we will add to the radius of the Sun. We take the difference between the result and the distance between the centers at the eclipse mid-time, and we obtain the eclipsed part of the diameter of the Sun. Thus, we will know from this the eclipsed part of the surface [of the Sun], according to what has been explained previously.

We will also know the distance between the centers at the eclipse mid-time and, thanks to the sum of the radii, the arc between the eclipse initial time and its middle time, and between its middle time to its end time, according to what we have explained previously. We add its twelfth part, which is the distance traversed by the Sun until the Moon reaches it. The resulting arc is the course of the Moon with its apparent motion from the initial time of the eclipse to its middle time, and from its middle time to the end time.

Since the lunar parallax in longitude is different in the three significant times (*azmina*) of the eclipse —I mean by that, in the initial, middle /Ea 66v/ and end times of the eclipse—, the apparent motion [of the Moon during the interval] between the eclipse initial time and the eclipse mid-time should not be equal to its apparent motion [during the interval] between the eclipse mid-time to its end time. Since both arcs are equal and the motions in both are different, both time intervals, i.e., the one that goes from the initial time [of the eclipse] to its middle time, and the one that goes from its middle time to its end time, must be different.

Let us clarify how these two phases are obtained with the greatest possible accuracy and give an example so that its demonstration would be clearer.

[See Figure 65.] Be arc AB the inclined orbit and point A the apparent position of the Moon at the initial time of the eclipse, point E its position at the middle time and point B its position in the end time. Be point T the center of the Sun at the initial time of the eclipse, point D at the middle time and point K at the end time. Arcs AT and BK are the sum of the radii of the Sun and the Moon. Since they are approximately equal, arcs AE and EB are equal. Be arc AZ the lunar parallax in longitude at the initial time of the eclipse and the arc EH at the middle time; both being different. The true positions of the Moon are points Z and H, and the apparent positions /Eb 80r/ points A and E. During the time in which the Moon apparently traverses arc AE, its true course is arc HZ. The difference between arcs AE and ZH is the difference between arcs AZ and HE, which are the parallaxes in longitude [at the initial and middle times]. If /B 68v/ the parallax takes place in the direction of the zodiacal signs across the ecliptic, the parallax at the initial time of the eclipse is greater than [the parallax] at the middle time, so that arc ZH is greater than arc AE. Therefore, the apparent motion is slower than the true one. If the parallax takes place in the opposite direction of the zodiacal signs across the ecliptic, the parallax at the initial time of the eclipse is smaller than at the middle time. Therefore, arc ZH is also greater than arc AE. Thus, the apparent motion is always slower than the true motion. The exact same thing happens for arc EB. If we take the difference between arcs AZ and EH and add it to arc AE, we obtain arc ZH. We divide it by the true

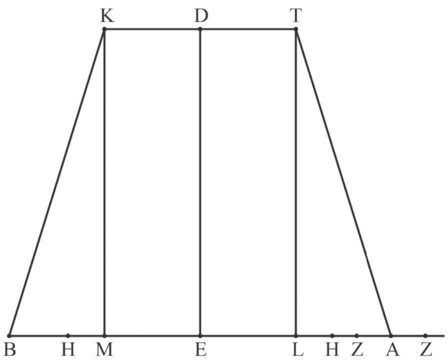


Figure 65. Ea 66v.

motion of the Moon and the resulting value is the time that the Moon takes to traverse, with its apparent motion, arc AE. The exact same thing happens in the case of arc EB, when adding the parallax on points E and B to arc EB.

Since the variation (*tafāḍul*) of parallaxes in longitude is maximum in areas close /Ea 67r/ to the mid-heaven of the ascendant and minimum in areas close to the ascendant (*al-ṭāliʿ*) or the descendant (*al-ghārib*) —and this is explained by what we have mentioned about the variation (*tafāḍul*) of the angles of the equation (*ikhtilāf*) relative to the eccentricity (*al-falak al-khārij al-markaz*)—, the phase of immersion (*zamān wuqūʿ fī l-kusūf*), if the distance of the Moon to the ascendant during the total duration of the eclipse, is less than 90°, is smaller than the phase of emersion (*zamān tarājuʿ al-imtilāʾ*). If the distance [of the Moon] to the ascendant is greater than 90°, the matter is the opposite; that is, the phase of immersion is greater than the phase of emersion. And if the Moon is at the eclipse mid-time in the mid-heaven of the ascendant, the two phases are equal.

This matter is not as Ptolemy thinks, for he said that if the middle time of the eclipse takes place at noon, both times are equal. But this is a mistake, for between the degree [in longitude] of the mid-heaven and the degree [in longitude] of the mid-heaven of the ascendant in the northern countries /Eb 80v/ there may be an arc with a value [which cannot be neglected] and which in the seventh climate reaches up to 37°. Thus, if the Moon during the eclipse is on this arc, after noon its distance from the ascendant would be less than 90°, or before noon its distance from the ascendant would be greater than 90°. Hence, the matter about the duration of the phases (*azmina*) [of solar eclipses] differs from what [Ptolemy] mentioned.

Likewise, Ptolemy makes a mistake when he states that the addition of the times that correspond to the arcs of the parallaxes in longitude always depends on the distance of the true conjunction to the meridian, be it before it or after it. This is never the case except in an eclipse whose ascendant would be the head of Aries or Libra. [Only] in this case, the degree [in longitude] of the mid-heaven is [the same of that of] the mid-heaven of the ascendant. In turn, when the ascendant is not one of these two points, these two degrees [in longitude] are different. If the position of the true conjunction is between these two degrees [in longitude], as [if the true conjunction] takes place before noon and its distance to the degree [in longitude] of the ascendant is greater than 90°, or [as if it] takes place after noon and its distance to the ascendant is less than 90°, then the time interval which corresponds to the parallax in longitude should be subtracted from the time interval which corresponds to the distance [in longitude] between the true conjunction and the meridian, although [Ptolemy] adds it. Therefore, there is a mistake in the apparent conjunction with a [non negligible] error, since the parallax in longitude in the northern countries has a significant value. Thus, the error (*khilāf*) [introduced by Ptolemy] in the apparent conjunction is a time [difference] that corresponds to the double of the parallax in longitude.

Likewise, the same thing happened to him in the delimitation of the side of the parallax in latitude to obtain from it the apparent latitude of the Moon. Ptolemy pointed out:

> If the parallax in latitude is northwards with respect to the ecliptic, we consider the matter. If [the position of] the Moon moves towards the node of the head [of the dragon], we add [this value], and if it moves towards the node of the tail [of the dragon], we subtract [it]. If the parallax in latitude is southwards with respect to the ecliptic, we will act in the opposite way.[56]

56. «If the effect of the latitudinal parallax is northwards with respect to the ecliptic, we add the result to the previously determined true position in [argument of] latitude at the moment of apparent conjunction when the moon is near the ascending node, but subtract it when the moon is near the descending node. Contrariwise, if the effect of the latitudinal parallax is southwards with respect to the ecliptic, we subtract the distance derived from the parallax from the apparent conjunction, when the node is near the ascending node, but add it when the moon is near the descending node». See Toomer p. 311.

Thus, he added (*aḍāfa*) the parallax in latitude in this position to the ecliptic. However, it must be added (*yajib an yuḍīfahu*) to the Moon itself, not to the ecliptic. Therefore, he introduces an error (*khilāf*) in the distance to the node, and he enters the table with [an argument] smaller or greater than the one that should be entered with in reality. Hence, for its apparent latitude —i.e., the [value] opposite the [argument] with which one enters the table— there will necessarily be a large error (*khilāf kathīr*). The same goes for the degrees (*ajzā'*) of the phase of immersion and the phase of emersion [obtained with the table]. And it is for this reason that we have drawn attention to it here.

6. Edition

This working edition is based on the following manuscripts:

Ea = MS Escorial, Real Biblioteca del Monasterio de San Lorenzo, ár. 910;
Eb = MS Escorial, Real Biblioteca del Monasterio de San Lorenzo, ár. 913; and
B = MS Berlin, Staatsbibliothek Preußischer Kulturbesitz, Landberg 132.

\Ea 64v, Eb 78r, B 66v\

في كسوف[57] الشمس

وأمّا الكسوفات الشمسيّة فإنّ تحصيل مقادير المنكسف ومقادير أزمان الكسوف فيها[58] تكون من قبل تحصيل[59] القوس المارّة بمركزي النيرين اللذين يريان[60] أعني الاجتماع المرئي

57. كسوف] كسوفات B Eb.
58. فيها] منها B Eb.
59. تحصيل] *om.* Ea.
60. اللذين يريان] *i.m.* Eb.

وذلك يكون بأن تقوّم[61] زمان الاجتماع الحقيقي وموضعه وزمان[62] الاجتماع المرئي وموضعه في البلد المطلوب ذلك فيه ومواضع القمر الحقيقيّة[63] في الطول والعرض والاختلاف لزمان ذلك الاجتماع المرئي وتقويم[64] ذلك يحتاج إلى توطئة ما أنا واصفه.

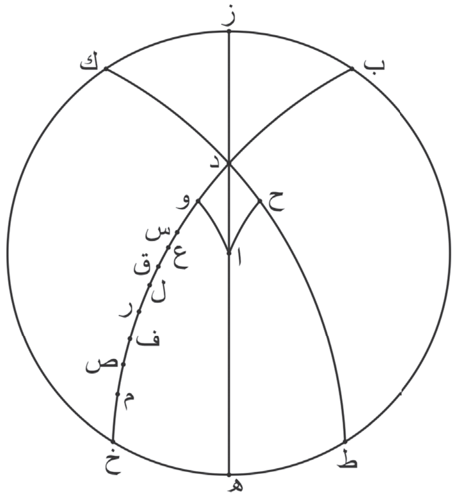


Figure 66. Ea 66r.

فلتكن \Ea 65r\ دائرة باء زاي جيم هاء دائرة أفق وسمت الرأس لذلك الأفق نقطة ألف وخطّ نصف النهار له خطّ زاي ألف هاء ولتكن كلّ واحدة من قوسي باء دال جيم وطاء دال[65] كاف نصف دائرة فلك البروج وليكن الطالع في وقت الاجتماع الحقيقي إحدى[66] نقطتي جيم وكاف ولتمرّ على قطبي كلّ واحدة من قوسي باء دال جيم وطاء دال كاف وعلى سمت الرأس قوسان[67] من دائرتين عظيمتين وهما قوسا ألف واو <و>ألف حاء فتكون كلّ واحدة من نقطتي حاء[68] واو تقسم نصف دائرتها بنصفين فإن كان موضع الاجتماع الحقيقي على إحدى قوسي جيم واو <و>حاء كاف أعني أن يكون بعده من الطالع أقلّ من تسعين جزءا

61. تقوّم] *dub et suppl. i.m.* Eb.
62. وزمان] في زمان Ea زمان Eb.
63. الحقيقيّة] الحقيقي B.
64. تقويم] تحقيقه *sup. lin.* Eb.
65. وطاء دال] د ط Eb.
66. إحدى] الحقيقي أحد B الحقيقي أحد Eb.
67. قوسان] قوس B قوسين Eb.
68. حاء] ج Eb.

فإنّ اختلاف المنظر في الطول يقع إلى[69] توالي البروج وإن كان على إحدى[70] قوسي باء واو <و>حاء طاء أعني إن كان بعده من الطالع أكثر من تسعين جزءا[71] فإنّ[72] \B 67r\ اختلاف المنظر في الطول يقع إلى خلاف توالي البروج واختلاف المنظر في الطول يكون إذا كان على الأفق الشرقي أعظم ما يكون ولا يزال يصغر بصعود القمر بالحركة الكلّيّة إلى أن يصل[73] القمر إلى وسط سماء الطالع أعني إلى إحدى[74] نقطتي واو حاء فحينئذ ينعدم اختلاف المنظر في الطول ويصير موضعه المرئي هو موضعه الحقيقي بعينه فإذا[75] تحرّك القمر بالحركة الكلّيّة وصار بعده من الجزء الطالع أكثر من تسعين جزءا ابتدأ اختلاف المنظر يعظم بالحركة[76] الكلّيّة ولا يزال كذلك إلى أن يصير إلى الأفق الغربي فمتى[77] وقع اختلاف المنظر في الطول في زمان الاجتماع[78] إلى توالي البروج كان زمان[79] اجتماع[80] المرئي متقدّما لزمان الاجتماع الحقيقي قبله ويكون اختلاف المنظر في الطول في وقت الاجتماع المرئي أعظم منه[81] في وقت الاجتماع الحقيقي وإن وقع اختلاف المنظر إلى خلاف توالي البروج كان[82] زمان الاجتماع المرئي متأخّرا عن زمان الاجتماع الحقيقي بَعده وكان اختلاف \Eb 78v\ المنظر في الطول في زمان الاجتماع المرئي أعظم منه في زمان الاجتماع الحقيقي فيكون لذلك اختلاف المنظر في الطول في زمان الاجتماع المرئي أعظم أبدا منه في زمان الاجتماع الحقيقي.[83]

فلنفرض موضع الاجتماع الحقيقي على أحد الموضعين[84] أعني أن يكون بُعده من الطالع أقلّ من تسعين أو بُعده من الطالع أكثر من تسعين فكأنّه[85] على قوس واو جيم وكأنه نقطة

69. إلى] *i.m.* B.
70. إحدى] أحد Eb.
71. جزءا] *om.* Ea B.
72. فإنّ] *bis scr.* B.
73. يصل] يصير Ea.
74. إحدى] أحد Eb.
75. فإذا] وإذا B.
76. بالحركة] حركته B بحركته Eb.
77. فمتى] فإن B.
78. في زمان الاجتماع] *om.* Ea.
79. فمتى وقع اختلاف المنظر في الطول في زمان الاجتماع إلى توالي البروج كان زمان] وزمان Eb.
80. كان زمان اجتماع] فزمان الاجتماع B.
81. في وقت الاجتماع المرئي أعظم منه] *i.m.* Eb.
82. كان] وكان Eb.
83. المنظر في الطول في زمان الاجتماع المرئي أعظم أبدا منه في زمان الاجتماع الحقيقي] بيان القمر في الاجتماع الحقيقي على أحد الموضعين B.
84. فلنفرض موضع الاجتماع الحقيقي على أحد الموضعين] *om.* B.
85. فكأنّه] فلنفرض موضع القمر في الاجتماع الحقيقي كأنّه B.

لام وليكن اختلاف منظره في الطول قوس لام ميم وموضعه المرئي نقطة ميم وموضع الشمس المرئي نقطة راء[86] واختلاف منظرها في الطول قوس لام راء[87] ونريد أن نعلم النقطة من فلك البروج التي إذا كان القمر فيها بالحقيقة كان بالرؤية على نقطة راء[88] فإن أخذنا اختلاف منظر القمر في الطول[89] لنقطة لام وأسقطنا منه \Ea 65v\ اختلاف منظر الشمس الكلّي وقوّسنا من الباقي اختلاف منظر القمر الكلّي[90] في الطول كان ذلك قوس راء[91] ميم فإن فصلنا من لدن نقطة لام مثل قوس راء[92] ميم إلى ضدّ جهتها كأنّها قوس لام قاف تكون قوس قاف راء[93] مثل قوس لام ميم التي هي اختلاف منظر نقطة لام فإن توهّمنا القمر على نقطة قاف فلو كان اختلاف منظره في الطول في نقطة قاف[94] مثل اختلاف منظره في نقطة لام الذي هو قوس لام ميم لكان موضعه المرئي على نقطة راء[95] ولكان ما أردنا لاكن اختلاف منظره في نقطة قاف أعظم منه في نقطة لام فكأنّه قوس قاف صاد فيكون قد[96] زاد على بغيتنا[97] قوس راء[98] صاد فإن فصلنا من لدن نقطة قاف مثل قوس راء[99] صاد[100] وهي قوس قاف عين فستكون قوس راء[101] عين مثل قوس صاد قاف[102] فإن توهّمنا القمر على نقطة عين فلو كان اختلاف منظره في نقطة عين مثل اختلاف منظره في نقطة قاف الذي هو قاف صاد لكان موضعه المرئي نقطة راء[103] وكان ذلك مطلوبنا لاكن اختلاف منظره \B 67v\ في نقطة عين أعظم منه في نقطة قاف فليكن اختلاف منظره في نقطة عين هو قوس عين فاء فإن أضفنا إلى

86. راء] ز Eb.
87. راء] ز Eb.
88. راء] ز Eb.
89. في الطول] الكلّي وقوسا من الباقي اختلاف منظر القمر الكلّي Ea.
90. الكلّي] *om.* Ea B لنقطة لام وأسقطنا منه اختلاف منظر الشمس الكلّي وقوّسنا من الباقي اختلاف منظر القمر الكلّي] *i.m.* Eb.
91. راء] ز Eb.
92. راء] ز Eb.
93. راء] ز Eb.
94. في نقطة قاف] *i.m.* Eb.
95. راء] ز Eb.
96. في نقطة قاف أعظم منه في نقطة لام فكأنّه قوس قاف صاد فيكون قد] *i.m.* Eb.
97. بغيتنا] بغيتها Ea B.
98. راء] ز Eb.
99. راء] ز Eb.
100. فإن فصلنا من لدن نقطة قاف مثل قوس راء صاد] *i.m.* Eb.
101. راء] ز Eb.
102. قاف] *om.* Eb.
103. راء] ز Eb.

قوس ر ف[104] الجزء منها إن كان محسوسا مثل جزءها من قوس ر ص[105] وحملنا ذلك على نقطة عين كأنّه قوس سين عين كانت على التقريب نقطة سين هي النقطة المطلوبة وهي التي إذا كان القمر عليها بالحقيقة كان بالرؤية على نقطة راء[106] التي هي موضع الشمس المرئي فتكون نقطة راء[107] هي موضع الاجتماع المرئي وذلك ما أردنا أن نبيّن.[108]

\Eb 79r\ ولنذكر العمل في هذا الكسوف الشمسي ليتقرّر به ما ذكرناه ويسهل فيه[109] ويستبين به أيضا ما أخطأ فيه بطلميوس في عمله[110] الذي ذكره لهذا الكسوف وفي تحديد أزمنته.

فنقول إنّا نخرج أوّلا اختلاف منظر القمر الكلّي في وقت الاجتماع الحقيقي ونسقط منه اختلاف منظر الشمس الكلّي فما بقي علمنا منه اختلاف منظر القمر في الطول وهو قوس راء[111] ميم من الشكل فنقسمه على حركة القمر الحقيقيّة في وقت الاجتماع الحقيقي فما خرج من أزمان الساعات نظرنا فإن كان اختلاف المنظر في الطول وقع إلى توالي البروج وقد تقدّم لنا تبيين[112] ذلك نقصنا تلك الأزمان من زمان ذلك الاجتماع الحقيقي وإن وقع اختلاف المنظر إلى خلاف توالي البروج زدناها على زمان ذلك الاجتماع الحقيقي فما كان بعد الزيادة أو النقصان[113] من الساعات استخرجنا به[114] اختلاف منظر القمر في الطول ثانية وهو قوس قاف صاد فنأخذ فصل ما بين اختلافي[115] المنظرين وهو قوس صاد راء[116] ونعلم في كم من الزمان يقطع القمر بحركته الحقيقيّة قوس صاد راء[117] فزدنا تلك الأزمان على الزمان الذي هو القمر فيه على \Ea 66r\ نقطة قاف أو نقصناها منه بحسب ما يعطيه بُعد موضع الاجتماع الحقيقي من الجزء[118] الطالع في ذلك الوقت فما كان من الأزمان استخرجنا بها[119]

104. ر ف] فاء صاد Ea Eb.
105. ر ص] فاء ميم Ea Eb.
106. راء] ز *i.m.* Eb انظر *gl.* Eb.
107. راء] ز Eb.
108. وذلك ما أردنا أن نبيّن] *om.* Ea Eb.
109. فيه] فهمه Eb.
110. عمله] علمه Ea.
111. راء] ز Eb.
112. تبيين] تمييز Ea Eb.
113. أو النقصان] والنقصان B Eb.
114. به] بها Eb.
115. اختلافي] اختلاف B.
116. صاد راء] ز ص B Eb.
117. صاد راء] ز ص Eb.
118. الاجتماع الحقيقي من الجزء] *dub* Ea.
119. بها] فيها B.

اختلاف المنظر في الطول ثالثة فكان[120] ذلك قوس عين فاء ونأخذ الفضل بينهما وبين اختلاف منظر نقطة قاف الذي هو مثل قوس[121] عين راء[122] يكون ذلك قوس راء[123] فاء فنضيف إليها جزءها إن كان محسوسا مثل جزءها من قوس[124] راء[125] صاد ونحمل[126] ذلك على قوس عين راء[127] تكون قوس سين راء[128] فتكون نقطة سين هي على التقريب النقطة التي إذا كان القمر عليها بالحقيقة كان بالرؤية على نقطة راء[129] فإذا علمنا هذه النقطة أعني قوس سين لام قسمناها على حركة \B. 68r\ القمر الحقيقيّة في وقت ذلك الاجتماع الحقيقي فما خرج من الأزمان نظرنا فإن كان اختلاف المنظر في الطول إلى توالي البروج نقصنا تلك الأزمان من زمان الاجتماع الحقيقي وإن كان وقع إلى خلاف[130] توالي البروج زدناها عليه فما كان \Eb 79v\ بعد الزيادة أو النقصان[131] فهو زمان الاجتماع المرئي.

فنعلم[132] مواضع القمر في الطول والعرض والاختلاف لذلك الزمان فنعلم[133] من ذلك عرضه الحقيقي واختلاف منظره الكلّي ونسقط منه اختلاف منظر الشمس ونعلم من الباقي اختلاف منظره[134] في العرض فنعلم من ذلك عرضه المرئي ثمّ نعلم[135] على ما تقدّم[136] من[137] عرضه المرئي مقدار[138] ما بين المركزين في وسط الكسوف ثمّ نعلم من موضع القمر في فلك تدويره مقدار نصف قطر القمر ونضيفه إلى نصف قطر الشمس فما كان أخذنا

120. في الطول ثالثة فكان] *dub* Ea.
121. قاف الذي هو مثل قوس] *dub* Ea.
122. راء] ز Eb.
123. راء] ز Eb.
124. مثل جزءها من قوس] *dub* Ea.
125. راء] ز Eb.
126. ونحمل] *dub* Ea.
127. راء] ز Eb.
128. راء] ز Eb.
129. راء] زاي Ea ز Eb.
130. خلاف] اختلاف Eb.
131. أو النقصان] والنقصان B.
132. فنعلم] يُعلم Eb.
133. فنعلم] يُعلم Eb.
134. الكلّي ونسقط منه اختلاف منظر الشمس ونعلم من الباقي اختلاف منظره] *om.* B Eb.
135. نعلم] يُعلم Eb.
136. ثمّ نعلم على ما تقدّم] فنعلم B فيُعلم Eb.
137. عرضه الحقيقي واختلاف منظره الكلّي ونسقط منه اختلاف منظر الشمس ونعلم من الباقي اختلاف منظره في العرض فنعلم من ذلك عرضه المرئي ثمّ نعلم على ما تقدّم من] *i.m.* Eb.
138. مقدار] بمقدار Eb.

الفضل بينه وبين ما[139] بين المركزين في وسط زمان الكسوف فما كان فهو المنكسف من قطر الشمس فنعلم[140] من ذلك المنكسف[141] من صفحتها على ما تقدّم.

وكذلك نعلم أيضا ما[142] بين المركزين في وسط زمان الكسوف ومن مجموع نصفي القطرين القوس التي من أوّل الكسوف إلى وسطه ومن وسطه إلى آخره على ما تقدّم فنزيد عليها جزءها من اثني عشر وهو ما تتحرّكه[143] الشمس حتّى يلحقها القمر فما كان فهي القوس التي يقطعها القمر بحركته[144] المرئية من أوّل الكسوف إلى وسطه ومن وسطه إلى آخره.

ولما كان اختلاف منظر القمر في الطول يختلف في أزمنة الكسوف الثلاثة أعني بذلك[145] أوّله ووسطه \Ea 66v\ وآخره وجب لذلك أن تكون حركته[146] المرئية من أوّل الكسوف إلى وسطه غير مساوية لحركته المرئية من وسطه إلى آخره ولما كانت هاتان القوسان متساويتين والحركتان[147] فيهما مختلفتان[148] وجب أن يكون الزمان أعني الذي من أوّله إلى وسطه والذي من وسطه إلى آخره مختلفين.

فلنبيّن كيف نستخرج كلّ واحد من هذين الزمانين على غاية ما تقدّر عليه من التحقيق ولنمثل لذلك مثالا كي يكون بالبرهان[149] عليه أبين وأوضح.

فليكن[150] الفلك المائل قوس ألف باء ونقطة ألف هي موضع القمر المرئي في أوّل الكسوف ونقطة هاء هي موضعه في وسطه ونقطة باء هي موضعه في آخره ومركز الشمس في أوّل الكسوف على نقطة طاء وفي وسطه على نقطة دال[151] وفي آخره على كاف وكلّ واحدة من قوسي ألف طاء وباء كاف مجموع نصفي قطري النيّرين ومن أجل أنّهما متساويان على التقريب تكون قوسا ألف هاء وهاء باء متساويتين وليكن اختلاف منظر القمر في الطول

139. الذي وجدناه [ما B Eb.
140. فيُعلم [فنعلم Eb.
141. من قطر الشمس فنعلم من ذلك المنكسف] *om.* Ea.
142. وكذلك نعلم أيضا من الذي B ولذلك أيضا نعلم من الذي [وكذلك نعلم أيضا ما Eb.
143. تحرّكه [تتحرّكه B.
144. بحركتها [بحركته Ea.
145. بذلك] *om.* Ea Eb.
146. حركة القمر [حركته Eb.
147. والحركة [والحركتان B Eb.
148. مختلفة [مختلفتان B Eb.
149. البرهان [بالبرهان Eb.
150. قطعة من [فليكن *i.m.* Eb.
151. ولتكن الشمس في وسط الكسوف [ومركز الشمس في أوّل الكسوف على نقطة طاء وفي وسطه على نقطة دال على نقطة دال وفي أوّله على طاء Ea Eb.

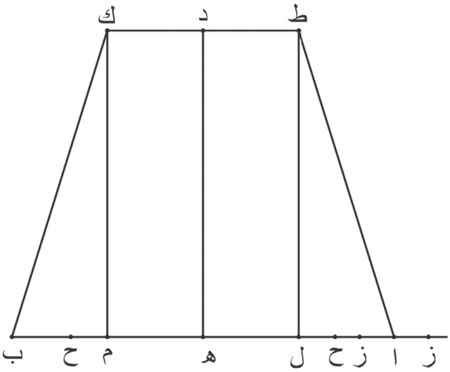


Figure 67. Ea 66v.

في أوّل الكسوف قوس ألف زاي وفي وسطه قوس هاء حاء وهما مختلفتان فيكون القمر بالحقيقة على نقطتي زاي وحاء وبالرؤية \Eb 80r\ على نقطتي ألف وهاء ففي الزمان الذي يقطع القمر قوس ألف هاء بالرؤية فيه يقطع بالحقيقة قوس حاء زاي والفضل بين قوسي ألف هاء وزاي حاء هو الفضل بين قوسي ألف زاي وحاء هاء[152] اللذان[153] هما اختلافا[154] المنظر في الطول فإن وقع \B 68v\ اختلاف المنظر في الطول إلى توالي البروج يلزم أن يكون اختلاف المنظر في أوّل الكسوف أعظم منه في وسطه فتكون لذلك قوس زاي حاء أعظم من قوس ألف هاء فتكون لذلك الحركة المرئية أبطأ من الحقيقية وإن[155] وقع اختلاف المنظر[156] إلى خلاف توالي البروج كان اختلاف المنظر في أوّل الكسوف أصغر منه في وسطه فتكون أيضا قوس زاي حاء أعظم من قوس ألف هاء فتكون الحركة المرئية أبدا أبطأ من الحركة الحقيقيّة وهذا بعينه يلزم في قوس هاء باء فإن أخذنا الفضل بين قوس ألف زاي وهاء حاء وزدناه على قوس ألف هاء كان ذلك قوس زاي حاء فنقسمها على حركة القمر الحقيقيّة فما خرج فهو الزمان الذي يقطع فيه القمر بالحركة[157] المرئية قوس ألف هاء ومثل ذلك بعينه يعرض في قوس هاء باء بأن نزيد الفضل بين اختلافي المنظرين[158] في نقطتي هاء وباء على قوس هاء باء.

152. وحاء هاء] ه ح B Eb.
153. اللذان] اللتان Eb.
154. اختلافا] اختلاف B.
155. الحقيقية وإن] الحقيقية فإن B الحقيقة وإن Eb.
156. اختلاف المنظر] *om*. Eb.
157. بالحركة] بحركته Eb.
158. المنظرين] المنظر Eb.

ولما كان تفاضل اختلاف المنظر في الطول أعظم ما يكون عند \Ea 67r\ وسط سماء الطالع وأصغر ما يكون عند الجزء الطالع أو الغارب وهذا يستبين ممّا ذكرناه في تفاضل زوايا الاختلاف الذي يعرض في الفلك الخارج المركز فيجب إن كان في[159] زمان الكسوف بأسره بعد القمر فيه[160] من الطالع أقلّ من تسعين أن يكون زمان الوقوع في الكسوف أصغر من زمان تراجع الامتلاء وإن كان بعده فيه من الطالع أكثر من تسعين كان الأمر بضدّ ذلك أعني أن يكون زمان الوقوع في الكسوف أعظم من زمان تراجع الامتلاء وإذا كان القمر في وسط الكسوف في[161] وسط سماء الطالع فحينئذ يكون الزمانان متساويين.

وليس يكون الأمر على ما ذكره[162] بطلميوس وذلك أنّه قال إن كان وسط زمان الكسوف في وقت نصف النهار يكون الزمانان متساويين وهذا خطاء لأنّه قد يكون بين الجزء المتوسّط للسماء وبين الجزء الذي هو وسط[163] سماء الطالع في البلاد الشمالية \Eb 80v\ قوس لها قدر وتبلغ في الإقليم السابع نحو سبعة وثلاثين جزءا فإن كان القمر في الكسوف في هذه القوس فيكون بَعد نصف النهار وبُعده من الطالع أقلّ من تسعين أو قبل نصف النهار[164] وبُعده[165] من الطالع أكثر من تسعين فيكون الأمر في الأزمنة[166] حينئذ[167] على خلاف ما ذكر

وكذلك ما ذكره[168] من زيادة الأزمان التي تجب لقسي اختلافات المنظر في الطول أبدا على بعد زمان الاجتماع الحقيقي من دائرة نصف النهار قبله أو بَعده خطاء ليس يصحبه ذلك دائما إلاّ في كسوف يكون الطالع فيه رأس الحمل أو الميزان[169] فيكون حينئذ الجزء المتوسّط للسماء هو وسط سماء \B 69r\ الطالع وأمّا متى كان الطالع غير هاتين النقطتين فيكون هذان الجزءان متغيّرين[170] فإن كان موضع الاجتماع الحقيقي فيما بين[171] هاذين[172]

159. في] *om.* Ea.

160. فيه] *om.* B Eb.

161. وسط الكسوف في] *om.* Eb.

162. ذكره] ذكر Ea Eb.

163. وسط] *i.m.* Eb.

164. وبُعده من الطالع أقلّ من تسعين أو قبل نصف النهار وبُعده] ويكون بعده من الطالع أقلّ من تسعين أو قبل نصف النهار ويكون *i.m.* Eb.

165. وبُعده] بعده Eb.

166. فيكون الأمر في الأزمنة] فتكون الأزمنة *gr. i.m.* Eb.

167. حينئذ] العظم والقصر *add. i.m.* Eb.

168. ذكره] ذكر Eb.

169. أو الميزان] والميزان B.

170. متغيّرين] متغايرين Eb.

171. بين] *om.* Ea.

172. هاذين] هذين Eb.

الجزءين وهو أن يكون قبل نصف النهار وبُعده من الجزء[173] الطالع أكثر من تسعين جزءا أو بَعد نصف النهار وبُعده من الطالع أقلّ من تسعين جزءا فيلزم حينئذ أن تنقص الأزمان التي تجب لاختلاف المنظر في الطول من أزمان البعد الذي للاجتماع الحقيقي من دائرة نصف النهار وهو يزيدها فيقع من ذلك من[174] الخطاء في زمان الاجتماع المرئي خلاف له قدر لأنّ اختلاف المنظر في الطول في البلاد الشماليّة يكون له حينئذ مقدار صالح[175] فيقع الخلاف في زمان الاجتماع المرئي بما يجب من الزمان لضعف اختلاف المنظر في الطول.

وكذلك عرضه[176] أيضا في تحديد[177] جهة اختلاف منظر القمر في العرض ليستخرج منه عرض القمر المرئي وذلك أنّه قال:

> إن كان اختلاف المنظر في العرض ممّا يلي الشمال عن فلك البروج نظرنا فإن كان القمر نحو عقدة الرأس زدنا وإن كان نحو عقدة الذنب نقصنا وإن[178] كان اختلاف المنظر في العرض ممّا يلي الجنوب عن فلك البروج فعلنا ضدّ ذلك.

فأضاف اختلاف المنظر في العرض في هذا الموضع إلى فلك البروج وإنّما يجب[179] أن يضيفه إلى القمر نفسه لا إلى فلك البروج ويدخل من ذلك في الأجزاء \Ea 67v\ التي هي البعد من العقدة خلاف فيدخل في الجدول بأقلّ أو أكثر ممّا يجب أن يدخل به على الحقيقة \ \Eb 81r فيلزم من ذلك أن يكون[180] في عرضه المرئي وهو الذي يوجد بإزاء ما[181] يدخل به في الجدول[182] خلاف كثير وكذلك يكون أيضا[183] في أجزاء السقوط في الكسوف وتراجع الامتلاء ولهذا ما نبهنا عليه في هذا الموضع.

173. الجزء] *i.m.* Ea.
174. من] *om.* B Eb.
175. صالح] *ras.* B.
176. عرضه] عرض له B.
177. تحديد] *i.m.* Eb.
178. وإن] فإن B.
179. وإنّما يجب] ويجب B.
180. أن يكون] *om.* Ea.
181. ما] *om.* B.
182. به في الجدول] *bis scr.* B.
183. يكون أيضا] أيضا يكون B.

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